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STUDIES REGARDING THE KINEMATIC AND DYNAMIC ANALYSIS OF PARALLEL MINI-MANIPULATORS

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ABSTRACT
One of the most important approaches in the scientific research concerning the increase of the positioning accuracy is the use of parallel structures. Parallel kinematic structures have a series of advantages that makes them adequate for the mini-robots construction: actuator positioning on the seating, miniaturization, stiffness, positioning precision and repeatability, actuators separation from the workspace. The problems concerning the kinematics and the dynamics of parallel robots are, as a rule, more complicated than those of serial ones. The dynamic modeling of parallel robots presents an inherent complexity due to their closed-loop structure and kinematic chains [3]. The paper presents the algorithms for the kinematic and dynamic analysis of two 6 DOF parallel mini-manipulators: 3-PPRS and 3-PPSR (so called guided in three points mechanisms). The obtained simulation results have shown that the developed models have a compact structure and could be used for the real-time control of the parallel mini-manipulators.

Keywords: Parallel robot, mini-robot, kinematics, dynamics, graphical simulation.

1. INTRODUCTION
Nowadays, it can be noticed in the world that product miniaturization is the main research topic in new product development for a large variety of application fields. One of the most important interests in the world scientific research concerning the increase of accuracy in positioning is the use of parallel structures. Parallel mini- and micro-robots are the direct result of the parallel robots knowledge, being a miniaturization of a structure or a succession of one or more conventional elements, which has as purpose the obtaining of the same function as from classical parallel robots.

Modeling and simulation are the first steps – initial stages of designing and construction of parallel min and micro-robots, using specialized software in 3D modeling (e.g. SolidWorks, SolidEdge, Unigraphics, CATIA, Inventor) and in simulation (e.g. LabView, Matlab, Adams).

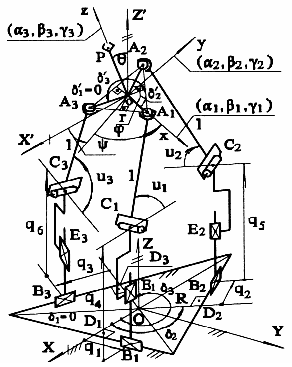

Figure 1. Kinematic scheme of the 3-PPRS mini-manipulator

Figure 2. The CAD model of the 3-PPRS mini-manipulator

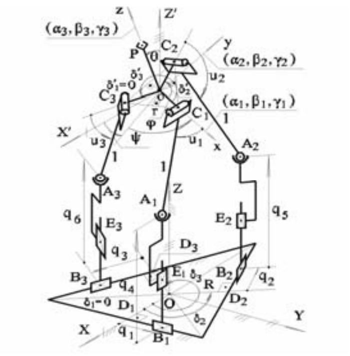

Figure 3. Kinematic scheme of the 3-PPSR mini-manipulator

Figure 4. The CAD model of the 3-PPSR mini-manipulator

The studied 3-PPRS and 3-PPSR parallel mini-manipulators belong to the spatial closed-kinematic chains guided in three points mechanisms, so-called A type parallel mechanisms, respectively B type parallel mechanisms [7]. The kinematic schemes of these parallel mechanisms are represented in the figures 1 and 3, and their CAD models in the figures 2 and 4. The six prismatic joints are actuated while the rest of them are passive. Both structures are characterized by the following parameters: R-radius of fixed base, r-radius of the working platform (WP), l-length of the guiding rods C_iA_i, $\delta_i' = \delta_i = 120^0\ (i-1)$ - the placement angles of the kinematic chains.

2. KINEMATIC MODELING OF THE PARALLEL MINI-MANIPULATORS

The kinematic models of these two types of parallel manipulators are obtained using the input-output equations [5]. The input-output equations represent the mobile circle equations on which the spherical joints centers are moving with respect to the fixed base in the 3-PPRS mechanisms case and in the 3-PPSR mechanisms, respectively with respect to the WP but expressed on the fixed base frame:

$$\left(\overline{P}_i - \overline{P}_{C_i}\right)^2 - l^2 = 0; \quad \left(\overline{P}_i - \overline{P}_{C_i}\right)\cdot \overline{N}_i = 0; \quad i = 1,2,3 \tag{1}$$

where: $\overline{P}_i$ is the position vector of the A_i guiding point with respect to the O centre of the fixed base; $\overline{P}_{C_i}$ is the position vector of the C_i centre of the rotational joint with respect to the O center of the fixed base; $\overline{N}_i$ is the unit vector of the C_i rotational joint axis.

2.1. Geometric model

For the 3-PPRS manipulator, to the equations (1) are added the equations of the fixed planes, in which the points C_i are imposed to remain:

$$X_{C_i} \cos\delta_i + Y_{C_i} \sin\delta_i - R = 0 \tag{2}$$

and the C_i points coordinates are computed with respect to the known A_i points coordinates. In this case, the actuated coordinates are obtained easily:

$$q_i = Y_i \cos\delta_i - X_i \sin\delta_i\,; \quad q_{i+3} = Z_i - \sqrt{l^2 - (R - X_i \cos\delta_i - Y_i \sin\delta_i)^2} \tag{3}$$

For the 3-PPSR manipulator, to the equations (1) are added the equations of the fixed planes, in which the points A_i are imposed to remain:

$$X_i \cos\delta_i + Y_i \sin\delta_i - R = 0 \tag{4}$$

and the A_i points coordinates are computed with respect to the known C_i points coordinates from the second-degree algebraic equation.

The actuated coordinates are obtained from the relations:

$$q_i = Y_i \cos\delta_i - X_i \sin\delta_i\,; \quad q_{i+3} = Z_i \tag{5}$$

2.2. Kinematic model

To find the kinematic model (KM), the I-O equations are derived with respect to time yielding to:

$$[A]\,\dot{\overline{q}} = [B]\,\dot{\overline{X}} \tag{6}$$

where: $\dot{\bar{q}}(\dot{q}_1,\dot{q}_2,\dot{q}_3,\dot{q}_4,\dot{q}_5,\dot{q}_6)$ - for 3-PPRS and $\dot{\bar{q}}(\dot{q}_1,\dot{q}_4,\dot{q}_2,\dot{q}_5,\dot{q}_3,\dot{q}_6)$ - for 3-PPSR is the driving velocity vector, $\dot{\bar{X}}(\dot{X},\dot{Y},\dot{Z},\omega_X,\omega_Y,\omega_X)$ is the end-effector velocity vector, $[A]$ is the inverse Jacobi matrix, $[B]$ is the direct Jacobi matrix.

For the 3-PPRS manipulator [A] is a diagonal matrix and [B] contains 9 constant elements, 3 of them are zero, while for 3-PPSR manipulator [A] is a block diagonal matrix and [B] has all elements variable.

While the inverse kinematic model ($\dot{\bar{q}} = [A]^{-1}[B]\dot{\bar{X}}$) has an analytical solution, the direct kinematic model ($\dot{\bar{X}} = [B]^{-1}[A]\dot{\bar{q}}$) can only be numerically solved, that means it requires the discrete definition of the matrix [B] for the mobile platform pose parameters and then its numerical inversion.

3. DYNAMIC MODELING OF PARALLEL MINI-MANIPULATORS

The dynamic modeling of parallel robots presents an inherent complexity due to their closed-loop structure and kinematic constraints. Many of the mechanics classical methods cannot be successfully applied for parallel robots.

In order to study the dynamic model (DM) using the Lagrange formulation ([2, 4]) several simplifying hypothesis were adopted: all joint are frictionless; the inertia of guiding arms C_iA_i is neglected; the manipulated object is reduced to a material point and located in the centroid of the *WP*.

Since a robot has 6 degrees of freedom, one is naturally inclined to select 6 generalized coordinates, for example q_i, (i=1,2,3,4,5,6) and then evaluate a set of 6 equations of the second type for these coordinates. However, due to the complexity of the geometric model, the evaluation of Lagrange derivatives with six coordinates only, is found to be extremely tedious. A much better approach is to choose 12 generalized coordinates: q_i, q_{i+3}, (i=1,2,3), X,Y,Z, α, β, γ.

In both cases of the parallel mini-manipulators, the Lagrange equations with multipliers take the following form:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = Q_j + \sum_{k=1}^{6}\lambda_k A_{kj} \ ; \quad j = 1,2,\dots,12 \tag{7}$$

The Lagrange function, accordingly to the hypothesis made, is:

$$\begin{aligned} L = {} & \frac{1}{2}(M+m)\left(\dot{X}^2+\dot{Y}^2+\dot{Z}^2\right) + \frac{1}{2}\left(I_x\omega_x^2 + I_y\omega_y^2 + I_z\omega_z^2\right) + \frac{1}{2}\sum_{i=1}^{3} m_i\dot{q}_i^2 + \\ & + \frac{1}{2}\sum_{i=1}^{3} m_{i+3}\left(\dot{q}_i^2 + \dot{q}_{i+3}^2\right) - \tau\left[(M+m)gZ + \sum_{i=1}^{3} m_i gh + \sum_{i=1}^{3} m_{i+3}g\left(q_{i+3}+d\right)\right] \end{aligned} \tag{8}$$

where: *M* – mass of the traveling (working platform); *m* – mass of the manipulated object; I_x, I_y, I_z – moments of inertia of WP about the axes of the *oxyz* frame; m_i, m_{i+3} – mass of “i” and “i+3” link; *h, d* – constant dimensional parameters; *g* – gravitational acceleration; $\dot{X}, \dot{Y}, \dot{Z}$ - scalar components of the velocity of the point “o” (WP center of mass) in the global coordinate system *OXYZ* ; $\omega_x, \omega_y, \omega_z$ - scalar component in *oxyz* frame of angular velocity of *WP*; λ_k - Lagrange multipliers; Q_j – generalized external force; A_{kj} - the coefficients of the generalized velocities from the equation $[J]\dot{\bar{q}} - \dot{\bar{X}}^* = 0$, $[J] = \begin{bmatrix} [I]_{3\times3} & [0]_{3\times3} \\ [0]_{3\times3} & [R] \end{bmatrix}^{-1} [B]^{-1}[A]$ is the system Jacobi matrix, $[R]$ is the rotation matrix, $\dot{\bar{X}}^* = [\dot{X},\dot{Y},\dot{Z},\omega_x,\omega_y,\omega_z]$, τ = +1 for the upper WP, τ =-1for the down WP.

Since the formula for the Lagrange function is not complicated, the evaluation of its derivatives with respect to the generalized coordinates and velocities is relatively simple. Eliminating the six multipliers from the Lagrange equations, one can derive only six equations for the dynamic model. Thus for the 3-PPRS mini-manipulator, these differential equations are:

$$[Q_1,Q_2,Q_3,Q_4,Q_5,Q_6] =$$

$$= diag(m_1+m_4, m_2+m_5, m_3+m_6, m_4, m_5, m_6)\cdot[\ddot{q}_1,\ddot{q}_2,\ddot{q}_3,\ddot{q}_4+\tau g,\ddot{q}_5+\tau g,\ddot{q}_6+\tau g]^T +$$

$$+[J]^T \cdot \begin{bmatrix} (M+m)\ddot{X}-R_X, (M+m)\ddot{Y}-R_Y, (M+m)(\ddot{Z}+\tau g)-R_Z, \\ I_x\varepsilon_x + (I_z - I_y)\omega_y\omega_z - M_x, I_y\varepsilon_y + (I_x - I_z)\omega_z\omega_x - M_y, I_z\varepsilon_z + (I_y - I_x)\omega_x\omega_y - M_z \end{bmatrix}^T \quad (9)$$

R_X, R_Y, R_Z , M_x, M_y, M_z are the scalar components of the applied to the WP forces torsor.

4. SIMULATION PROGRAM

A complex simulation program was developed in order to study the geometric, kinematic and dynamic characteristics of parallel robots [6]. For the graphical modeling of the parallel structures it was used Solid Edge™, one of the most advanced software for computer aided design, available on the market. This program was selected especially for its outstanding performances in terms of stability and user-friendly interface, and even more for its total compatibility with Visual Basic. The geometric parameters of the parallel structure can be modified within the 3D modeling software (Solid Edge Assembly) influencing the simulation environment. The assembly relations between parts, between subassemblies or between parts and subassemblies can be also modified.

These facilities of the simulation software enable the possibility to develop a complex study about the kinematics and dynamics in order to optimize the parallel structure.

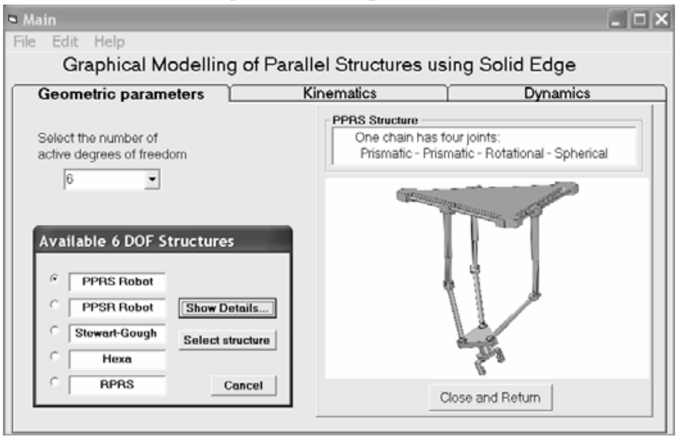

Figure 5. The simulation program

5. CONCLUSION

The paper presents the algorithms for the kinematic and dynamic analysis of two 6 DOF parallel mini-manipulators: 3-PPRS and 3-PPSR. The obtained simulation results have shown that the developed models have a compact structure and could be used for the real-time control of the parallel mini-manipulators. In the 3-PPRS mini-manipulator case, the kinematical model is simpler.

6. ACKNOWLEDGEMENTS

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