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NUMERICAL SOLUTION TO THE HEAT TRANSFERENCE PROBLEM, IN GRINDING

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ABSTRACT
The objective of this article is obtaining a mathematical model that describes the thermal problem in Grinding. A numerical solution of the equation of bi-dimensional heat diffusion is obtained. The numerical solution is made by the finite differences method, incorporating a scheme of fixed-point type Picard to solve the non-linealities introduced by the thermal properties of the material, which are considered depending on the temperature. The obtained models are also compared, and they are validated with information reported by another authors.
Keywords: Grinding, Heat Transference, residual tensions.

1. INTRODUCTION
One of the most important problems that it is necessary to control in high precision mechanizing is the optimization of the distribution of superficial residual tensions that remain in the workpiece material and the heat distribution that is generated on it because of the process itself, as well as the interaction between both phenomenons. These phenomenons can change the microstructure and, consequently, the thermal and mechanical properties of the superficial layer of a material. Therefore, to assure a high degree of superficial finish in a piece, factors that influence on the development of residual tensions should be controlled. First, to find a solution to the problem of heat transfer, the method of finite differences will be use. This one has shown important results in the solution of engineering problems that, due to its geometry and / or boundary conditions have a great difficulty to give quickly an exact result for analytical solutions. This is a way to solve the thermal problem in a precise manner.

2. HEAT TRANSFER PROBLEM IN GRINDING
A surface grinding process is assumed to be two-dimensional. The head source profile is rectangular, moving along the positive direction of the X-axes on the work-piece surface, as show in Figure 1. For a stationary frame of reference (X-Y), the general governing equation is:

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)=\frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{1}$$

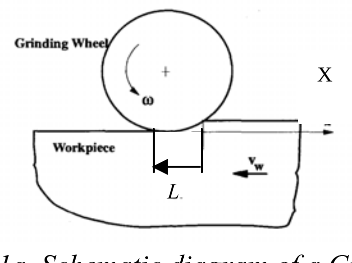


Figure 1a. Schematic diagram of a Grinding process.

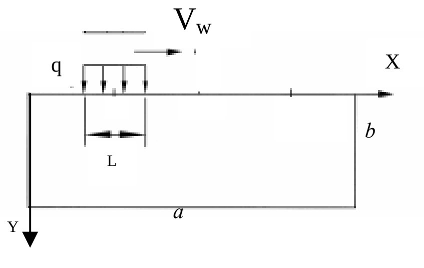


Figure 1b. A model of used thermal profile

If the frame of reference moves with a constant speed V_w, wich is equal to the table speed of the grinding machine, the equation (1) becomes.

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)=-\frac{V_w}{\alpha}\frac{\partial T}{\partial x} \tag{2}$$

where x is the horizontal coordinate of the moving reference frame.
The boundary conditions in *x=0, x=a,* and *y=0* (excepting in the zone where exist heat flow) are of heat flow by natural convection. In *y=b* there is a condition of constant ambience temperature.

3. FINITE DIFFERENCE

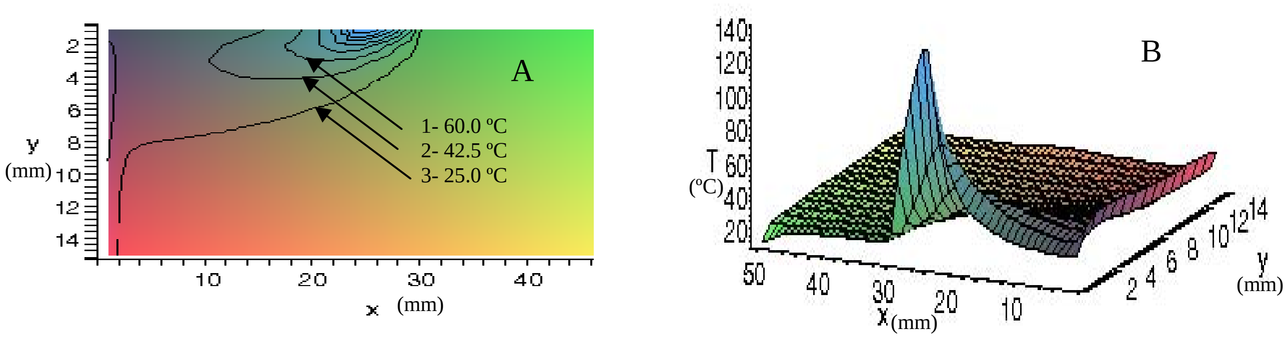
The numerical schema developed to predict the steady head flux is briefly described in this section.
The relation between thermal conductivity and temperature, specific head capability and temperature for steel is non-linear. The expression for the thermal conductivity or head capability in terms of the temperature in many cases is not even differentiable. In such a situation, an iterative strategy of the Picard type can be used. According to this approach the thermal conductivity and head capability evaluated with temperature T^{P-1}. Keeping in mind the previous consideration, the linear approximation can be constructed in the following way: given estimation for the value of the temperature T^{p-1} find T^p as a solution of the problem (3). The finite difference method is applied for approach the differential, to the eq.(2). The domain is discretized by imposing a grid defined for a set of nodes x_i and y_j . Eq. (2) can be written in discrete form as following.

$$k_{i,j}^{p-1}\left[\left(\frac{T_{i+1,j}^{p}-2T_{i,j}^{p}+T_{i-1,j}^{p}}{\Delta x^2}\right)+\left(\frac{T_{i,j+1}^{p}-2T_{i,j}^{p}+T_{i,j-1}^{p}}{\Delta y^2}\right)\right]=-\frac{V_w}{\alpha}\left[\frac{T_{i+1,j}^{p}-T_{i-1,j}^{p}}{\Delta x}\right] \tag{3}$$

We introduced the equation in the computer using MAPLE software, and we developed an application to find the solutions to the problem.

4. MODEL VERIFICATION

With the obtained application we have the temperature distribution in the geometry of the workpiece (Figure 2). In this way, we can know how the heat flow behaves in the material.



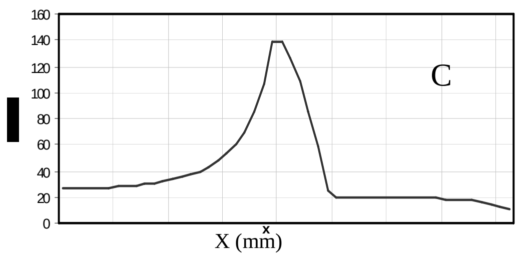
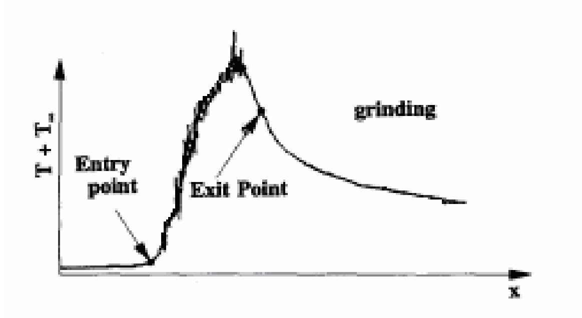
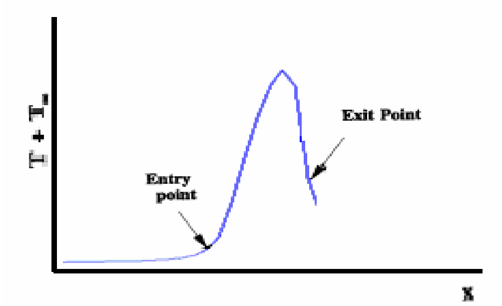


Figure 2. Temperature distribution around the geometry of the workpiece in the Grinding process.
A- Isotherms distribution
B- Temperatures distribution in the workpiece configuration.
C- Temperature distribution in the external face of the workpiece (y=0).

The obtained results are compared with those obtained by Liangchi et al. in [1]. In this case we are comparing the curve type and the entry and exit point of the tool in the workpiece during the process. The results of the comparison are shown below (Fig. 3 A and B).

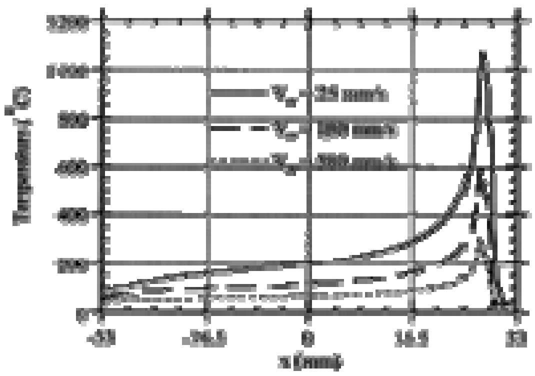


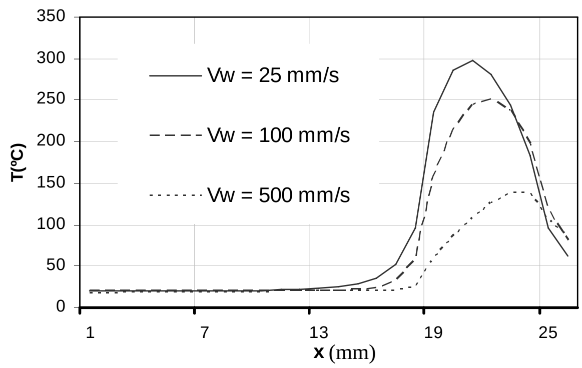


A- Results of reference paper *B- Our results*

Figure 3. Temperature distribution on the workpiece surface (y=0). $T+T_\infty$ and x are undimensionals.

How it can observe the curve configurations are very similar in booth cases.
We also made another comparison. We compare our temperature distribution for various heat source velocities with the results obtained by P.N. Moulik et al. in [2].





A- Results of reference paper *B- Our results*

Figure 4. Temperature distribution on the workpiece surface (y=0) for various heat source velocities.

In the graphics, the obtained results are very similar. When the heat velocity is higher, the generated temperatures are lowers. We assumed that the difference between curve forms and temperature values is due to the nature of the materials and the scale used in x-axis.
As we can observe, the results shown on the graphics are very similar in both cases. They represent the solution of the same problem by different methods. Using this positive conclusion as a start point, we tried to solve another problem: the temperature dependences that have the material properties. It means that the material of the workpiece has a thermal conductivity that changes with temperature. In the previous case, we assumed that thermal conductivity had a constant value. That's what was considered in the papers that we used to compare our results, but it's not too real.

5. RESULTS INTRODUCING VARIABLES PROPERTIES

We continued making tests to analyze the influence of the variation of thermal conductivity and head capability on the material behaviour in the grinding process. We obtain now the results that are shown in Figure 5.

Comparing the results shown in Figure 2 and Figure 5 (Temperature distributions in case of constant (A) and variable (B) thermal conductivity) we can observe the following aspects:

- The temperature value in the contact point between tool and workpiece is higher in the case (B).
- The curve form is very similar in case (A) and case (B).
- The heat flow velocity influence is bigger in case (A).

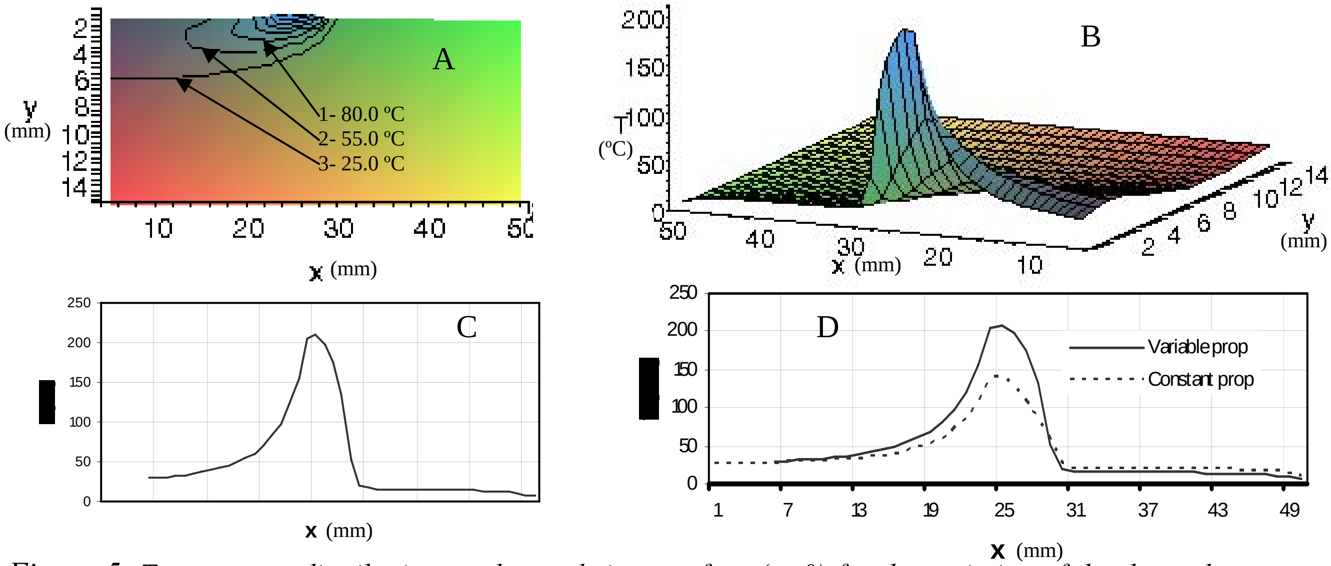


Figure 5. Temperature distribution on the workpiece surface (y=0) for the variation of the thermal conductivity due to the variation of the temperature.
A- Isotherms distribution (Corresponding to Figure 5 B).
B- Temperature distribution in the workpiece configuration.
C- Temperature distribution in the external face of the workpiece.
D- Temperature distribution in the external face of the workpiece. Comparison between Figure 2C and 5C

6. CONCLUSIONS

We obtained an appropriate model to define the problem of heat transference in the Grinding process. The model incorporates the variables properties.

Curves in cases of constant and variable thermal conductivity are very similar. They have the same morphology, but there are differences between the temperature values in the existence heat flow zone, it means that the properties variability is a very important factor.

7. NOTATION

X, Y	Co-ordinates	q_{sx}	Out heat flow in X direction
V_w	Workpiece velocity	q_{sy}	Out heat flow in Y direction
q	Heat flow	α	Thermal expansion coefficient
L	Length of the contact zone between wheel and workpiece	T $T(amb)$	Temperature Ambience temperature
q_{ex}	Entrance heat flow in x direction	$T(i)$	Temperature in i position in x direction
q_{ey}	Entrance heat flow in y direction	K	Thermal conductivity coefficient
a / b	Total length / wide of the workpiece	$T_{i,j}^{p}$	Temperature in i, j position in p iteration
Δx	X grid spatial size	Δy	Y grid spatial size

8. REFERENCES

[1] Liangchi, Z. And Mofid, M.: Applied mechanics in Grinding-IV. The Mechanism of Grinding induced phase transformation, Int. J. Mach. Tools Manufact. Vol. 35. No. 10, pp. 1397-1409, 1995.,
[2] Moulik, P.N., Yang, H.T.Y. and Chandrasekar, S.: Simulation of thermal stresses due to grinding, International Journal of Mechanical Sciences 43, p 831-851, 2001.,
[3] Mahdi and Liangchi Zhang: The finite element thermal analysis of Grinding processes By Adina, Computers & Structures Vol. 56, No. Z/3, pp. 313-320, 1995.,
[4] Ozisik, M.N., Heat transfer. A basic approach, 1985.