10th International Research/Expert Conference
"Trends in the Development of Machinery and Associated Technology"
TMT 2006, Barcelona-Lloret de Mar, Spain, 11-15 September, 2006

CALCULUS OF LONG RECTANGLE PLATE WITH A CONCENTRATE LOAD OF ONE OF ITS BORDER BY TRANSFER-MATRIX METHOD

Mihaela SUCIU, Liviu SUCIU, Anton POPA, Liana HANCU,
Mihai TRIPA, Gavril BALC, Augustin CRETU, Mircea BEJAN
Mechanical Faculty, Technical University of Cluj-Napoca, ROMANIA
E-mail: mihaelaica@yahoo.fr

ABSTRACT
This paper presents the analytical calculus of the long rectangulary plate, embeded at one of its lengths and with a concentrated load of an another length, using the Transfer-Matrix Method, in according to Strength of Materials hypotheses. The method consist in discretizing in units parts the length of the plate, each part is as a beam. The Transfer-Matrix of the plate links the state vectors of the left side at the right side. With the conditions of the bearings, we are calculated the state vector at the origin and the general expression for the deformation.
Keywords: Transfer-Matrix Method, state vector, long rectangle plate, Dirac's fonction, Heaviside's fonction.

1. INTRODUCTION
The rectangulary plate with long lengths can be calculated by the Transfer-Matrix Method, the plate is discretized, of its lengths into parts. One part is equal at the unit and it is as a beam, with a Transfer-Matrix associate. The state vector at the origin is calculated after the conditions of the bearings. The general formula for the deformation give the deformations in all sections of the rectangle plate.

2. THE EQUATIONS FOR A LONG RECTANGLE PLATE
We have a long plate rectangle (Figure 1.). The method consist in discretizing the plate into parts, each part have the width equal at a unit. We consider the load on a line parallel with thelength and due to bending. The plate is now a curve surface (Figure 1., b)). The mathematical expression of the load on the unit's length is: **q(x)** [1].

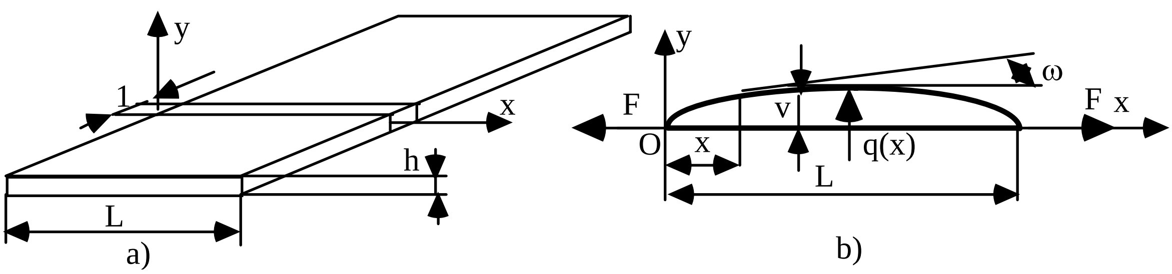

Figure 1.

The hickness of each part is h and the beam bending's rigidity is D:

$$D = \frac{Eh^3}{12\left(1-\nu^2\right)} \tag{1}$$

when: E is the Young's modulus and ν is the Poisson's coeficient. In the point x, we have: T(x)-catting force, M(x)-total flexion moment, v(x)-deformation, (x)-angle deformation or rotation of the midle fiber, m(x)-bending moment due to a single exterior load. F is the extension reaction in the baundary unit (fonction of v(x)). For the total bending moment M(x), we can write:

$$M(x) = m(x) + F \cdot v(x) \tag{2}$$

with:

$$\frac{d^2 m(x)}{dx^2} = q(x) \tag{3}$$

The differentiate equation for the deformation is:

$$\frac{d^4 v(x)}{dx^4} - \frac{F}{D}\frac{d^2 v(x)}{dx^2} = \frac{q(x)}{D} \tag{4}$$

We note:

$$\frac{F}{D} = \alpha^2 \tag{5}$$

and we have the associate equation, of the equation (4), without the second member:

$$v(x) = Ach\alpha x + Bsh\alpha x + C_1 x + C_2 \tag{6}$$

With Dirac's fonction and Heaviside's fonction and operators, the mathematical approach gives the particularly solution:

$$v^*(x) = \frac{1}{D\alpha^3}\int_0^x [sh(x-t) - \alpha(x-t)]q(t)dt \tag{7}$$

and the conditions:

$$v^*(0)=v'^*(0)=v''^*(0)=v'''^*(0)=0 \tag{8}$$

In the origin, we have:

$$\begin{cases} v(0) = v_0 \\ \omega(0) = \omega_0 \\ M(0) = M_0 \\ T(0) = T_0 \end{cases} \tag{9}$$

and we can write :

$$\begin{cases} v_0 = A + C_2 \\ \omega_0 = \alpha B + C_1 \\ M_0 = \alpha^2 AD \\ T_0 = -\alpha^3 BD \end{cases} \tag{10}$$

The integration constants are:

$$\begin{cases} A = \dfrac{1}{\alpha^2 D} M_0 \\ B = -\dfrac{1}{\alpha^3 D} T_0 \\ C_1 = \omega_0 + \dfrac{1}{\alpha^2 D} T_0 \\ C_2 = v_0 - \dfrac{1}{\alpha^2 D} M_0 \end{cases} \tag{11}$$

The general solution is:

$$v(x) = v_0 + \omega_0 x + \frac{ch\ \alpha x - 1}{\alpha^2 D} M_0 + \frac{\alpha x - sh\ \alpha x}{\alpha^3 D} T_0 + \\ + \frac{1}{\alpha^3 D} \int_0^x [sh\ \alpha(x-t) - \alpha(x-t)] q(t) dt \tag{12}$$

This is the calculus formula for the deformation.

3. THE TRANSFER-MATRIX FOR A LONG RECTANGULARY PLATE

In the poit 0, at the origin, the state vector is:

$$\{U\}_0 = \{v_0, \omega_0, M_0, T_0\}^{-1} \tag{13}$$

and in the point x:

$$\{U(x)\} = \{v(x), \omega(x), M(x), T(x)\}^{-1} \tag{14}$$

The Transfer-Matrix for the plate, with the Heaviside's fonction and operators, linking the two state vectors, is:

$$\begin{Bmatrix} v(x) \\ \omega(x) \\ M(x) \\ T(x) \end{Bmatrix} = \begin{bmatrix} 1 & x & \frac{ch\ \alpha x - 1}{\alpha^2 D} & \frac{\alpha x - sh\ \alpha x}{\alpha^3 D} \\ 0 & 1 & \frac{sh\ \alpha x}{\alpha D} & \frac{1 - ch\ \alpha x}{\alpha^2 D} \\ 0 & 0 & \frac{ch\ \alpha x}{D} & -\frac{sh\ \alpha x}{\alpha D} \\ 0 & 0 & \frac{\alpha sh\ \alpha x}{D} & -\frac{ch\ \alpha x}{D} \end{bmatrix} \begin{Bmatrix} v_0 \\ \omega_0 \\ M_0 \\ T_0 \end{Bmatrix} + \begin{Bmatrix} \frac{1}{\alpha^3 D} \int_0^x [sh\ \alpha(x-t) - \alpha(x-t)] q(t) dt \\ \frac{1}{\alpha^2 D} \int_0^x [ch\ \alpha(x-t) - 1] q(t) dt \\ \frac{1}{\alpha D} \int_0^x sh\ \alpha(x-t) q(t) dt \\ \frac{1}{D} \int_0^x ch\ \alpha(x-t) q(t) dt \end{Bmatrix} \tag{15}$$

More simply, we can write:

$$\{U(x)\} = [T]_x \{U\}_0 + \frac{1}{D}\{U\}_c \tag{16}$$

where: $\{U(x)\}$ is the state vector at an one current section, $[T]_x$ is the Transfer-Matrix betwin the side 0 and x, $\{U\}_0$ is the state vector of the origin, $\{U\}_c$ is a coefficients vector.

4. THE RECTANGLE PLATE EMBEBDED AT ITS RIGHT LONG BORDER WITH A CONCENTRATED VERTICAL LOAD AT A LEFT LONG BORDER

The section of a long plate rectangle at its right long boundary embebded, challkanged with a concentrated vertical load at a left long border is in Figure 2.

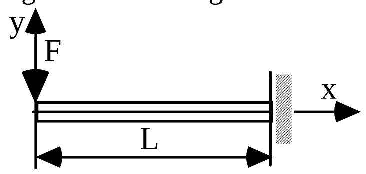

Figure 2

The distribution's function, with the Heaviside's fonction Y, for this load F is: is:

$$q(x) = -FY(x) \tag{17}$$

The coeficients vector $\{U\}_c$ is:

$$\{U\}_c = \begin{Bmatrix} -\dfrac{F}{\alpha^3 D}(sh\,\alpha x - \alpha x) \\ -\dfrac{F}{\alpha^2 D}(ch\,\alpha x - 1) \\ -\dfrac{F}{\alpha D} sh\,\alpha x \\ -\dfrac{F}{D} ch\,\alpha x \end{Bmatrix} \tag{18}$$

For the bearings of the embeded boundary and for the free border, the conditions are:

$$\begin{cases} M(0) = M_0 = 0 \\ T(0) = T_0 = F \\ v(L) = 0 \\ \omega(L) = 0 \end{cases} \tag{19}$$

We can write for the last section at the right border, for x=L:

$$\begin{Bmatrix} v(L) \\ \omega(L) \\ M(L) \\ T(L) \end{Bmatrix} = \begin{bmatrix} 1 & L & \dfrac{ch\,\alpha L - 1}{\alpha^2 D} & \dfrac{\alpha L - sh\,\alpha L}{\alpha^3 D} \\ 0 & 1 & \dfrac{sh\,\alpha L}{\alpha D} & \dfrac{1 - ch\,\alpha L}{\alpha^2 D} \\ 0 & 0 & \dfrac{ch\,\alpha L}{D} & -\dfrac{sh\,\alpha L}{\alpha D} \\ 0 & 0 & \dfrac{\alpha\, sh\,\alpha L}{D} & -\dfrac{ch\,\alpha L}{D} \end{bmatrix} \begin{Bmatrix} v_0 \\ \omega_0 \\ M_0 \\ T_0 \end{Bmatrix} - \begin{Bmatrix} \dfrac{F}{\alpha^3 D}(sh\,\alpha L - \alpha L) \\ \dfrac{F}{\alpha^2 D}(ch\,\alpha L - 1) \\ \dfrac{F}{\alpha D} sh\,\alpha L \\ \dfrac{F}{D} ch\,\alpha L \end{Bmatrix} \tag{20}$$

The expressions (20) and (19) gives:

$$\begin{Bmatrix} 0 \\ 0 \\ M(L) \\ T(L) \end{Bmatrix} = \begin{bmatrix} 1 & L & \dfrac{ch\,\alpha L - 1}{\alpha^2 D} & \dfrac{\alpha L - sh\,\alpha L}{\alpha^3 D} \\ 0 & 1 & \dfrac{sh\,\alpha L}{\alpha D} & \dfrac{1 - ch\,\alpha L}{\alpha^2 D} \\ 0 & 0 & \dfrac{ch\,\alpha L}{D} & -\dfrac{sh\,\alpha L}{\alpha D} \\ 0 & 0 & \dfrac{\alpha\, sh\,\alpha L}{D} & -\dfrac{ch\,\alpha L}{D} \end{bmatrix} \begin{Bmatrix} v_0 \\ \omega_0 \\ 0 \\ F \end{Bmatrix} - \begin{Bmatrix} \dfrac{F}{\alpha^3 D}(sh\,\alpha L - \alpha L) \\ \dfrac{F}{\alpha^2 D}(ch\,\alpha L - 1) \\ \dfrac{F}{\alpha D} sh\,\alpha L \\ \dfrac{F}{D} ch\,\alpha L \end{Bmatrix} \tag{21}$$

We obteind:

$$\begin{cases} v_0 = aF \\ \omega_0 = bF \end{cases} \tag{22}$$

with notes:

$$\begin{cases} a = \dfrac{2(sh\alpha L - \alpha L)}{\alpha^3 D} \\ b = \dfrac{2(ch\alpha L - 1)}{\alpha^2 D} \end{cases} \tag{23}$$

This values gives, for the expression of deformation (12), for a long plate rectangle with a vertical concentrated load F:

$$v(x) = aF + bFx + \frac{\alpha x - sh\,\alpha x}{\alpha^3 D} F + \frac{sh\,\alpha x - \alpha x}{\alpha^3 D} F \tag{24}$$

5. CONCLUSION

The formulas (24) we give all the deformations for all the sections of the long rectangle plate with a concentrate load of the left free border and embebded at its right boundary.

6. REFERENCES

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