INDUSTRIAL ULTRASONIC HORNS OPTIMISATION

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ABSTRACT
In this paper it is presented a simulation method for pulse propagation in a media with periodical nonhomognities. There are described the mathematical concept, the solutions of the wave propagation and the results of simulation. This method can be used for industrial horn analysis.
Keywords: pulse propagation, modeling.

1. THEORETICAL CONSIDERATIONS
The progressive waves propagate with a constant velocity only if the properties, geometrical and physical, of the media are constant [2]. In case of an ultrasonic horn the cross section has a continuous variation along the its longitudinal axe. The general shape of a progressive wave is:

$$u(x,t)=u(x)\cdot e^{i(\omega t-kx)}, \tag{1}$$

where
$$\varphi(x,t)=\omega t-kx, \tag{2}$$

is defined as phase function of a progressive sinusoidal wave that propagates in the positive sense of longitudinal axe.
An analysing of wave propagation in nonhomogeneous media can cover the following aspects: a variation of the Young modulus E, a variation of the material density ρ, and a variation of the cross section S [1] and [3]. From all three media nonhomogenities the first two are more or less theoretical and the third is a practical one having the possibility to be checked by a set-up.
It was considered the case of a cross section variation with o periodicity of $500\,mm$ for a horn, considered as a beam with the length of $250\,mm$.
The equation that describes the wave propagation is given y:

$$f(x)\frac{\partial^2 u}{\partial t^2}=\frac{\partial}{\partial x}\left(g(x)\frac{\partial u}{\partial x}\right), \tag{3}$$

.where $f(x)$ and $g(x)$ are two functions that describe the nonhomogenities of the propagation media:

$$\begin{cases} f(x)=\rho(x)S(x), \\ g(x)=E(x)S(x). \end{cases} \tag{4}$$

There were introduced the following assumptions:

a) The solution of the wave propagation (3) admits Fourier transform. This yields to:

$$u(x,t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\tilde{u}(x,\omega)e^{-j\omega t}d\omega\,, \tag{5}$$

b) The media nonhomogenities are very small comparing with a homogeneous one and these can be described by the following functions:

$$\begin{cases} f(x)=f_0\left[1+\eta_1 p(x)\right]; \\ g(x)=g_0\left[1+\eta_2 q(x)\right], \end{cases} \tag{6}$$

where functions $p(x)$ and $q(x)$ are considered to carry out the needed conditions to be written in a Taylor expansion shape, in a point x_0:

$$\begin{cases} p(x)=\sum_{n=0}^{\infty}\frac{p^{(n)}(x_0)}{n!}(x-x_0)^n=\sum_{n=0}^{\infty}a_n(x-x_0)^n\,, \\ q(x)=\sum_{n=0}^{\infty}\frac{q^{(n)}(x_0)}{n!}(x-x_0)^n=\sum_{n=0}^{\infty}b_n(x-x_0)^n\,, \end{cases} \tag{7}$$

where

$$\begin{cases} p^{(n)}(x_0)=\left.\frac{\partial^n p}{\partial x^n}\right|_{x=x_0}, \\ q^{(n)}(x_0)=\left.\frac{\partial^n q}{\partial x^n}\right|_{x=x_0}. \end{cases} \tag{8}$$

Introducing the solution (5) in wave equation (3) results:

$$\frac{\partial}{\partial x}\left[g(x)\frac{\partial\tilde{u}(x,\omega)}{\partial x}\right]+\omega^2 f(x)\tilde{u}(x,\omega)=0\,, \tag{9}$$

or:

$$g(x)\bar{u}''+g'(x)\bar{u}'+\omega^2 f(x)\bar{u}=0\,. \tag{10}$$

The solution $\tilde{u}(x,\omega)$ can be written as a Fourier expansion:

$$\tilde{u}(x,\omega)=\sum_{n=0}^{\infty}\frac{\tilde{u}^{(n)}(x_0)}{n!}(x-x_0)^n=\sum_{n=0}^{\infty}c_n(\omega)(x-x_0)^n\,. \tag{11}$$

In case of the ultrasonic horns, with a variable cross section, there is a point where the section is maximum in one point x_0, and it can be written the following relationship:

$$\left.\frac{d\left[g(x)\right]}{dx}\right|_{x=x_0}=0\,. \tag{12}$$

It is considered o solution $\bar{u}_0(x,\omega)$ in the neighbourhood of a point x_0. Then, the equation (9) becomes:

$$g_0\left[1+\eta_2 q(x_0)\right]\bar{u}_0''+\omega^2 f_0\left[1+\eta_1 p(x_0)\right]\bar{u}_0=0\,. \tag{13}$$

Dividing the above relationship by $g_0[1+\eta_2 q(x_0)]\neq 0$ and taking considering the phase velocity of the waves in an homogeneous media (described by the constants f_0 and g_0):

$$c=\sqrt{\frac{g_0}{f_0}}\ , \tag{14}$$

it is obtained:

$$\bar{u}_0''+\left(\frac{\omega}{v_0}\right)^2 \bar{u}_0=0\ , \tag{15}$$

where:

$$v_0=c\sqrt{\frac{1+\eta_2 q(x_0)}{1+\eta_1 p(x_0)}}\ , \tag{16}$$

and v_0 represents the new value of the propagation velocity of a wave that takes into consideration the nonhomogeneities of the media.

The solutions of the wave equation (13) are:

$$\bar{u}_0(x,\omega)=\bar{u}_0^{(+)}+\bar{u}_0^{(-)}=c_0^{(+)}e^{j\frac{\omega}{v_0}x}+c_0^{(-)}e^{-j\frac{\omega}{v_0}x}\ , \tag{17}$$

where $\tilde{u}_0^{(+)}$ represents the progressive component and $\bar{u}_0^{(-)}$ is the regressive one.

2. ACOUSTIC PULSE PROPAGATION IN A MEDIA WITH PERIODIC VARIABLE PROPERTIES

It is considered a media with a periodic variation of the properties λ_p described by:

$$\begin{cases} f(x+n\lambda_p)=f(x), \\ g(x+n\lambda_p)=g(x), \\ g'(x_0)=0. \end{cases} \tag{18}$$

The wave propagation impose the previous analysis of some particularities of the model:

a) Considering the variable change $x\to x+\lambda_p$, the propagation equation (9) becomes:

$$\frac{\partial}{\partial x}\left[g(x+\lambda_p)\frac{\partial \bar{u}(x+\lambda_p,\omega)}{\partial x}\right]+\omega^2 f(x+\lambda_p)\bar{u}(x+\lambda_p,\omega)=0\ , \tag{19}$$

with the following solutions dependence:

$$\begin{cases} \bar{u}(x+\lambda_p,\omega)=C\bar{u}(x,\omega), \\ |C|=1. \end{cases} \tag{20}$$

The condition $|C|=1$ is necessary on a hand for having limited oscillations and, on the other hand it is considered that the media is a conservative one.

b) Based on the above considerations, there must be considered the following periodical functions, with a period of λ_p:

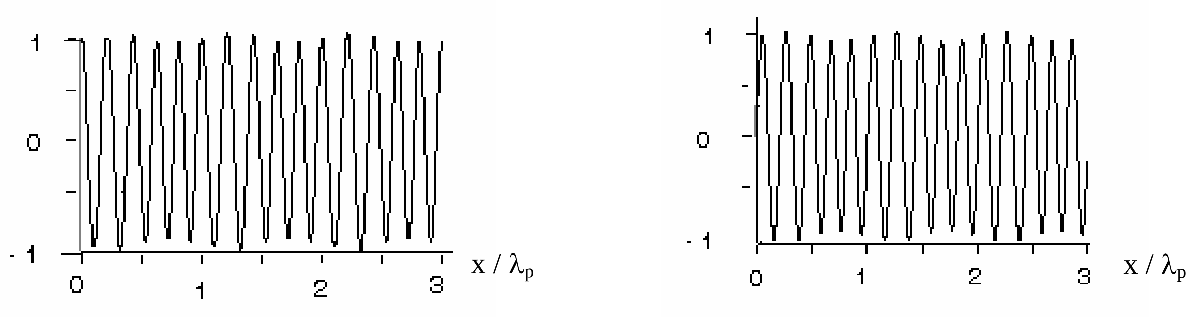
$$\bar{u}(x+n\lambda_p,\omega)=\bar{u}(x,\omega)\ . \tag{21}$$

c) The necessity to carry out the condition: $\bar{u}(x_0+\lambda_p,\omega)=\bar{u}(x_0,\omega)$,. (22)

d) The main components of the wave have the following shapes:

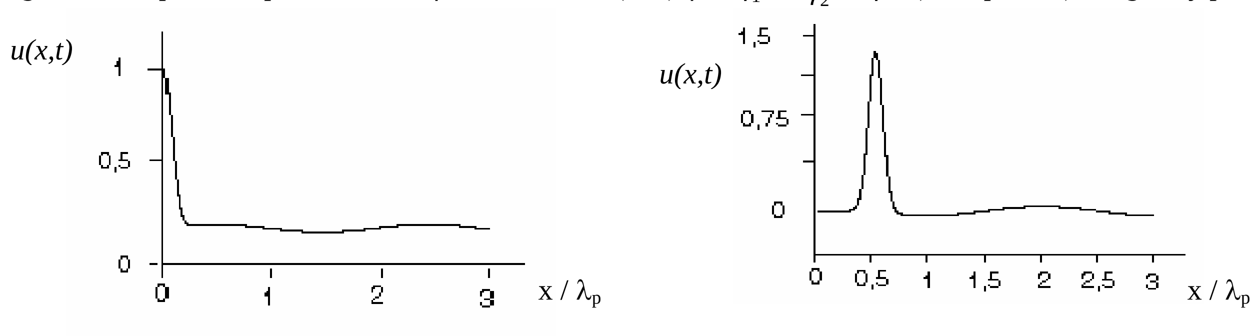
$$\begin{cases} A\left(x+\lambda_p,\omega\right)=A\left(x\right), \\ \varphi\left(x+\lambda_p,\omega\right)=\varphi\left(x\right), \\ k(x+\lambda_p,\omega)=k(x), \\ v_\varphi(x+\lambda_p,\omega)=v_\varphi(x). \end{cases} \tag{4.120}$$

The simulations were done in Labview and are presented in the next plots.



a) b)

Figure 1. Graphical representation of the solution $\bar{u}(x,\omega)$ for η_1 = η_2 = η : a)real part; b) imaginary part



a) b)

Figure 2 Gaussian pulse for η_1 = η_2 =0,2 at the horn beginning (a), and at its end (b).

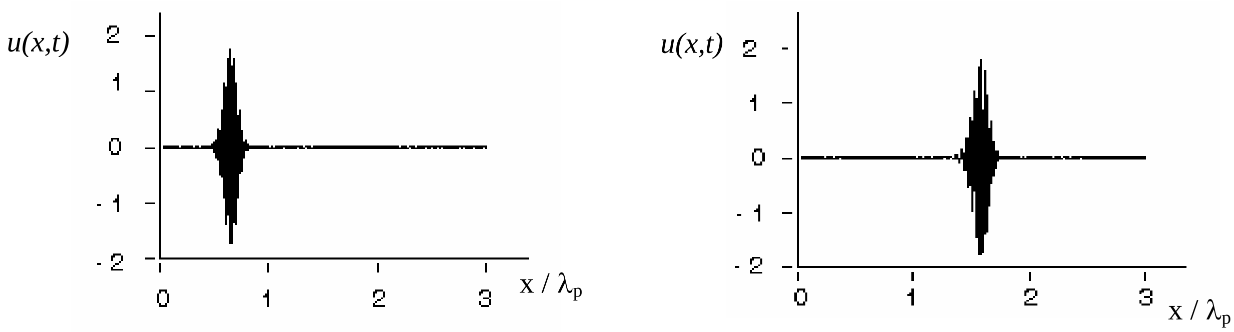


Figure 3. The Gaussian pulse evolution with initial conditions.

3. REFERENCES

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