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THE DETERMINATION OF BRAKE TORQUE UNDER UNEVEN PRESSURE DISTRIBUTION IN DISC BRAKES

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ABSTRACT
It’s generally accepted that the pressure distribution between rotor and pads interface is uneven in disc brakes. In traditional brake torque calculation methods, the uniform interface pressure and the uniform pad wear are validated. However, at the beginning of brake process, the leading and trailing edge have been occurred at the rotor and pads interface depending on rotating direction of disc. There’s higher pressure between the rotor and the pad on the leading side than the trailing side. The pressure changes can’t be determined certainly because there’s no experimental method available for directly measuring disc brake pad pressure while it’s operating. In this study, it has been represented a new method by using uneven pressure distribution at the rotor and pads interface. It has been determined to describe the pressure distribution in mathematical equations using linear and non linear changes.
Keywords: Disc brake, brake torque, leading-trailing edge, pressure distribution

1. INTRODUCTION
Disc brakes are the most common equipment to stop vehicles like cars. A disc brake of floating caliper design consists of pads, caliper and disc (rotor). The caliper and disc are assembled together. One of the major requirements of the caliper is to press the pads against the rotor. Normal forces are generated by the hydraulic pressure in order to press the brake pads against the rotor and the piston presses the pad against disc. A reaction takes place where the caliper transfers the force to the pad on the other side of the disc. The investigations of disc brake pressure distributions are important because of effects on the life of pads and squeal. The non uniform pad pressure distribution causes uneven wear and shortens life of pads. Under normal circumstances, disc rotates with wheel at the same angular velocity. The brake lining is fixed. At the beginning of brake process, the normal force and friction force occur at the rotor and pads interface. The frictional forces acting at the contact interface are parallel in the surfaces. The direction of the frictional force is the same as the motion of the disc for the pad. The tilting moment that acts the brake pad against disc occurs due to the friction forces. This moment makes up additional force pressing the brake pads against the disc on the leading side while decreases the pressing force on the trailing side and the moment effects the pads moving opposite direction of disc on the trailing side. Therefore, there’s higher pressure between the disc and the pad on the leading side than the trailing side. The tilting moment can be calculated around the point of A which is assumed as the center of tilting effect. There’s no specific suggestion for the position point A. The distance between A and friction interface t_p is based on the assumptions. The tilting moment is getting higher with the increasing pad thickness; t_p. The interface pressure

distribution and uneven wear due to uneven pressure distribution have been investigated by a number of people. It was observed that significantly sever wear appeared on the leading side comparing the trailing side of a pad. There's no specific investigation related to the distributions of contact pressure. And also there's no experimental method available for directly measuring disc brake pad contact pressure while it's operating. In all traditional brake torque calculation methods, the uniform interface pressure and the uniform pad wear are validated. [1,2,3,4]

The aim of this study is to examine the effect of the pressure distributions at the rotor and the pad interface on the brake torque by using linear and non linear pressure distribution assumptions stated by mathematical equations.

2. THE ANALYSIS OF THE EFFECT OF PRESSURE DISTRIBUTION

The non uniform pressure distribution represented by a linear change is illustrated in Figure 1. This approach is called by pressure triangle analysis. The average force pressing the pad against the disc is indicated by F_o and the average pressure is p_o when the car is stationary. At the beginning of brake process, the leading and trailing edge have been occurred at the rotor and pads interface depending on rotating direction of disc. Let's assume that at the leading side the pressure increases as much as Δp, at the trailing edge, the pressure decreases as the same ratio. When a linear pressure variation is assumed, the values will change between $p_{max} = p_o + \Delta p$ and $p_{min} = p_o - \Delta p$ along the contact area. The tilting moment about point A will be calculated by using pressure triangle.

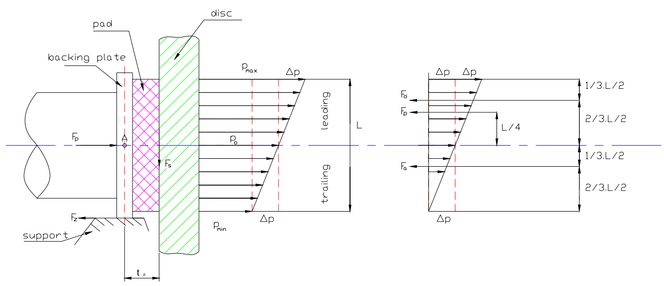

Figure 1. The pressure changes in linear pressure distribution

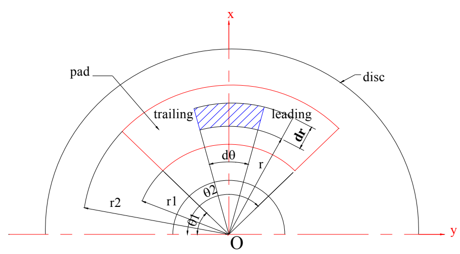

Figure 2. Disc brake

The tilting moment at the clock direction;

$$F_s.t_p + F_z.\frac{L}{2} + F_t.\frac{1}{3}.\frac{L}{2} \tag{1}$$

The balancing moment at the opposite direction is,

$$F_p.\frac{L}{4} + F_l.\frac{2}{3}.\frac{L}{2} \tag{2}$$

Where F_z is the friction force between support and backing plate, F_l is the normal force at the leading side, F_t is the normal force at the trailing side, F_p is the average force for this area, F_s is the friction force between disc and brake lining.

If the average pressure is taken $\Delta p/2$ at both sides, it can be written absolutely for unit weight $F_t = F_l$

$$F_l = \left| F_t \right| = \frac{L}{2}.1.\frac{\Delta p}{2} = \frac{\Delta p.L}{4} \tag{3}$$

Application of moment balance about point A yields;

$$\mu_p.\ p_0.L.t_p + \mu_z.\ \mu_p.\ p_0.L.\frac{L}{2} + \Delta p.\frac{L}{4}.\frac{L}{6} = \frac{L}{2}.\Delta p.\frac{L}{4} + \Delta p.\frac{L}{4}.\frac{L}{3} \quad (4)$$

Solving for pressure change Δp results ;

$$\Delta p = \frac{6p_0}{L}(\mu_p.t_p + \mu_z.\ \mu_p.\frac{L}{2}) \quad (5)$$

The maximum pressure at the leading side and the minimum pressure at the trailing side can be written respectively;

$$p_{max} = p_0 + \Delta p = p_0 [1 + \frac{6}{L}(\mu_p.t_p + \mu_z.\ \mu_p.\frac{L}{2})] \quad (6)$$

$$p_{min} = p_0 - \Delta p = p_0 [1 - \frac{6}{L}(\mu_p.t_p + \mu_z.\ \mu_p.\frac{L}{2})] \quad (7)$$

where L=pad length (mm), t_b=pad thickness (mm), t_p=pad thickness/support distance (mm), μ_p=pad/disc friction coefficient, μ_z=backing plate/support friction coefficient, p_o=average pressure (N/mm^2), $t_b=t_p$.

If we remove point A from middle of the backing plate to plate-lining interface, we can take t_b instead of t_p. This will provide directly the effect of lining thickness on tilting moment.

3. THE BRAKE MOMENT ANALYSIS OF DISC BRAKES

Case 1, the pressure is uniform. The elementary area dA=r.dθ.dr is taken in to the consideration at the distance r from the disc center O inside the pad-disc contact area. The pressure between pad and disc is p, the normal force effected on this area is dF_N = p.dA. The friction force is dF_s = μ.p.dA.

The elementary braking moment at the considered area can be written as;

$$dM_{fr} = r.\ dF_s = \mu.\ p.r.r.d\theta.dr \quad (8)$$

The braking moment can be expressed as follows;

$$M_{fr} = z.\mu \int_{r_1}^{r_2}\int_{\theta_1}^{\theta_2} p.r^2.d\theta.dr \quad (9)$$

After performing the integrations, the braking moment will be;

$$M_{fr} = z.\mu.p_o.(\frac{r_2^3 - r_1^3}{3}).(\theta_2 - \theta_1) \quad (10)$$

Where z is the number of contact surfaces which is 2 in our case due to the pads press on both sides of the disc.

Case 2, the pressure is varying linearly depending on θ. Although, in traditional calculations of brake moment, the uniform pressure is validated, the pressure is differ on the leading and trailing side in reality. The pressure varies from P_{min} to P_{max} linearly in θ direction but constant in r direction. The equation of pressure is given by the following expression,

p(θ) = aθ+b

The boundary conditions for $\theta=\theta_1$, $p=p_{min}$ and for $\theta=\theta_2$, $p=p_{max}$ then $p_{max}-p_{min} = a(\theta_2 - \theta_1)$ can be written.

After applying arrangement in the equation, a and b constants are determined.

$$a = \frac{p_{max}-p_{min}}{\theta_2 - \theta_1} \quad (11)$$

$$b = \frac{p_{min}.\ \theta_2 - \theta_1.\ p_{max}}{\theta_2 - \theta_1} \quad (12)$$

By putting the constants a and b in to the equation;

$$p(\theta) = \frac{p_{max}-p_{min}}{\theta_2 - \theta_1}\theta + \frac{p_{min}.\ \theta_2 - \theta_1.\ p_{max}}{\theta_2 - \theta_1} \quad (13)$$

After using this pressure equation in equation 9, the brake moment will be,

$$M_{fr} = z.\mu.(\frac{r_2^3 - r_1^3}{3})[(\frac{p_{max}-p_{min}}{\theta_2 - \theta_1}).(\frac{\theta_2^2 - \theta_1^2}{2}) + (\frac{p_{min}.\ \theta_2 - \theta_1.\ p_{max}}{\theta_2 - \theta_1})(\theta_2 - \theta_1)] \quad (14)$$

Case 3, the pressure distribution between disc and pad can be more complicated than linear change. In this case it’s assumed that pressure is varying parabolically.

$p(\theta) = a\theta^2 + b\theta + c$

The boundary conditions for $\theta=\theta_1$, $p=p_{min}$ and for $\theta=\theta_2$, $p=p_{max}$.
If it's accepted that the derivative of function $p(\theta)$ is zero in $\theta=\theta_1$,
$p'(\theta) = 0$
$2a\theta_1+b=0$ and $a=-b/2\theta_1$
The constants a,b and c can be determined.
By putting the constants a,b and c in to the equation;

$$p(\theta) = \frac{(-p_{min}+p_{max})}{\theta_1^2-\theta_2^2-2\,\theta_1.\theta_2}\,\theta^2 + \frac{2\theta_1(p_{min}-p_{max})}{\theta_1^2-\theta_2^2-2\,\theta_1.\theta_2}\,\theta + p_{min} - \frac{\theta_1^2(p_{min}-p_{max})}{\theta_1^2-\theta_2^2-2\,\theta_1.\theta_2} \quad (15)$$

After using this pressure equation in equation 9, the brake moment will be,

$$M_{fr} = z.\mu\,\frac{1}{\theta_1^2-\theta_2^2-2\,\theta_1.\theta_2}\,\left(\left(\frac{\theta_2^3-\theta_1^3}{3}\right)p_{max} - \left(\frac{\theta_2^3-\theta_1^3}{3}\right)p_{min} - \theta_1^2.p_{min}.\theta_1 - \theta_2^2.p_{max}.\theta_1 - \theta_2^2.p_{min}.\,\theta_2 - \theta_1^2.p_{max}.\theta_2\right)\frac{(r_2^3-r_1^3)}{3} \quad (16)$$

It's possible varies pressure, θ, r, μ values, the magnitude of brake moment can be calculated numerically. The effect of pressure distribution has been shown in Figure 3. Figure 3 has been obtained by using the numerical values θ_1=1,04 rad, θ_2=1,92 rad, r_1=107,5 mm, r_2=70,5 mm, μ=0,3 and z=2 as near to the real automotive brake system.The pressure changes between p_{max} and p_{min} is taken as $2.\Delta p$=0,04. p_{max} and p_{min} are related to the pressure value of p_{pad}.

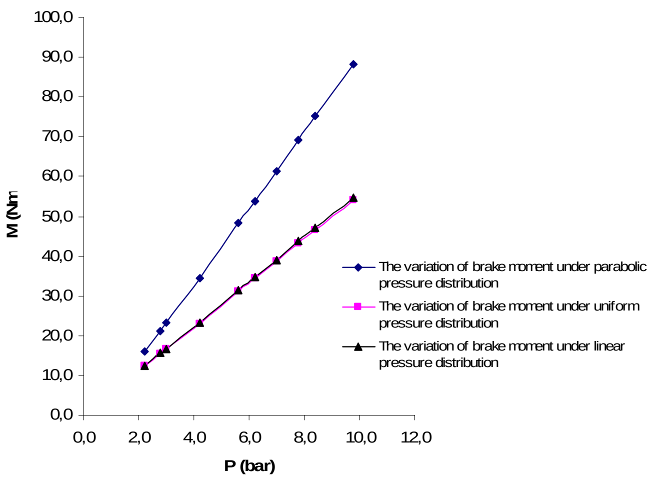

Figure 3. The variation of brake moment with pressure distribution

4. CONCLUSION

The pressure change at the leading and trailing areas is effected seriously from lining thickness. As the lining thickness, the tilting moment is being increased considerably. Since the pressure increase at the leading side and pressure decrease at the trailing side are simetrical, the total braking moment won't be changed. But for parabolic pressure change is effecting the braking moment significantly. The linear and non linear pressure changes are a small overview of what is possible if the pressure is uniform. It's verified that this mathematical method can predict the pressure changes in θ direction effecting the braking moment.

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