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ASYMMETRICAL BENDING IN THE ELASTIC – PLASTIC REGION

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ABSTRACT:
The contribution treats the asymmetrical bending of beams with constant cross- section in the elasto-plastic domain. The beams were made of material with an elastic-linear hardening rheological model and were loaded by a constant bending moment. The elasto-plastic deflection state of the beam after unloading is analytically determined. Numerical and experimental results are given and the difference between them is shown.
Keywords: elastoplastic bending, radius of curvature, plastic deformations, beams

1. PHYSICAL MODEL FORMULATION

- the longitudinal x-axis of the beam is straight or curved prior to bending;
- the vector of the inner bending moment (M_{pl}=*const.*) is constant along the longitudinal x-axis;
- the elasto-plastic bending process with constant bending moment was performed by bending beams on a circular plate with constant radius r_1.
- the rheological model of material has elastic-linear hardening mechanical properties.
- the radius of curvature *R* is constant along the longitudinal x-axis.
- cross-section of the beam is asymmetrical according to axis in which the bending moment vector is acting.
- after loading and unloading process, the equilibrium state between loading elasto-plastic and unloading elastic bending moments is re-established again.

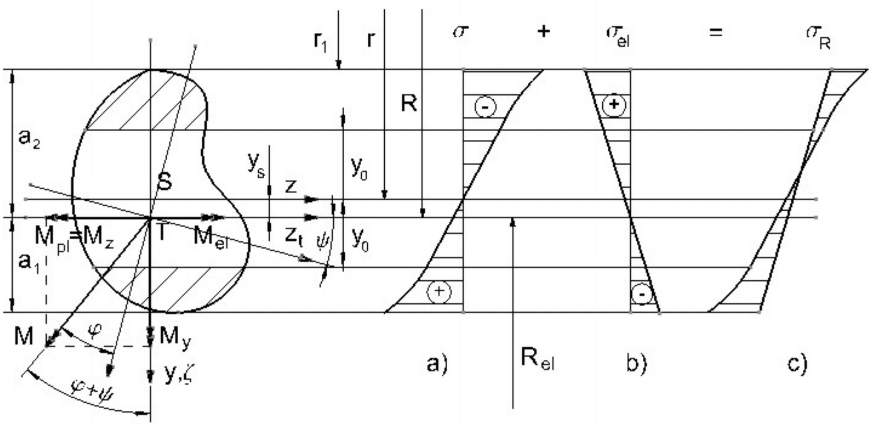

Figure 1. The elasto-plastic bending process: a) loading, b) unloading, c) resulting stress state.

It is well known that in the general form of the cross section of the beam, the neutral axis does not coincide with the axis of gravity. The difference between them is y_s., Fig. 1. Due to the action of the bending moment M_{pl}, an elastoplastic stress state σ_{pl} (y) occurs.

For determination of the final deflection state of the beam we have to determine the position of the neutral axis *z,* which is defined with angle ψ (Fig. 1). The position of the neutral axis is determined from force equilibrium state $N=0$. The final radius of curvature and y-axis are perpendicular to the neutral axis.

The loading stress state for the material with elastic–linear hardening rheological model can be defined as (Fig. 1a):

$$\sigma_{pl}(y)=\begin{cases}-\sigma_0\left[1+\dfrac{E_t}{E}\left(-\dfrac{y}{y_0}-1\right)\right]; -a_2+y_s\le y\le -y_0\\ \sigma_0\dfrac{y}{y_0}; |y|\le y_0\\ +\sigma_0\left[1+\dfrac{E_t}{E}\left(\dfrac{y}{y_0}-1\right)\right]; y_0\le y\le a_1+y_s\end{cases} \quad ...(1)$$

Equilibrium state of axial forces can be written as follows:

$$N=-\int_{-a_2+y_s}^{-y_0}\sigma_0\left[1+\frac{E_t}{E}\left(-\frac{y}{y_0}-1\right)\right]b(y)dy+\int_{-y_0}^{y_0}\sigma_0\frac{y}{y_0}b(y)dy+\int_{y_0}^{a_1+y_s}\sigma_0\left[1+\frac{E_t}{E}\left(\frac{y}{y_0}-1\right)\right]b(y)dy=0 \quad ...(2)$$

Components M_y and M_z of the vector of the plastic bending moment can be written as follows:

$$\begin{aligned}M_y=&\int_{-a_2+y_s}^{-y_0}\sigma_0\left[1+\frac{E_t}{E}\left(-\frac{y}{y_0}-1\right)\right]z(y)b(y)dy-\int_{-y_0}^{y_0}\sigma_0\frac{y}{y_0}z(y)b(y)dy-\\&-\int_{y_0}^{a_1+y_s}\sigma_0\left[1+\frac{E_t}{E}\left(\frac{y}{y_0}-1\right)\right]z(y)b(y)dy\end{aligned} \quad ...(3)$$

$$\begin{aligned}M_z=&-\int_{-a_2+y_s}^{-y_0}\sigma_0\left[1+\frac{E_t}{E}\left(\frac{-y_0}{y}-1\right)\right]yb(y)dy+\int_{-y_0}^{y_0}\sigma_0\frac{y^2}{y_0}b(y)dy+\\&+\int_{y_0}^{a_1+y_s}\sigma_0\left[1+\frac{E_t}{E}\left(\frac{y}{y_0}-1\right)\right]yb(y)dy\end{aligned} \quad ...(4)$$

The position of the neutral axis, Fig. 1, can be determined from the following equation:

$$tg(\psi+\varphi)=\frac{M_z}{M_y}\Rightarrow\psi=\psi(\varphi,y_o,y_s) \quad ...(5)$$

Components of plastic bending moments M_y and M_z are defined by using equations (3) and (4). In all equations, y_0 means depth of the plastification part of the cross section and is: $y_0=r\varepsilon_0=\sigma_0 r/E$.

The final deflection state after the loading and unloading process of the beam is determined from the equilibrium condition of bending moments:

$$\left(\frac{\sigma_0}{y_0}-\frac{E}{R}\right)\cdot\left[\int_0^{-a_2}\zeta^2 dA(\zeta)-\int_0^{a_1}\zeta^2 dA(\zeta)\right]-\int_{-a_2+y_s}^{-y_0}\sigma_0\left[1+\frac{E_t}{E}\left(-\frac{y}{y_0}-1\right)\right]\cdot yb(y)dy+$$
$$+\int_{-y_0}^{y_0}\sigma_0\frac{y^2}{y_0}b(y)dy+\int_{y_0}^{a_1+y_s}\sigma_0\left[1+\frac{E_t}{E}\left(\frac{y}{y_0}-1\right)\right]yb(y)dy=0 \qquad \ldots(6)$$

The final radius of curvature *R is determined by solving* equations (5) and (6), and is perpendicular to the neutral z-axis.

2. ANALYTICAL EXAMPLES

For analytical and numerical examples the following beams were chosen:
Unequal angles cross section L 15x10x1.5 mm. Chemical analysis of material: Fe 0.19 %, Si 0.42 %, Mg 0.48 %, the rest is aluminium with no detrimental effects. Mechanical properties are: yield stress σ_0=197.4 N/mm^2, Young's modulus in elastic region E= 63716 N/mm^2, modulus in plastic region E_t= 394.5 N/mm^2.

2.1 In-plane deflection state of the beam with unequal legs cross section L 15x10x1.5

It is well known that a beam with unequal-legs cross–section does not deform in-plane in the case when the bending moment is acting along the main axis of inertia *η*. In this example we are looking for the axis, along which the bending moment should be acting for the final deflection state to appear in-plane.
To get the in-plane deflection state, the beam has to be loaded with a bending moment as shown in Fig. 2 and the following relationship of the angles has to be fulfilled $\psi+\varphi=90^\circ$. In this case we get the following results:

$$\psi=-4.376^\circ, y_s=-0.156, y_0=0.325, R=119.067mm$$

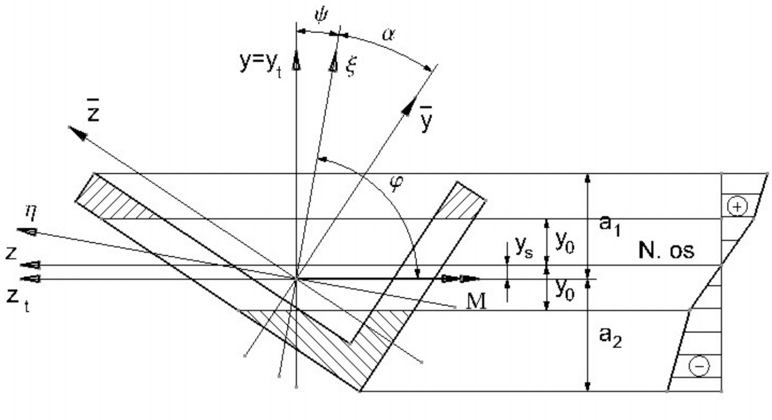

Figure 2. Loading state of cross section L 15x10x1.5.

2.2 Non-planar deflection state of the beam with unequal-legs cross section L 15x10x1.5

In this case the bending moment has to act in $\bar{z}$ -axis, Fig. 3. The final deflection state of the beam is in this case out-of- plane.

According to Fig. 3 and by using the equations (2), (5), (6) and expression:

$$y_0=\frac{\sigma_0}{E}\left(\frac{r_1}{\cos(\alpha+\psi)}+(y_t+z_t tg(\alpha+\psi))\cos(\alpha+\psi)+y_s\right) \qquad \ldots(7)$$

in this case we get the following results:

$$\psi=10.97^\circ, y_s=0.0069, y_0=0.327, R=118.1mm$$

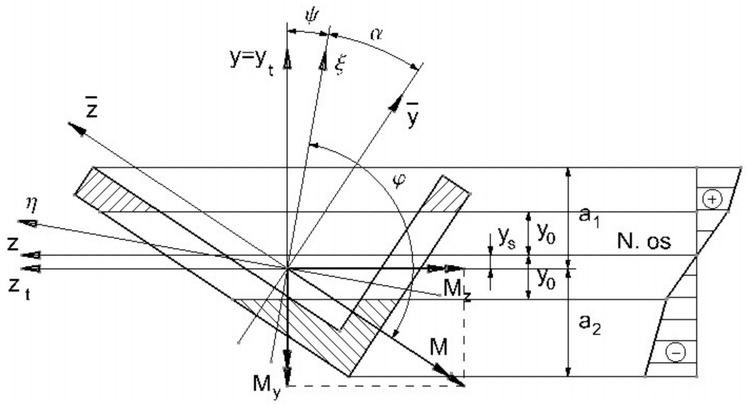

Figure 3. Loading state of cross section L 15x10x1.5.

3. COMPARISON BETWEEN ANALYTICAL AND EXPERIMENTAL RESULTS

For both analytical cases, the experiments were made. The beams were bent on a special testing device on a tool with constant radius of curvature r_1. Special tools were made for each of the examples, Fig.4.

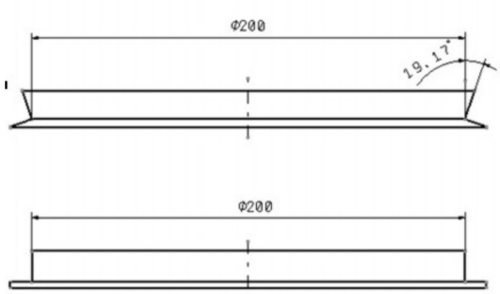

Figure 4. Special tools for bending device.

In the case of the non-planar deflection state in the beam with unequal-legs cross–sections L 15x10x1.5 measurements were made with 3D-measuring device. Cross sections of beams were scanned. The final results of deflection states were obtained by using the CAD software CATIA V5. The comparison between the analytical and experimental determined results is shown in Table 1.

Table 1. Analytically and experimentally determined radii of curvature R.

Shape of cross section	**R [mm]**	**Re [mm]**	**ε [%]**
L 15x10x1.5 (planar deflection state)	119.07	120.34	1.07
L 15x10x1.5 (non-planar deflection state)	118.1	120.02	1.65

4. CONCLUSIONS

From the comparison between the analytical and experimental results, Table 1, one can see that the maximal error is 1.65 % for the case of non-planar deflection state of the beam with unequal legs cross–sections L 15x10x1.5.

5. REFERENCES

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