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THE DISTRIBUTION OF THE EQUIVALENT STRESSES IN THE DANGEROUS SECTION OF THE STRENGHT ELEMENTS MADE BY U - PROFILE WITH THIN WALLS

Galaftion Sofonea
"Lucian Blaga" University of Sibiu,
"Hermann Oberth" School of Eneneering
Str. Emil Cioran nr. 4, 550025 - Sibiu
Romania

ABSTRACT
The worth presents the research on the optimization of the material distribution on the thin wall sections made by the U profile taking into consideration the sections dimensions, by maintaining the same area. The variation diagrams of the sections geometrical characteristics are shown below as well as the distribution of the equivalent stresses in the dangerous segment taking into consideration also the lundrauce twist.
Keywords*: optimization, thin wall sections, equivalent stresses, lundrauce twist.*

1. NOTES AND TERMINOLOGY

- b – section`s width, $[mm]$;
- h – section`s height, $[mm]$;
- $l.\ l_1, l_2$ -rad`s length, $[mm]$;
- t – the wall`s thickness, $[mm]$;
- $k^2 = \dfrac{G \cdot I_t}{E \cdot I_\omega}, [mm^2]$;
- I_t- the geometrical characteristic of pure twist rigidity, $[mm^4]$;
- I_z - axial inertia moment compared to the Oz axis, $[mm^4]$;
- I_ω- sector inertia moment, $[mm^6]$;
- S_ω- static sector moment, $[mm^4]$.
- B_ω - bimoment, $[N \cdot mm^2]$;
- N – axial force, $[N]$;
- M_z, M_y- bending moments, $[N \cdot mm]$;
- M_t - pure twisting moment, $[N \cdot mm]$;
- M_ω - twisting-bending moment, $[N \cdot mm]$;
-E, G - longitudinal, transversal module, $[MPa]$;
- T_y, T_z - cutting forces, $[N]$;
- σ, τ - normal, tangential stresses, $[MPa]$;
- ω - sector coordinate, $[mm^2]$.

2. GENERAL CONSIDERATION

The mathematical solution to this optimization problem would be the most indicated way, offering remarkable advantages, but such an approach is not always accessible in the case of strength structures because it must take into consideration a number of factors, objective and subjective ones.

Not being possible so far to make a general optimization to lead to a general solution capable to satisfy the multitude of aspects and existent request, the work refers only to the optimization of the material distribution on the section

3. THE VARIATION OF THE GEOMETRICAL CHARACTERISTICS OF THIN –WALLED SECTIONS MADE BY THE U PROFILE

Using U profile sections with thin walls in building the strength elements having the form and dimensions as shown in figure1 means to make use of the calculating relations for determining the geometrical characteristics of the sections:

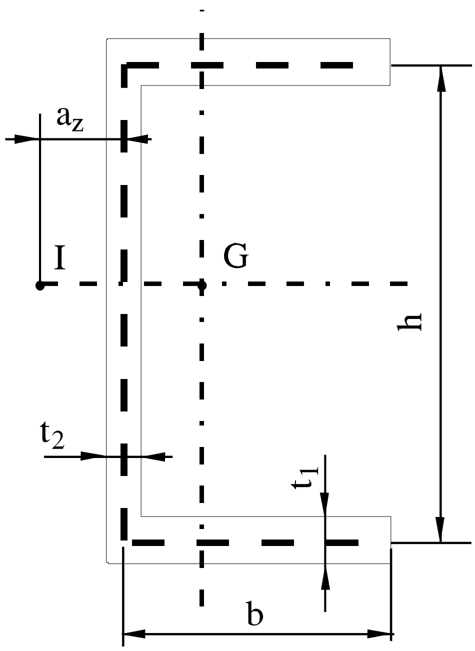

Figure 1. The form and dimension of the section

$$a_z = \frac{b^2 \cdot t_1}{2 \cdot b \cdot t_1 + \frac{h \cdot t_2}{3}}, \qquad (1)$$

$$I_z = \frac{\left(b + \frac{t_2}{2}\right) \cdot (h + t_1)^3 - \left(b - \frac{t_2}{2}\right) \cdot (h - t_1)^3}{12}; \qquad (2)$$

$$I_t = \frac{1}{3} \cdot \left(2 \cdot b \cdot t_1^3 + h \cdot t_2^3\right); \qquad (3)$$

$$I_\omega = \frac{1}{6} \cdot (b - 3 \cdot a_z) \cdot h^2 \cdot b^2 \cdot t_1 + a_z^2 \cdot \left(\frac{b \cdot h^2 \cdot t_1}{2} + \frac{h^3 \cdot t_2}{12}\right) \qquad (4)$$

$$S_{\omega \max} = \frac{h \cdot (b - a_z)^2}{4} \cdot t_1 . \qquad (5)$$

The variation of these characteristics for $A = 1800\ mm^2$, constant, has been used in analyzing some variants of sections, as a result of the thin strip width used. The visualization of these variations is shown in figure2.

As compared to the theoretical solutions obtained and shown in the above pictures we have to impose also the local stability conditions of the sole and the heart [4]:

$$\frac{b}{t_1} \leq k_1, \text{ where } k_1 = 13. \qquad (6)$$

Condition 6 imposes the limitation of the maximum thickness $b_{\max}$ of the section in the cases of figure 2 as of $b \leq 104\ mm$. This is shown on diagrams through shaded rectangles.

4. VARIATION OF EQUIVALENT STRESSES {N THE DANGEROUS SECTION

The calculation for the metal bars made from thin-walled profiles, taking into consideration the hindrance twist, out of exterior motives or as a result of various hindered deplanes across the metal bar are:

$$\sigma = \frac{M_z \cdot I_y + M_y \cdot I_{zy}}{I_y \cdot I_z - I_{zy}^2} \cdot y - \frac{M_y \cdot I_z + M_z \cdot I_{zy}}{I_z \cdot I_y - I_{zy}^2} \cdot z + \frac{N}{A} + \frac{B_\omega}{I_\omega} \cdot \omega; \qquad (7)$$

$$\tau = \frac{T_y \cdot I_y - T_z \cdot I_{zy}}{b \cdot \left(I_y \cdot I_z - I_{zy}^2\right)} \cdot S_z + \frac{T_z \cdot I_z - T_y \cdot I_{zy}}{h \cdot \left(I_y \cdot I_z - I_{zy}^2\right)} \cdot S_y + \frac{M_t}{I_{td}} \cdot t + \frac{M_\omega}{t \cdot I_\omega} \cdot S_\omega, \qquad (8)$$

where the last terms from each relation represents the values of the normal stress, σ, respectively the values of the tangential stresses, τ, produced by the twisted hindrance.

In the case of thin-walled metal bar (figure 3), required by an eccentric concentrated force as compared to the metal bar's axis, the static $(M_t, M_\omega, B_\omega)$ and geometrical (φ, φ') sizes are:

- *on the interval* $0 \le x \le t_p$:

$$\begin{Bmatrix} \varphi(x) \\ \varphi^I(x) \\ \frac{B(x)}{G \cdot I_t} \\ \frac{M(x)}{G \cdot I_t} \end{Bmatrix} = \begin{bmatrix} 1 & \frac{1}{k} \cdot shkx & 1 - chkx & x - \frac{1}{k} \cdot shkx \\ 0 & chkx & -k \cdot shkx & 1 - chkx \\ 0 & -\frac{1}{k} \cdot shkx & chkx & \frac{1}{k} \cdot shkx \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{Bmatrix} \varphi_0 \\ \varphi_0^I \\ \frac{B_0}{G \cdot I_t} \\ \frac{M_{x0}}{G \cdot I_t} \end{Bmatrix}, \tag{9}$$

- on the interval: $t_p \le x \le l$:

$$\begin{Bmatrix} \varphi(x) \\ \varphi^I(x) \\ \frac{B(x)}{G \cdot I_t} \\ \frac{M(x)}{G \cdot I_t} \end{Bmatrix} = \begin{bmatrix} 1 & \frac{1}{k} \cdot shkx & 1 - chkx & x - \frac{1}{k} \cdot shkx & \frac{1}{k} \cdot shk(x - t_p) - (x - t_p) \\ 0 & chkx & -k \cdot shkx & 1 - chkx & chk(x - t_p) - 1 \\ 0 & -\frac{1}{k} \cdot shkx & chkx & \frac{1}{k} \cdot shkx & -\frac{1}{k} \cdot shk(x - t_p) \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix} \begin{Bmatrix} \varphi_0 \\ \varphi_0^I \\ \frac{B_0}{GI_t} \\ \frac{M_{x_0}}{GI_t} \\ \frac{Pe}{GI_t} \end{Bmatrix} \tag{10}$$

The resistance condition imposed according to the theory of form variation energy will be:

$$\sigma_{ech} = \sqrt{\sigma^2 + \tau^2} \le \sigma_a \ . \tag{11}$$

The drawn out program allows the dimensioning through successive trials, so that conditions (11) could be deserved as well as the drawing of the equivalent stresses in the most important points of the section for A = constant when width b and h vary, maintaining a constant area (figure 2.).

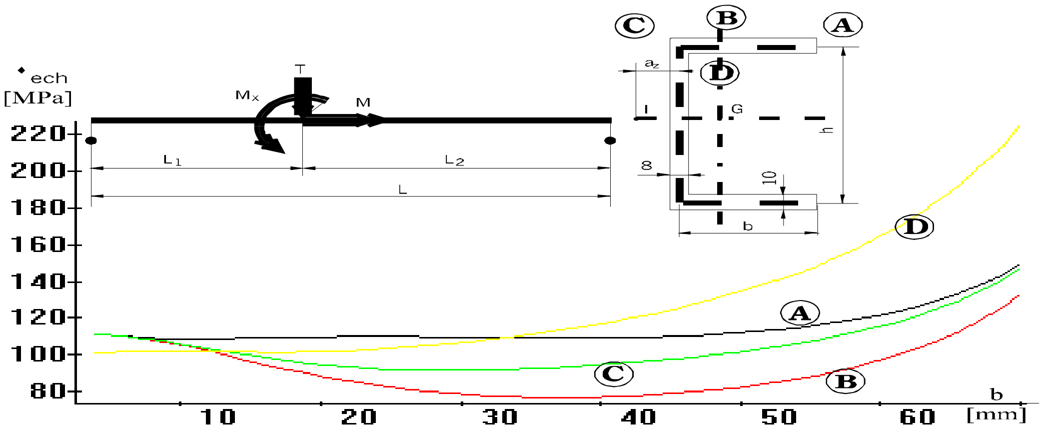

Figure 2. The variation of the equivalent stresses on the dangerous section for A = 1800 mm^2*,* $t_1 = 10\ mm,\ t_2 = 8\ mm$.

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As compared to the theoretical solutions obtained and shown in the above figures we have to impose also the local stability conditions of the heart. Condition (6) imposes the limitation of the maximum height of the section in the cases shown in figure9 at $b \leq 104 \ mm$.

4. CONCLUSIONS

The results of the research lead to the visualization of the geometrical characteristics variation so as to lead to choosing the best thickness of the walls and depending the best ratio between width b and height h. The research allows the establishing of the best form the bar of the section needs to have in order to efficiently use the material. On the basis of these determinations we can obtain up to 9,7% material economy as compared to the sections used. It has been noticed that the bars analyzed are not in danger of losing their general stability.
The calculation program allows the solving of other loading-leaning of the bars state. The method presented in this study care be extended to other forms of the bars sections with thin walls.

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