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MULTIPLIER-FREE FILTER DESIGN USING LT COMB FILTERS

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ABSTRACT
This paper present on simple method for multiplier-free finite impulse response (FIR) filter design based on the cascade of comb and linearly tapered comb filters. The stopband frequency determines the length of the comb filter. The method is used for narrowband filters where the passband frequency is not more than 0.1 of the stopband frequency. The sharpening technique is used to satisfy the desired stopband attenuation and the minimum passband ripple.
Keywords: multiplier-free, FIR filter, Sharpening.

1. INTRODUCTION
There are two different digital filter types to satisfy a given specification: infinite impulse response (IIR) filters and finite impulse response (FIR) filters. In many applications it is often advantageous to employ FIR filters, than IIR filters, since they can be designed with exact linear phase and exhibit no stability problems [1]. However FIR filters have a computationally more intensive complexity compared to IIR filters with equivalent magnitude responses. Linear phase FIR filters of length N require $(N+1)/2$ multipliers, N-1 adders and N-1 delays. The complexity of the implementation increases with the increase in the number of multipliers.
During the past several years, many design methods have been proposed to reduce the complexity of the FIR filters (reduce the number of multipliers). Another approach is a true multiplier-free design where the coefficients are reduced to simple integers or to simple combinations of powers of two, [2, 3, 4]. Bhattacharya & Saramaki [4] state that "the major approach is based on optimizing the filter coefficient values such that the resulting filter meets the given specification with its coefficient values represented in minimum number of signed powers-of-two (MNSPT) or canonic signed digits (CSD) representations of binary digits." However "one may not assure or guarantee that the optimal solution will always be found under the given constraints" [4]. One alternative approach is based on combining simple sub-filters, [5].
In this article we present the simple method for an efficient multiplier-free FIR filter design based on linearly tapered comb filters. The rest of the paper is organized as follows. In next section we describe linearly tapered comb filters. The Section 3 describes the procedure which is illustrated with two examples.

2. LINEARLY TAPERED COMB FILTER
Consider a tapered comb filter with R-1 tapered elements on both sides, making the total number of tapered elements equal to $2(R-1)$. Let $h_M[n]$ and $h_R[n]$ denote the impulse responses of two comb filters of length M and R, respectively

$$h_M[n] = \begin{cases} 1/M & 0 \le n \le M-1 \\ 0 & otherwise, \end{cases} \qquad (1)$$

$$h_R[n] = \begin{cases} 1/R & 0 \le n \le R-1 \\ 0 & otherwise, \end{cases} \quad . \qquad (2)$$

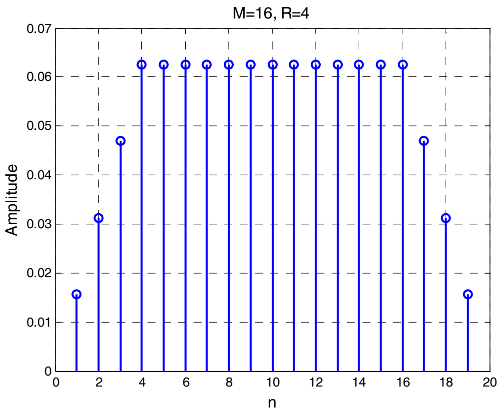
The general form of the impulse response of the linearly tapered comb (LTC) filter is given by the convolution of $h_M[n]$ and $h_R[n]$,

$$h_{LT,R}[n] = h_M[n] * h_R[n], \qquad (3)$$

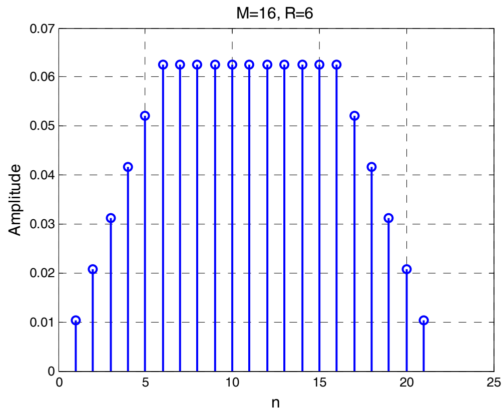
where * indicates the convolution.
The corresponding transfer function is

$$H_{LT,R}(z) = \frac{1}{M}\frac{1-z^{-M}}{1-z^{-1}} \times \frac{1}{R}\frac{1-z^{-R}}{1-z^{-1}} . \qquad (4)$$

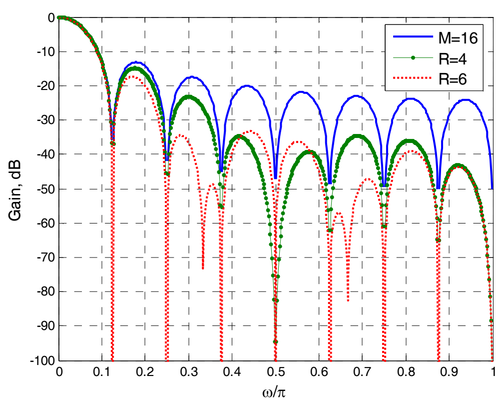
Figures 1 (a) and (b) show the impulse responses of LTC filters for two values of R, 4 and 6, respectively, and M=16, while Figure 1 (c) presents the corresponding gains in dBs along with the gain of the comb filter $h_M[n]$. Figure 1. (d) compares the gain responses of the cascade of k=3 comb filters , and the cascade of two LTC filters with R_1=4 and 6, and M=16.



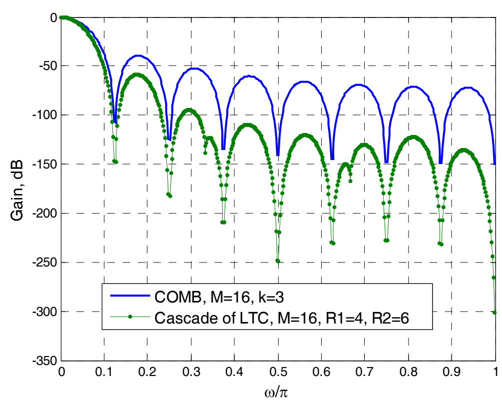
a. R=4.



b. R=6.



c. Gain responses.



d. Gain responses.

Figure 1. LTC filters.

We can notice the following:

- One can control the stopband attenuation by changing the values of R.
- Additionally, LTC filters have the same zeros as the corresponding comb filter.
- LTC filters are also multiplier-free.
- The cascade of LTC filters has a lowpass characteristic with an improved stopband attenuation.

The designed filter is the cascade

$$H(z)=\prod_{i=1}^{N} H_{LT,R_i}(z)\,, \tag{5}$$

where $N<M$ and $R_1=2$. If filter (5) does not satisfy the specification we propose to apply the sharpening technique [6].
The designed filter is

$$Sh\{H(z)\}=Sh\left\{\prod_{i=1}^{N} H_{LT,R_i}(z)\right\}, \tag{6}$$

where Sh{.} means sharpening.

3. DESCRIPTION OF THE METHOD

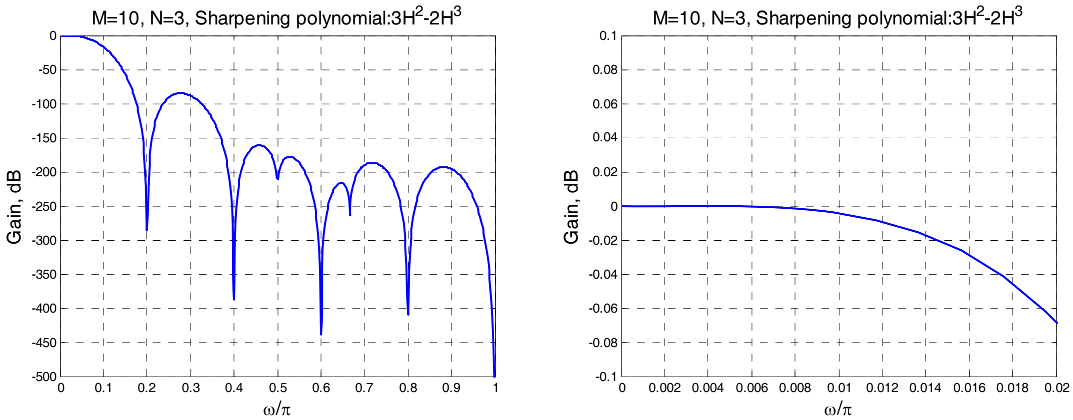
The filter specification is given by the normalized passband and stopband frequencies, passband ripple in dB and the stopband attenuation in dB. Usually the passband frequency is not more than 0.1 of the stopband frequency.
The method is described in the following steps:
1. The value of M is determined by the stopband frequency as

$$M=\lfloor 2/\omega_s \rfloor, \tag{7}$$

where $\lfloor \cdot \rfloor$ means the nearest integer.
2. i=0.
3. $i = i+1$. If $i = N$, go to step 5.
4. Chose the value R_i. If the specification of the filter (5) is satisfied go to step 6. If not, go to step 3.
5. Apply the sharpening technique starting with the simplest sharpening polynomials. Increase the order of the sharpening polynomial until the specification is satisfied. Otherwise increase the value of N and go to step 3.
6. End of procedure.

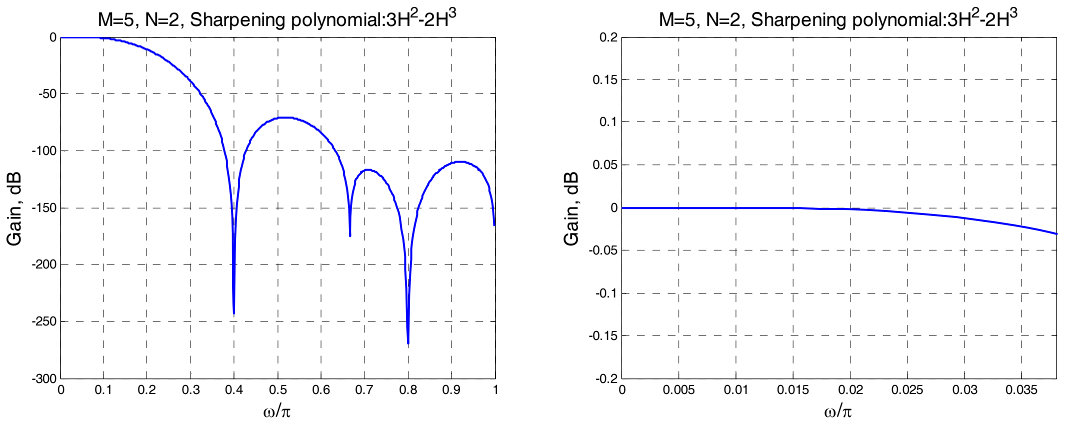
The method is illustrated with the following two examples.
Example 1:
The passband and the stopband frequencies are 0.02 and 0.2, respectively. The passband ripple is 0.1dB, and the minimum stopband attenuation is 80 dB.
The corresponding equiripple filter has an order of 36 and has 18 multipliers.
According to (7) we have M=10.
We chose N=3, and we have $R_1=2$, $R_2=3$, and $R_3=4$.
The filter does not satisfy the specification. We apply the sharpening ($3H^2-2H^3$). The corresponding magnitude response is shown in Fig. 2 (a). The passband zoom, shown in Fig. 2 (b) demonstrates that the specification is satisfied.

Example 2:
In this example the specification is the following: The passband frequency and the passband ripple are 0.038 and 0.05 dB, respectively. The stopband frequency is 0.41, while the minimum stopband attenuation is 70 dB. The corresponding equiripple filter has 9 multipliers.
From (7) it follows M=5. We choose N=2. Using the sharpening polynomial $3H^2-2H^3$ we get the magnitude response of the designed filter shown in Fig. 3. Note that the specification is satisfied.



a. Overall magnitude response. *b. Passband zoom.*

Figure 2. Example 1.



a. Overall magnitude response. *b. Passband zoom.*

Figure 3. Example 2.

4. CONCLUSIONS

The simple method for narrowband multiplier-free FIR filter design is presented. The designed filter is the cascade of linearly tapered comb filters. The corresponding length of the comb filter is determined by the stopband frequency. The number N of the cascaded LTC filters is typically less than M, where M is the length of the corresponding comb filter. If the cascade does not satisfy the given specification we use the sharpening technique.

5. REFERENCES

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