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DYNAMICS OF THE MECHANICAL AGGREGATE AT TRANSITIONAL REGIME

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ABSTRACT
In the paper analyses the oscillations in torsion for the adopted dynamic model of the mechanical aggregate presented as a two-mass system at transitional regime. The mathematic model describes torsional oscillations and the electro motor's torque influence too.
The system of nonlinear equations is solved with Runge-Kutta method using Matlab programming.
The concrete conclusions on behaviour of the dynamic system have been reached based on graphically presented results.
Key words: Mechanical Aggregate, Dynamic Absorber, Two-mass System, Transient Regime, Non-linear System

1. INTRODUCTION
Mechanical or machine aggregate as an assembly of driving machine, power transmitters and working machines represents a very complicated electromechanical system. During design process of such a system the characteristics of its elements and as well as coupling characteristics needs to be analysed.
In this paper, dynamics of a mechanical aggregate at transitional regime presented by two masses linked-up by a dynamic absorber is elaborated.
Especially the influence of the dynamic absorber's coefficient in variability of the difference between rotational angle of absorber and absolute rotational angle of the second mass, working machine $(\varphi - \varphi_2)$ and angular velocity difference $(\dot{\varphi} - \dot{\varphi}_2)$ of them was in details analysed.
For the adopted model the static characteristic of electromotor was taken into consideration, but not the dynamic one. Also, it is adopted that moment of working machine changes is sinusoidal function.
The results for the dynamic model of the mechanical aggregate are graphically presented for time interval $t \in [0,5]$ *s*, representing work of such a system at transitional regime.
The conclusions of dynamic response of the adopted dynamic system/model were based on graphical presentation of the results.

2. DYNAMIC AND MATHEMATICAL MODEL OF MECHANICAL AGGREGATE
The dynamic model for a system of *n*-masses is given in *Figure 1.* Such a dynamic model is build based in some basic assumptions: electromotor's moment changes depending on its static characteristics; masses of the shafts are neglected - they act as springs; deformations in torsion are within elasticity limits etc.

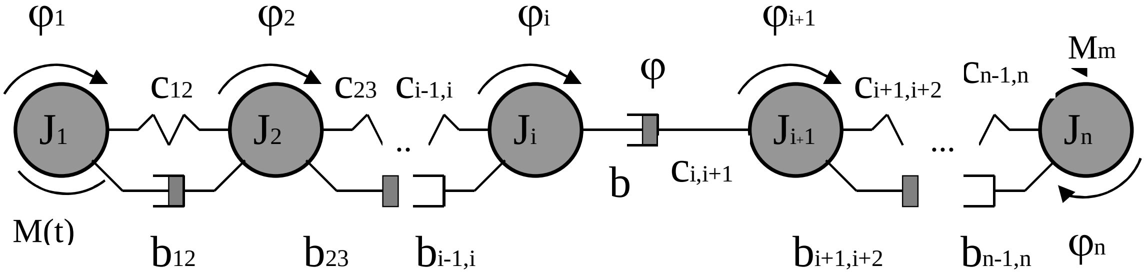

Figure 1. Dynamic model of a system with n-masses

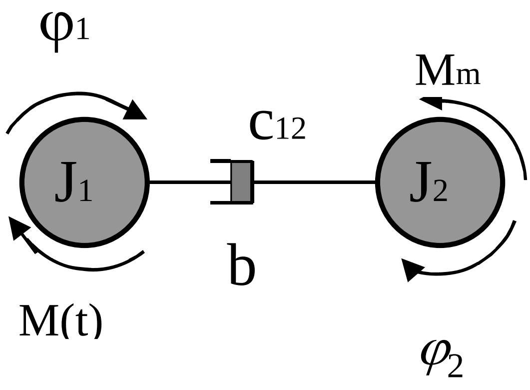

Figure 2.

The dynamic model for the mechanical aggregate subject of analysis in this paper is adopted as a two-mass system linked-up with dynamic absorber.

Where: $\varphi_1, \varphi_2 [rad]$ - absolute rotational angles of masses; $\varphi [rad]$ - dynamic absorber's rotational angle; $\dot{\varphi}_1, \dot{\varphi}_2 [rad/s]$ - angular velocities of masses; $\dot{\varphi} [rad/s]$ - dynamic absorber's angular velocity; $J_1, J_2 [kgm^2]$ - moments of inertia for masses; $c_{12} [Nm]$ - rigidity in torsion of the shaft between masses and absorber; $b [Nms]$ - dynamic absorber's coefficient; $M_0 [Nm]$ - initial circuit moment of electromotor; $M(t)[Nm]$ - circuit moment of electromotor; $M_{m0} [Nm]$ - initial *Figure 2. Two-mass model* moment of working machine; $M_m [Nm]$ - moment of working machine; $\beta [Nms]$ - stiffness coefficient for the electromotor's mechanical characteristic; $T_E [s]$ - time equivalent constant.

For the adopted dynamic model, *Figure 1.*, the general mathematical model is built taking into consideration torsional oscillations and the electro motor's torque influence:

$$
\begin{aligned}
& T_E \cdot \dot{M} + M = M(t) \\
& J_1 \cdot \ddot{\varphi}_1 + b_{12} \cdot (\dot{\varphi}_1 - \dot{\varphi}_2) + c_{12} \cdot (\varphi_1 - \varphi_2) = M \\
& \cdots\cdots\cdots \\
& J_i \cdot \ddot{\varphi}_i - b_{i-1,i} \cdot (\dot{\varphi}_{i-1} - \dot{\varphi}_i) + b \cdot (\dot{\varphi}_i - \dot{\varphi}) - c_{i-1,i} \cdot (\varphi_{i-1} - \varphi_i) = 0 \\
& J_{i+1} \cdot \ddot{\varphi}_{i+1} + b_{i+1,i+2} \cdot (\dot{\varphi}_{i+1} - \dot{\varphi}_{i+2}) - c_{i,i+1} \cdot (\varphi - \varphi_{i+1}) + c_{i+1,i+2} \cdot (\varphi_{i+1} - \varphi_{i+2}) = 0 \\
& \cdots\cdots\cdots \\
& J_n \cdot \ddot{\varphi}_n - b_{n-1,n} \cdot (\dot{\varphi}_{n-1} - \dot{\varphi}_n) - c_{n-1,n} \cdot (\varphi_{n-1} - \varphi_n) = -M_m \\
& b \cdot (\dot{\varphi}_i - \dot{\varphi}) = c_{i,i+1} \cdot (\varphi - \varphi_{i+1})
\end{aligned}
\tag{1}
$$

If in equation (1) we put *n=2*, then the mathematical model for mechanical aggregate as a two-mass system, *Figure 2.*, is given by:

$$
\begin{aligned}
& T_E \cdot \dot{M} + M + \beta \cdot \dot{\varphi} + \beta \cdot \frac{c_{12}}{b} \cdot (\varphi - \varphi_2) = M_0 \\
& J_1 \cdot \ddot{\varphi} + J_1 \cdot \frac{c_{12}}{b} \cdot (\dot{\varphi} - \dot{\varphi}_2) + c_{12} \cdot (\varphi - \varphi_2) = M \\
& [A + B \cdot (\sin\varphi_2 + 0.5 \cdot \lambda \cdot \sin 2\varphi_2)^2] \cdot \ddot{\varphi}_2 + 0.5 \cdot [2 \cdot B \cdot (\sin\varphi_2 + 0.5 \cdot \lambda \cdot \sin 2\varphi_2) \cdot \\
& \cdot (\cos\varphi_2 + \lambda \cdot \cos 2\varphi_2)] \cdot \dot{\varphi}_2^2 - c_{12} \cdot (\varphi - \varphi_2) = -M_{m0} \sin(\Omega \cdot t)
\end{aligned}
\tag{2}
$$

Where: $M(t) = M_0 - \beta \cdot \dot{\varphi}_1$ - moment of electromotor;

$M_m = M_{m0} \cdot \sin(\Omega \cdot t)$ - moment of working machine; and the coefficients given by relationas:

$$A = J_m + r^2 \cdot \frac{\rho_0}{l}; B = r^2 \cdot (m_k + m_0 \cdot \frac{l - \rho_0}{l}); \lambda = \frac{r}{l}$$

At the contact point of dynamic absorber with electric line, the moment of the electric forces is equal with moment of the frictional viscosity forces:

$$b \cdot (\dot{\varphi}_1 - \dot{\varphi}) = c_{12} \cdot (\varphi - \varphi_2) \qquad (3)$$

The non linear system of equations (2) was solved by Runge-Kuta method of fifth order and program executed in Matlab, *Figure 3*. The following substitutes for variables were adopted:

$$M = x(1); \varphi = x(2); \varphi_2 = x(3)$$

$$\dot{M} = \frac{dx(1)}{dt} = x(4); \dot{\varphi} = \frac{dx(2)}{dt} = x(5); \dot{\varphi}_2 = \frac{dx(3)}{dt} = x(6) \qquad (4)$$

And initial values:

$$M(0) = M_0; \varphi(0) = 0; \varphi_2(0) = 0; \dot{\varphi}(0) = 0; \dot{\varphi}_2(0) = 0; \qquad (5)$$

The system of equations (2) was solved for parameters values:

$$J_1 = 150kgm^2; T_E = 0.02s; M_0 = 110Nm; \beta = 17.5Nms; c_{12} = 10^4 Nm;$$
$$M_{m0} = 150Nm; \Omega = 100s^{-1}; J_m = 8.8kgm^2; m_0 = 36kg; m_k = 44kg;$$
$$r = 0.5m; l = 1.5m; \rho_0 = 0.665m \qquad (6)$$

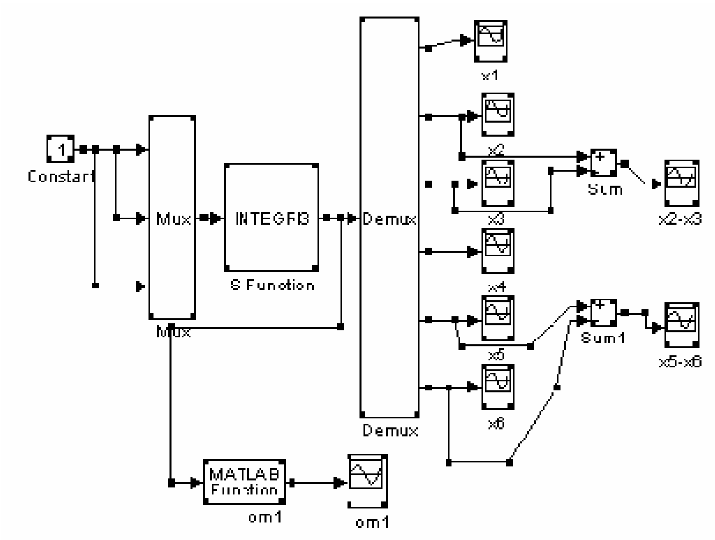

Figure 3. Block diagram for solving system of equation with Runge-Kuta method

The adopted model for mechanical aggregate, *Figure 2.*, was analysed in dependence of coefficient *b* which characterizes dynamic absorber. Therefore the variability of the difference between rotational angle of absorber and absolute rotational angle of the second mass, working machine $(\varphi - \varphi_2)$ and angular velocity difference $(\dot{\varphi} - \dot{\varphi}_2)$ is elaborated for: *1) b=0; 2) b=120Nms; 3) b=720Nms and 4) b=1200Nms at time interval t∈[0,5]s.* Graphical presentation of the results is given in *Figure 4*.

3. CONCLUSIONS

Analysing graphs given in *Figure 4.* and response of other variables, can be concluded that:

- For b=0 then $\varphi - \varphi_2 = 0$, equation (3) showing that working machine (mass 2) and dynamic absorbed will have the same angle of rotation;
- With increase of the values for coefficient of dynamic absorber *b*, the difference $(\varphi - \varphi_2)$ for rotational angles and rotational velocity $(\dot{\varphi} - \dot{\varphi}_2)$ will increase as well;
- For smaller values of *b*, the difference $(\varphi - \varphi_2)$ begin to increase more at time between *t=2.5s and t=3s;*
- With increase of the dynamic coefficient oscillations are stable;
- Amplitudes at the begin of the time period increases with increase of value for coefficient b.

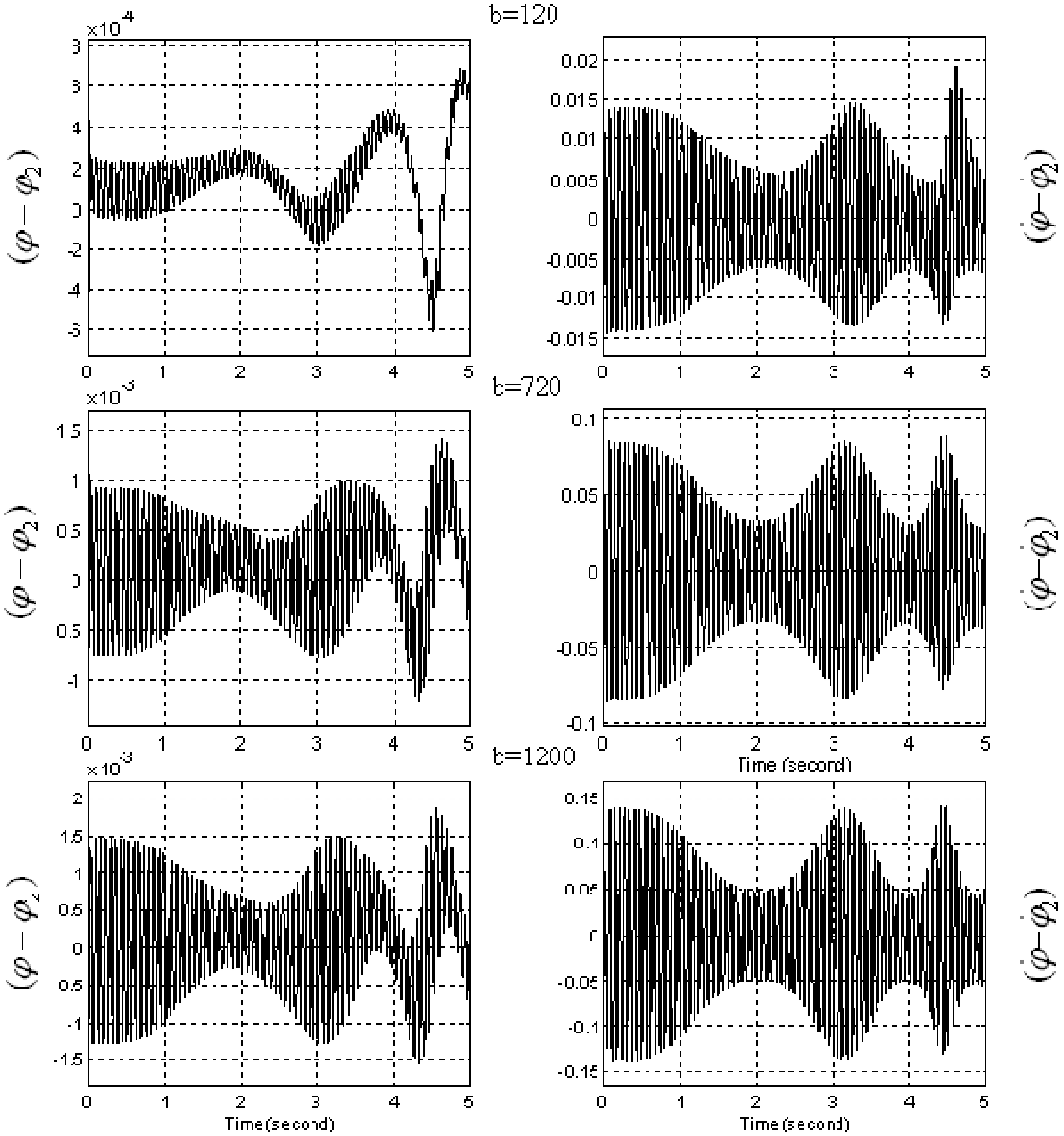

Figure 4. Graphically presented results for different values of dynamic absorber coefficient

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