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QUATERNION BASED KINEMATIC ANALYSIS FOR INDUSTRIAL ROBOT MANIPULATORS

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ABSTRACT
Formulating the suitable mathematical models and deriving the efficient algorithms for a robot kinematics mechanism are very crucial for analyzing the behavior of industrial robot manipulators. Direct kinematics that determines the Cartesian position and orientation of the end-effector for the specified values of joint parameters is generally performed using homogenous transforms. In this paper, quaternion-vector pairs are used as an alternative method for driving the direct kinematics of industrial robot manipulators equipped with Euler wrist to have 6-DOF robot manipulators. The successive screw displacements in this method provide a very compact formulation for the kinematic equations and also reduce the number of equations obtained in each goal position, according to the matrix counterparts. The kinematics model of CC cylindrical robot manipulator is given as example for illustrating the compactness of the method.
Keywords: Quaternions, forward kinematics, industrial robots.

1. INTRODUCTION
Robotics requires appropriate tools for manipulating 3-dimensional objects. Generally, homogenous transformations based on 4x4 real matrices are used for representing the spatial transformations of point vectors. Although such matrices are implemented to the robot kinematics readily, they include in redundant elements (such matrices are composed of 16 elements of which four are completely trivial) that cause numerical problems in robot kinematics and also increase cost of algorithms [1].

Although quaternions are suitable for the robot kinematics, they have not been used extensively by the robotics community. Dual quaternions can present rotation and translation in a compact form of transformation vector, simultaneously. While the orientation of a body is represented with nine elements in homogenous transformation, the dual quaternions reduce the number of elements to four. It offers considerable advantage in terms of computational robustness and storage efficiency for dealing with the kinematics of robot chains [1].

Quaternions were first introduced by an Irish mathematician William Rowan Hamilton [2] whose goal is to extend the notion of complex numbers represented as algebraic pairs to triplets [3]. So far, quaternions have been used by authors in many applications, such as, classical mechanics, quantum mechanics, aerospace and geometric analysis. Salamin [4] gave a brief analysis of advantageous of quaternions and matrices as rotational operators. General properties of quaternions as rotational operators were studied by Pervin and Webb [5] who also gave quaternion formulation of moving geometric objects. Gu and Luh [6] used quaternions for computing the Jacobians for robot kinematics and dynamics. Kim and Kumar [7] used quaternions for the solution of direct and inverse kinematics of six degrees of freedom robot manipulator.

In this paper, quaternion vector pairs are used as an alternative method for driving the direct kinematics of fundamental robot manipulators classified by Huang and Milenkovic [8]. The quaternion formulation and kinematics identification for the robot kinematics are presented. Finally, quaternion based direct kinematics analysis of CC cylindrical robot manipulator is given as example in detail.

2. QUATERNION FORMULATION

A quaternion is a quadrinomial expression, with a real angle θ and an axis of rotation *n = ix + jy + kz*, where *i, j* and *k* are imaginary numbers. A quaternion may be expressed as a quadruple *q = (θ, x, y, z)* or as a scalar and a vector *q = (θ, u)*, where *u= x, y, z*. In this paper a quaternion is denoted as,

$$q = [s, v] = [\cos(\frac{\theta}{2}),\ \sin(\frac{\theta}{2}) < k_x, k_y, k_z >] \tag{1}$$

where $s \in R$ and $v \in R^3$. In equation 1, θ and k, a rotation angle and unit axis, respectively. For a vector r oriented at an angle θ about the vector k, there is a quaternion that represents the orientation. Although unit quaternions are very suitable to express the orientation of a rigid body, they do not contain any information about its position in the 3D space. The way to represent both rotation and translation in a single transformation vector is to use dual quaternions. The point vector transformation using dual quaternions can be given as,

$$Q(q, p) = \left(\left[\cos(\frac{\theta}{2}),\ \sin(\frac{\theta}{2}) < k_x,\ k_y,\ k_z > \right], < p_x,\ p_y,\ p_z > \right) \tag{2}$$

where the unit quaternion $q = [\cos(\theta/2),\ \sin(\theta/2) < k_x,\ k_y,\ k_z >]$ indicates rotation and the vector $p =< p_x,\ p_y,\ p_z >$ encodes the translational displacement. Note that this convention requires only seven elements for representing the whole transformation of a single joint. Quaternion multiplication is vital to combining the rotations. Let, $q_1 = [s_1, v_1]$ and $q_2 = [s_2, v_2]$ denote two unit quaternions. In this case, multiplication process is shown as

$$q_1 * q_2 = [s_1 s_2 - v_1 \cdot v_2,\ \ s_1 v_2 + s_2 v_1 + v_2 \times v_1] \tag{3}$$

where ($\cdot$), ($\times$) and ($*$) are dot product, cross product and quaternion multiplication, respectively. In the same manner, the quaternion multiplication of two point vector transformations is denoted as,

$$Q_1 Q_2 = (q_1, p_1) * (q_2, p_2) = q_1 * q_2,\ \ q_1 * p_2 * q_1^{-1} + p_1 \tag{4}$$

where, $q_1 * p_2 * q_1^{-1} = p_2 + 2s_1(v_1 \times p_2) + 2v_1 \times (v_1 \times p_2)$.

3. KINEMATICS IDENTIFICATION

Based on the quaternion modeling convention, the forward kinematics vector transformation for an open kinematics chain can be derived as follows: Consider a 3 DOF serial robot manipulator having one prismatic and two revolute joints as illustrated in Figure 1a. A coordinate frame is attached to the base of the robot arbitrarily and the z-axis of the frame is assigned for pointing along the rotation axis of first joint. This frame does not move and, is considered as the reference coordinate frame. The position and the orientation vectors of all other joints are assigned in terms of this frame. Since the z-axis of the reference coordinate frame is the unit line vector along the rotation axis of the first joint, the quaternion vector that represents the orientation of the first joint is

$$q_1 = [\cos\overline{\theta}_1,\ \sin\overline{\theta}_1 < 0,0,1 >] \tag{5}$$

where $\overline{\theta}_1 = \theta_1 / 2$. As shown in Figure 1b, the unit line vector of the second joint is the opposite direction of the y-axis of the reference coordinate frame, in this case, the orientation of the second joint is,

$$q_2 = [\cos\overline{\theta}_2,\ \sin\overline{\theta}_2 < 0, -1, 0 >] \tag{6}$$

Because, the third joint is prismatic; there is only a unit identity quaternion that can be denoted as

$$q_3 = [1,\ < 0, 0,\ 0 >] \tag{7}$$

When the first joint is rotated anticlockwise direction around the z axis of reference coordinate frame by an angle of θ_1, the link l_1 traces a circle in the xy-plane which is parallel to the xy plane of the reference coordinate frame as given in Figure 1b. Any point on the circle can be determined by using the vector of $< p_x,\ p_y,\ p_z > = < l_1 \sin\theta_1,\ -l_1 \cos\theta_1,\ h_1 >$ that is the position vector of the first joint. If the second joint is rotated as in Figure 1c, in this case plane of the rotation will be xz-plane relative to the reference coordinate frame. The position vector of the second quaternion can be written as

$$p_2 = < p_x,\ p_y,\ p_z > = < -l_2 \sin\theta_2,\ 0,\ l_2 \cos\theta_2 > \tag{8}$$

Since, the third joint is prismatic; the direction of the translation is along the z-axis of the reference coordinate frame and stated as

$$p_3 = < p_x,\ p_y,\ p_z > = < 0,\ 0,\ d_3 > \tag{9}$$

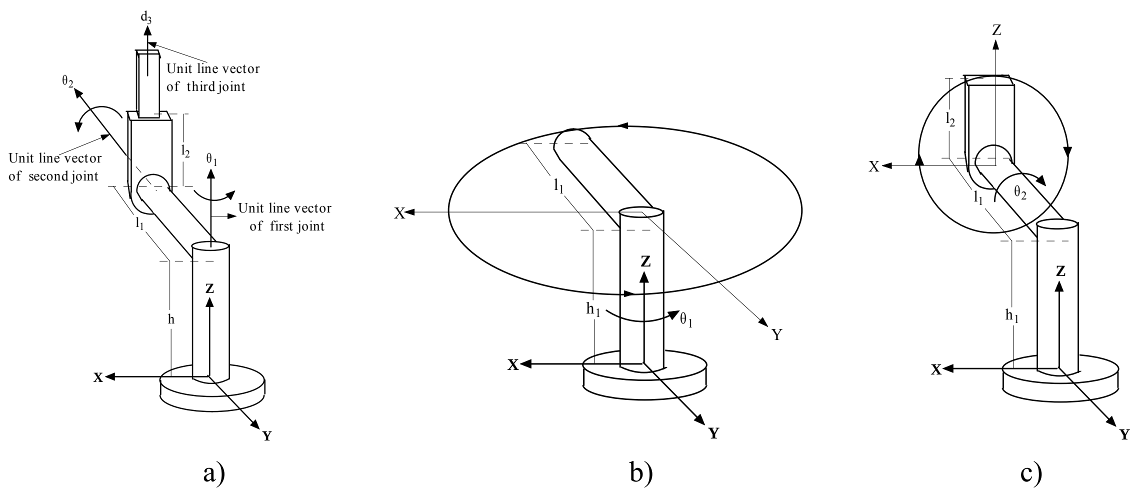

Figure 1 a) 3 DOF serial robot manipulator, b) the link l_1 traces a circle in the xy-plane, c) the link l_2 traces a circle in the xz-plane.

4. EXAMPLE

CC cylindrical robot manipulators with Euler wrist as in Figure 2 is considered as example to illustrate the application of the quaternions to the robot kinematics. θ_1, θ_3, θ_4, θ_5, θ_6 and d_2 are joint parameters for revolute and prismatic joints, respectively and l_1 and l_2 denote the link lengths. For reason of compactness, $\theta_i / 2$, $\sin(\theta_i / 2)$, $\cos(\theta_i / 2)$, $\sin(\theta_i)$, and $\cos(\theta_i)$ are represented as $\overline{\theta}_i$ $\overline{s}_i$, $\overline{c}_i$, s_i and c_i respectively. The kinematics transformations of CC robot manipulator are given as

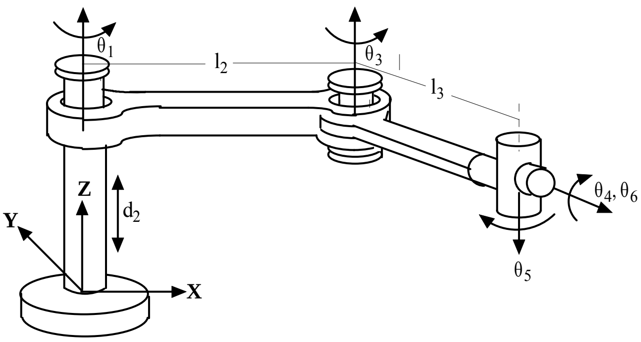

Figure 2. Rigid body model and coordinate frame attachment of the CC robot manipulator.

$$Q_1 = R(z,\theta_1)T(x,\ l_2c_1)T(y,\ l_2s_1) = [\bar{c}_1,\ \bar{s}_1 < 0,0,1 >], < l_2c_1,\ l_2s_1,\ 0 > \tag{10}$$

$$Q_2 = T(z, d_2) = [1, < 0,0,0 >], < 0,0, d_2 > \tag{11}$$

$$Q_3 = R(z,\theta_3)T(x,\ l_3s_3)T(-y,\ l_3c_3) = [\bar{c}_3,\ \bar{s}_3 < 0,0,1 >], < l_3s_3,\ -l_3c_3,\ 0 > \tag{12}$$

$$Q_4 = R(-y,\theta_4) = [\bar{c}_4,\ \bar{s}_4 < 0,-1,0 >], < 0,\ 0,\ 0 > \tag{13}$$

$$Q_5 = R(-z,\theta_5) = [\bar{c}_5,\ \bar{s}_5 < 0,0,-1 >], < 0,0,0 > \tag{14}$$

$$Q_6 = R(-y,\theta_6) = [\bar{c}_6,\ \bar{s}_6 < 0,-1,0 >], < 0,0,0 > \tag{15}$$

The forward kinematics of the CC robot manipulator can be written in terms of $Q(s,\ v,\ p)$ as

$$Q([s,v],p) = ([\bar{c}_4\bar{c}_6\bar{c}_a - \bar{s}_4\bar{s}_6\bar{c}_b, < \bar{c}_4\bar{s}_6\bar{s}_a + \bar{s}_4\bar{c}_6\bar{s}_b,\quad -\bar{c}_4\bar{s}_6\bar{c}_a - \bar{s}_4\bar{c}_6\bar{c}_b,\quad \bar{c}_4\bar{c}_6\bar{s}_a - \bar{s}_4\bar{s}_6\bar{s}_b >],$$
$$< l_3s_{1+3} + l_2c_1,\quad l_2s_1 - l_3c_{1+3},\quad d_2 >) \tag{16}$$

where $a = (\bar{\theta}_1 + \bar{\theta}_3 - \bar{\theta}_5)$, $b = (\bar{\theta}_1 + \bar{\theta}_3 + \bar{\theta}_5)$ and $s_{1+3} = \sin(\theta_1 + \theta_3)$, $c_{1+3} = \cos(\theta_1 + \theta_3)$.

5. CONCLUSION

In this paper, quaternion vector-pairs are presented as an alternative method for driving the direct kinematics of industrial robot manipulators. The compact and simple formulation for the robot kinematics makes the quaternion vector-pairs a very powerful alternative method. The kinematics model of CC robot manipulator is given as example for illustrating elegance of quaternion algebra.

6. REFERENCES

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