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DETERMINATION OF STRESSES AT THE BLOCK WITH TWO CYLINDRICAL CANALS IN “T” INTERSECTION

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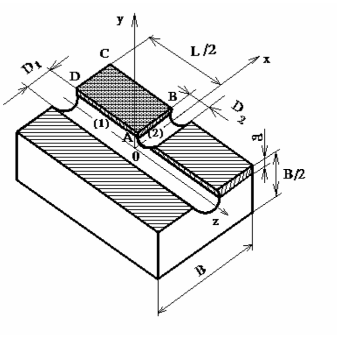
ABSTRACT
The work presents the analytical determination of stresses and deformations in a prismatic block of finite dimensions which has inside two cylindrical canals that form a “T” crossing ran by an under pressure fluid. .
Keywords: hydraulic blocks, thick walls tubes.

1. INTRODUCTION
The modular hydraulic blocks have a wide spread in hydrostatic operated installations. A fundamental characteristic of these components refers to the interior cylindrical canals whose axes cross under a 90° angle, offering thus many of circulation for the under pressure fluid. Among the very many constructive solution we chose the case of block which has inside two cylindrical canals forming a “T” crossing. The dimensional characteristics of the two canals and the block are shown in figure1.

2. CALCULATION HYPOTHESES
The stresses and deformations have been calculated for a “g” band, small thickness, figure 1, which is to be found across the symmetrical longitudinal plane of the block. We used the overlapping of effects device for two tubes having thick walls, corresponding to the two canals (having D_1 and D_2 as diameters D_2).

It follows: - tube I - which has the interior diameter D_1 and the exterior diameter B; - tube II - which has the interior diameter D_2 and the exterior diameter L. For the thick walled tube I, the dimensional factor $h_1 = \frac{B}{D_1}$ has values ranging within the interval $h_1 \in (2.5,...6)$ and for the thick walled tube II, the dimensional factor $h_2 = \frac{L}{D_2}$ has values respecting the condition $h_2 > 6$. For the “g” band in the case of tube II we used Lamé’s relations for the calculation of stresses and deformations. For the band having “g” thickness in the case of tube I we adopted the following calculation hypothesis:



Figur. 1. Modular hydraulic blocks

- we considered the area corresponding to the diameter D_2 as being the area of a ring with the interior diameter D_n and length $L_n = D_2$ fit without construction in tube I, figure 2.a. By eliminating the interior ring we obtain a thick walled tube having a constant diameter but with a pressure variation in the area of the ring which will be:

$$\Delta p_i = p_i - \sigma_r \tag{1}$$

where: - p_i is interior pressure of the tube; - σ_r is radial stress in the tube having the interior diameter D_n and the external one B when radius $r = \dfrac{D_1}{2}$.

If the interior diameter of the ring grows up to the value $D_n = B$ we obtain a gap corresponding to D_2 of the canal (2), figure 2b.

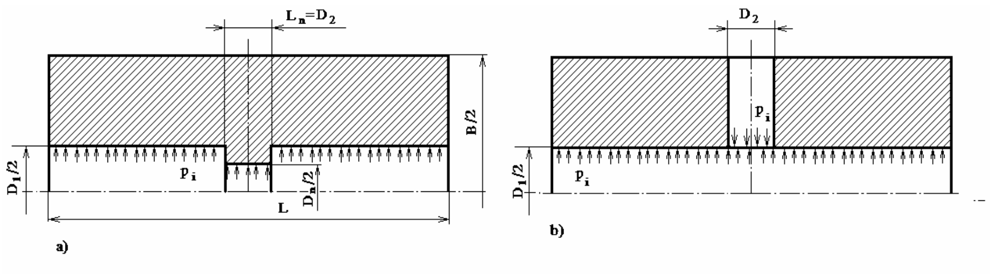


Figure 2. Thick walled tube

Taking into consideration [1,2] for the determination of the radial move we can obtain Lamé's relation for the thick-walled tube, as follows:

$$u(r,z) = A(z)\cdot r + \frac{B(z)}{r} \tag{2}$$

where: $A(z)$, $B(z)$ are unknown function which are to be determined.

If the adimensional coordinates $(r_e = \dfrac{B}{2},\ \zeta = \dfrac{z}{r_e},\ \rho = \dfrac{r}{r_e})$ are used "eqn"(2) becomes:

$$u(\zeta,\rho) = A(\zeta)\cdot \rho + \frac{B(\zeta)}{\rho} \tag{2a}$$

The w move toward the Oz axis can be determined from the condition that the glidings $\gamma_{rz} = 0$.

From:

$$\gamma_{rz} = \frac{1}{r_e}\cdot\left(\frac{\partial u}{\partial \xi} + \frac{\partial w}{\partial \rho}\right) = 0 \tag{3}$$

it results:

$$w = -\int\left(\frac{\partial u}{\partial \xi}\right)\cdot d\rho = -\int\left(A'(\xi)\cdot\rho + B'(\xi)\cdot\frac{1}{\rho}\right)\cdot d\rho = = -A'(\xi)\cdot\frac{\rho^2}{2} - B'(\xi)\cdot ln\,\rho + C(\xi) \tag{4}$$

where $C(\zeta)$ is a new unknown function. Function $C(\zeta)$ can be determined provided that the axial force should be zero. The stresses σ_z can be determined as follows:

$$\sigma_z = \frac{E}{(1+\mu)\cdot(1-2\mu)}\left[(1-\mu)\varepsilon_z + \mu(\varepsilon_r + \varepsilon_\theta)\right] \tag{5}$$

where: - ε_r is radial strain; - ε_z is axial strain; - ε_θ is circumferential strain; - μ is Poisson constant; - E is modulus of Young.
The axial force N_z can also be determined:

$$N = \int_{r_i}^{r_e} 2\pi \cdot r \cdot \sigma_z \cdot dr = 2\pi \cdot r_e^2 \cdot \int_k^1 \sigma_z \cdot \rho \cdot d\rho = 0 \tag{6}$$

where: $k = \frac{r_i}{r_e}$, $(r_i = \frac{D_1}{2})$.
For C' we have:

$$C' = -\frac{\mu}{(1-\mu)} \cdot 2 \cdot A + A'' \cdot \frac{1+k^2}{4} + B'' \cdot \left(-\frac{1}{2} - \frac{k^2 \cdot \ln k}{1-k^2} \right) \tag{7}$$

By using C' value we can determine the strain ε_z.
Knowing the values:

$$\varepsilon_r = \frac{1}{r_e} \cdot \frac{\partial u}{\partial \rho}, \qquad \varepsilon_\theta = \frac{1}{r_e} \cdot \frac{u}{\rho}, \qquad \varepsilon_z = \frac{1}{r_e} \cdot \frac{\partial w}{\partial \zeta} \tag{8}$$

we can determine the total energy of the system which is equal to the sum between the interior potential energy of deformation and the exterior forces potential.

$$E_t = U + U_f \tag{9}$$

The potential energy of deformation is:

$$U = \int_0^l \int_{r_i}^{r_e} 2\pi \cdot r \cdot W \cdot dr \cdot dz = 2\pi \cdot r_e^3 \int_0^l \int_{r_i}^{r_e} W \cdot \rho \cdot d\rho \cdot d\xi \tag{10,a}$$

where the specific energy W can be expressed by the help of relation:

$$W = \frac{E}{2 \cdot (1+\mu)} \cdot \left[\frac{1-\mu}{1+2\mu} (\varepsilon_r + \varepsilon_\theta + \varepsilon_z)^2 - 2 \cdot (\varepsilon_\theta \cdot \varepsilon_r + \varepsilon_\theta \cdot \varepsilon_z + \varepsilon_r \cdot \varepsilon_z) \right] \tag{10,b}$$

The pressure force potential is:

$$U_f = 2\pi \int_0^l (-r_i \cdot p_i \cdot u_i) \cdot dz \tag{10,c}$$

By replacing the values of deformations and strains in the "eqn" (9),(10) and integrating them in function of the ρ we can obtain the total energy of the system: On condition that the system's energy should be minimum we obtain thus the solutions for functions $A(\xi)$ and $B(\xi)$.

$$\begin{aligned} A(\xi) &= e^{-\alpha\xi} \cdot (C_1 \cdot \sin \alpha\xi + C_2 \cdot \cos \alpha\xi) + e^{\alpha\xi} \cdot (C_3 \cdot \sin \alpha\xi + C_4 \cdot \cos \alpha\xi) + \\ &+ e^{-\beta\xi} \cdot (C_5 \cdot \sin \beta\xi + C_6 \cdot \cos \beta\xi) + e^{\beta\xi} \cdot (C_7 \cdot \sin \beta\xi + C_8 \cdot \cos \beta\xi) + A^* \\ B(\xi) &= -m \cdot \begin{bmatrix} e^{-\alpha\xi} \cdot (C_1 \cdot \sin\alpha\xi + C_2 \cdot \cos\alpha\xi) + \\ = e^{\alpha\xi} \cdot (C_3 \cdot \sin\alpha\xi + C_4 \cdot \cos\alpha\xi) \end{bmatrix} + n \cdot \begin{bmatrix} e^{-\beta\xi} \cdot (C_5 \cdot \sin\beta\xi + C_6 \cdot \cos\beta\xi) + \\ + e^{\beta\xi} \cdot (C_7 \cdot \sin\beta\xi + C_8 \cdot \cos\beta\xi) \end{bmatrix} + B^* \end{aligned} \tag{11}$$

3. CALCULATION OF STRESSES AND DEFORMATIONS

The values α ,β, *m, n* depend on the *k* ratio and of Poisson's coefficient. The constants $C_1 \ldots C_8$ can be determined from the limit conditions. Stresses σ_z, σ_r, σ_θ can be determined by using the relations:

$$\sigma_z = \frac{1}{r_e} \cdot \frac{E \cdot (1-\mu)}{(1+\mu)\cdot(1-2\mu)} \cdot \cdot \left[A''(\xi) \cdot \left(\frac{1+k^2}{4} - \frac{\rho^2}{2} \right) - B''(\xi) \cdot \left(\ln \rho + \frac{k^2 \cdot \ln k}{1-k^2} + \frac{1}{2} \right) \right] \tag{12.a}$$

$$\sigma_r = \frac{1}{r_e} \cdot \frac{E \cdot (1-\mu)}{(1+\mu)\cdot(1-2\mu)} \cdot \left[A''(\xi) \cdot \left(\frac{1+k^2}{4} - \frac{\rho^2}{2} \right) - B''(\xi) \cdot \left(\ln \rho + \frac{k^2 \cdot \ln k}{1-k^2} + \frac{1}{2} \right) \right] + \frac{E}{r_2} \cdot \left[\frac{A(\xi)}{1-\mu} - \frac{B(\xi)}{1+\mu} \cdot \frac{1}{\rho^2} \right] \tag{12,b}$$

$$\sigma_\theta = \frac{1}{r_e} \cdot \frac{E \cdot (1-\mu)}{(1+\mu)\cdot(1-2\mu)} \cdot \cdot \left[A''(\xi) \cdot \left(\frac{1+k^2}{4} - \frac{\rho^2}{2} \right) - B''(\xi) \cdot \left(\ln \rho + \frac{k^2 \cdot \ln k}{1-k^2} + \frac{1}{2} \right) \right] + \frac{E}{r_2} \cdot \left[\frac{A(\xi)}{1-\mu} + \frac{B(\xi)}{1+\mu} \cdot \frac{1}{\rho^2} \right] \tag{12,c}$$

For the tube II the stresses $\sigma_z=0$ and σ_r, σ_θ stresses can be determined by using Lamé's relations. Taking into consideration the geometrical characteristics of the crossing we took into consideration only the segment limited by the *A, B, C, D* points, fig.1. by attaching a rectangular orthogonal system of axes, having the origin in the crossing point of the two canals (as shown in figure 1) we will have:

$$\sigma_x = -\sigma_{r1}, \quad \sigma_y = \sigma_{\theta 1} + \sigma_{\theta 2}, \quad \sigma_z = -\sigma_{r2} \tag{13}$$

where: $-\ \sigma_{r1}, \sigma_{\theta 1}$, are the stresses of tube I; - $\sigma_{r2}, \sigma_{\theta 2}$, are the stresses of tube II.

Knowing the values of the stresses corresponding to the three directions for a point in plane *ABCD*, having the coordinates *(x, z)* we can determine the equivalent stresses by using Tresca's criterion.

$$(\sigma_e)_{x,z} = max(\sigma_x, \sigma_y, \sigma_z) - min(\sigma_x, \sigma_y, \sigma_z) \tag{14}$$

4. RESULTS AND CONCLUSIONS

The stresses and deformations have been calculated considering the fact that at the interior of the two canals there is a fluid under a pressure of $p_i=1$ *[MPa]*.

Figures 3,4 show the stresses for the case when the dimensional factor is $h_1=2.5$ and *3*.

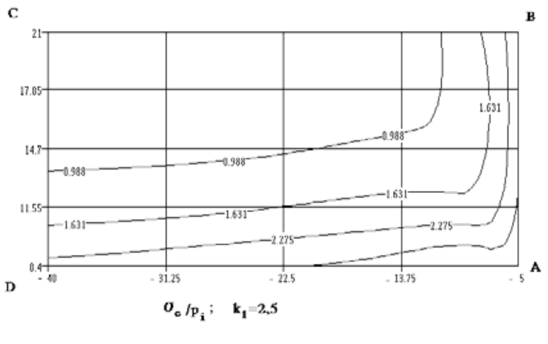


Figure 3 Stresses when the dimensional factor is $h_1=2.5$

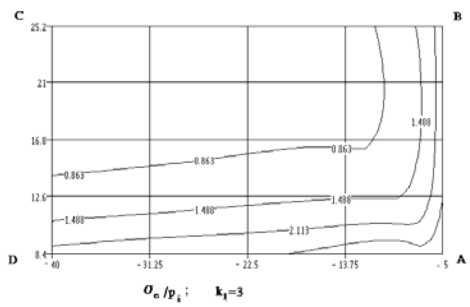


Figure 4 Stresses when the dimensional factor is $h_1= 3$

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