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THEORETICAL APPROXIMATION OF THE VIBRATION IN A REGENERATIVE CUTTING PROCESS

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ABSTRACT
In this paper, the phenomenon of chatter vibration in metal cutting is investigated. For this purpose, a theoretical one-degree of freedom model of the vibrations generated in an orthogonal cutting process is developed. The dynamic cutting forces are strongly affected by the variations of the cutting process parameters, and the model is based on the mechanical theoretical prediction of the cutting zone. The developed model considers the variation of the rake angle, depth of cut, and a nonlinear model of the cutting force. An analysis of the differential equation obtained enables to determine the different types of oscillatory patterns that develop. A numerical solution by means of the fourth-order Runge-Kutta method allows the determination of the variation of the cutting force and rake angle as a function of time.
Keywords: Surface roughness, chatter vibration.

1. INTRODUCTION
Chatter is a phenomenon that deserves attention in machining processes because it causes undesired effects. In this paper, the development of a theoretical one-degree of freedom model for predicting chatter is considered. Unlike the studies that have been previously carried out, this model is based on cutting parameters such as the rake angle, the cutting speed and the chip thickness. The paper is divided into the following sections: first, a description is given of how the model was developed; a numerical simulation using real values is then undertaken for model verification; this is followed by the presentation of the results and their discussion; and, finally, conclusions are drawn.

2. PHYSICAL-MATHEMATICAL MODEL OF THE CUTTING PROCESS
The model developed, for a turning process, is an orthogonal cutting model, in which the interface tool-piece is considered to have a single degree of freedom: movement only takes place in a horizontal direction (x-coordinate), (see Fig. 1).

$$m\ddot{x} + c\dot{x} + kx = \Delta F_x \tag{1}$$

Equation (1) governs a dynamic system with a single degree of freedom, where m is the equivalent mass of the system, c is the damping coefficient, k is the stiffness constant and ΔF_x is the oscillating force component. The determination of the ΔF_x function is fundamental in the model since it is responsible for stimulating the system that causes dynamic behaviour. This function depends solely on the dynamic characteristics of the cut, and, therefore, chip thickness and cutting speed must be functions of the cutting conditions. The undeformed chip thickness h, is an oscillating function that moves in a steady-state defined by h_0. The external force F_x that moves the system, is the sum of two components: the steady-state component F_{x0} plus an oscillating component called ΔF_x (Equation (2)).

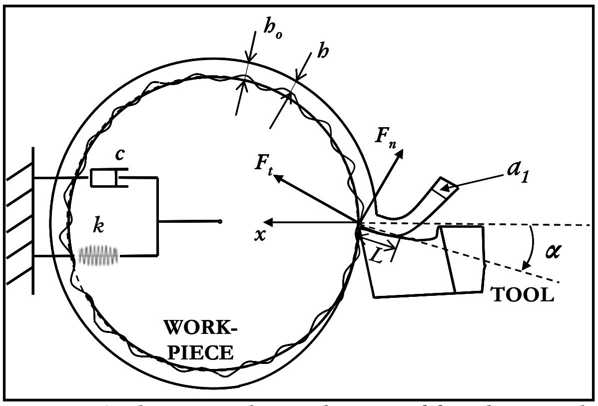

Figure 1.Theoretical one-degree of freedom model

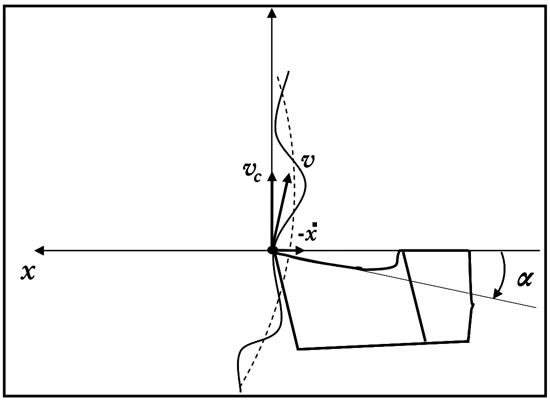

Figure 2. Kinematics of the Rake angle

If the dynamic force F_x and the F_{x0} component are known, it is possible to determine ΔF_x,, which is responsible for generating the oscillations in one-dimensional mechanical systems.

$$\Delta F_x = F_x - F_{x0} \tag{2}$$

For a stationary process, Equation (3) describes the force in x direction, where F_t is the friction force between the material and the tool, F_n is the normal force between the tool and the material, and α is the rake angle.

$$F_{x0} = F_t \cos(\alpha) - F_n \sin(\alpha) \tag{3}$$

Several authors [1-3] have carried out studies to determine stationary cutting parameters. In this paper, the results obtained by Toropov and others are used [2-3]. They serve to deduce the analytical expressions for determining the F_n y F_t , which act on the rake face of the tool. Equation (4) is obtained using these expressions, which makes it possible to evaluate the stationary force in x direction in an orthogonal cutting process. α_0 is the steady-state rake angle, e is chip width, which corresponds to the tool advance per revolution, S_f is the true fracture strength of the work material, and ε is the chip thickness coefficient.

$$F_{x0} = eS_f \varepsilon h_o \left[\left(\frac{\pi}{2} - 1 \right) \cos\left(\alpha_0\right) - \sin\left(\alpha_0\right) \right] \tag{4}$$

During an oscillating cutting process, the tool will follow a path defined by curve h, which oscillates around h_0 (Fig. 1). The existence of an oscillating movement in the cutting tool gives rise to oscillations on the rake angle around the stationary value α_0. Likewise, the angular motion that the tool undergoes may influence the shear angle. The relative movement of the tool in direction x is:

$$x = h - h_0 \tag{5}$$

Isolating h from Equation (5), we obtain displacement h_0 in addition to the displacement due to x oscillation, which is the independent variable in Equation (1). The cutting speed is a variable that is related to the changes experienced by the rake angle. As can be seen in Fig. 2, it is possible to deduce Expression (6) for the rake angle, where $\dot{x}$ is the speed of the oscillation in x direction, v_c is the cutting speed and α is the rake angle rate. The difference between the steady-state rake angle and the rake angle is the rake angle rate, which is similar to the conditions described for un-deformed chip thickness (5).

$$\alpha = \alpha_0 + \arctan\left(\frac{-\dot{x}}{v_c} \right) \tag{6}$$

Using these magnitudes, we can rewrite Equation (4), in which the component of the dynamic force is a slight oscillation that is approximately the same value as that for steady-state force F_{x0}. By replacing (5) and (6) in the force equation, (7) is obtained, similarly to how (4) is obtained. This method may be applied to predict the oscillating force, which is based on variables that are easy to define because they are the variables in the machining process. Finally, based on (7), (4) and (2), the oscillating component of the force ΔF_x is obtained.

$$F_x = eS_f \varepsilon \left(h_0 + x\right)\left[\left(\frac{\pi}{2} - 1\right)\cos\left(\alpha_0 + \arctan\left(\frac{-\dot{x}}{v_c}\right)\right) - \sin\left(\alpha_0 + \arctan\left(\frac{-\dot{x}}{v_c}\right)\right)\right] \tag{7}$$

3. NUMERICAL SIMULATION

Equation (1) is numerically solved by using ΔF_x (defined above). In order to do this, the fourth order Runge-Kutta numerical method is used. The conditions of cut and the constants of the differential equation correspond to the real values in a cutting process. In the differential equation, m = 0.1155 kg, c = 0.9 kg/s, y k = 589048 N/m. The equation is solved and the graphs are plotted using the values obtained. The graphs are presented and discussed in the following section.

4. RESULTS AND DISCUSSION

The first simulation was carried out for studying the behaviour of the model. Figure 3 shows the oscillations of the system under different conditions.

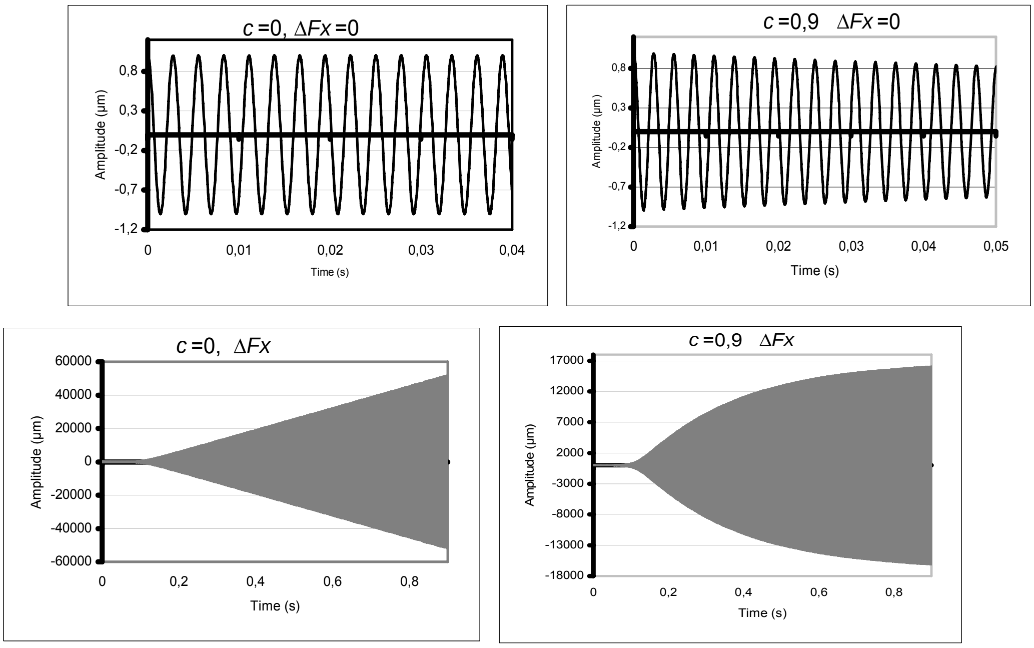

Figure 3. Model´s behaviour

The first graph shows the model's behaviour when c is zero, as well as the force involved, for which a harmonic signal was obtained. The second graph shows the behaviour when the value of c is different from zero, for which a damping signal is obtained. In the following graph, the damping coefficient is zero, but ΔF_x is introduced, for which a signal generated by the self-excited vibration force can be observed. The last graph represents the developed model. Once the graphs had been analyzed, it was possible to state that the model behaves satisfactorily, making it possible to predict the self-excited oscillating behaviour.

In order to study the effect of the different parameters e, h_o, v_c, α, on the model, simulations were carried out in which the different values were varied and their behaviour observed. When e was varied, a change in the amplitude of the oscillation was observed, which became greater when the value of e was increased. This change in the amplitude is bound to affect the surface quality. In addition, it was noticed that there was a change in the initial transition, so that when e increases, the steady-state takes place before (Fig. 4).

The value of h_0 was varied between 0.3 and 1 mm to demonstrate that its behaviour with respect to the amplitude and the steady-state is similar to the previous case (Fig 5). In the case of cutting speed (v_c)

and the rake angle (α), simulations were carried out, which resulted in graphs that do not present appreciable variations for the finishing conditions. Thus, it is possible to conclude that these parameters do not significantly influence the proposed model.

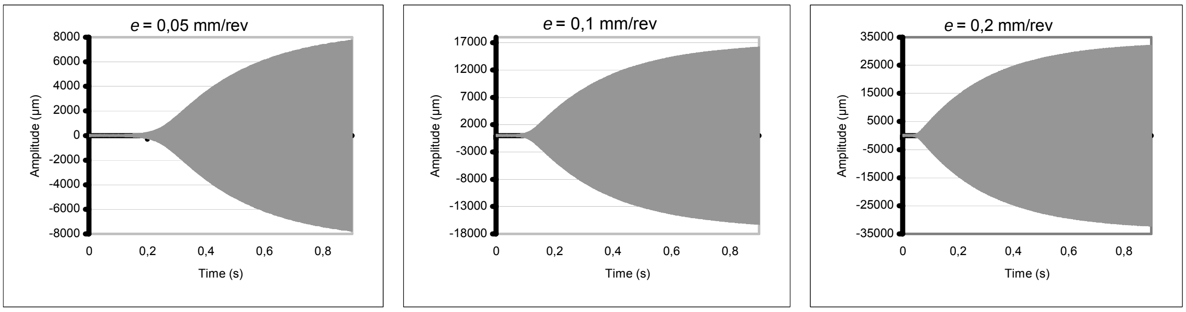

Figure 4. The response of the model to variation of the parameter e

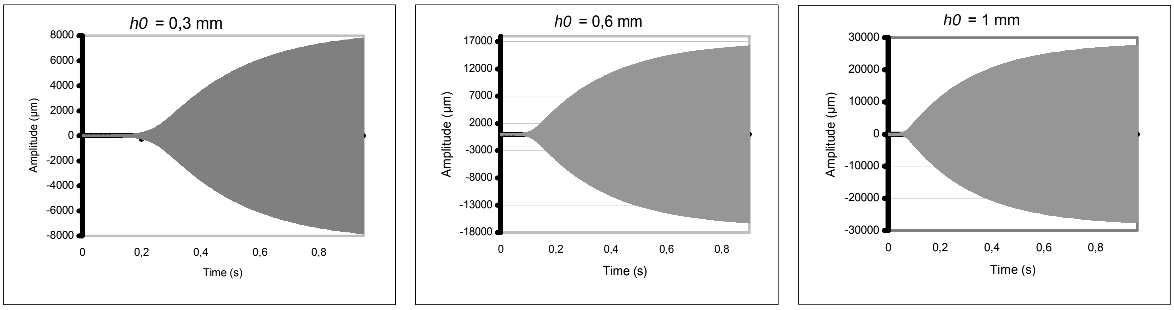

Figure 5. The response of the model to variation of the parameter h_0

Finally, in order to study the influence of the initial condition on the model, simulations using different values of $x(0)$ were carried out (Fig. 6). The initial condition in one of the simulations was 100 times greater than in the other. We noticed that although the cutting conditions were varied, the model predicted an oscillation amplitude that did not depend on the initial condition.

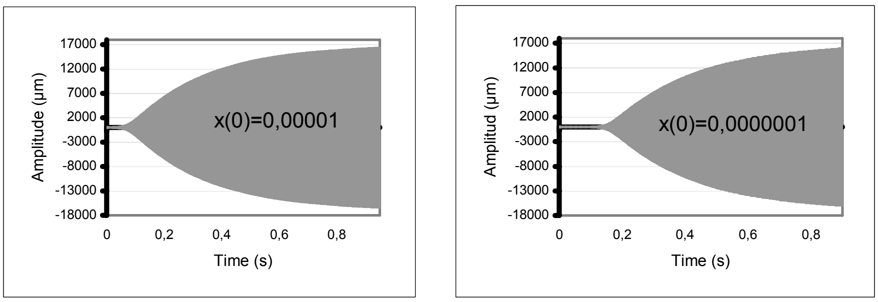

Figure 6. The response of the model to variation of the initial condition

4. CONCLUSIONS

A model for predicting chatter based on the cutting parameters in machining processes was successfully constructed. It has been demonstrated that the tendencies in the model's behaviour are correct.

5. REFERENCES

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