10th International Research/Expert conference
"Trends in the Development of Machinery and Associated Technology"
TMT 2006, Barcelona-Lloret de Mar, Spain, 11-15 September, 2006

ANALYSING OF HELICOID SURFACES HAVING THE SAME AXIS BY MATHEMATICAL MODEL

Illés Dudás [1]
Department of Production Engineering, University of Miskolc, Miskolc
Hungary
Zsuzsanna Balajti [2]
Department of Descriptive Geometry, University of Miskolc, Miskolc
Hungary

ABSTRACT
We have developed a summarized general kinematic model (Fig. 1) for examination of mating of cylindrical helicoid surfaces and their tools, furthermore conical helicoid surfaces and their tools on the base of Dudas type general mathematical model [3], which is suitable to deal with examination at the same time, when their axis are the same.
The aim of the paper to show a new model which is suitable for neglecting the errors of the other. All of these can be done in one mathematical model. This model supposes the existence of a new CNC machine tool, which is suitable for production of proper production in case of changing axis distance as well.
Keywords: Helicoid surfaces, production geometry, modelling.

1. GENERAL MATHEMATICAL MODEL FOR EXAMINATION OF CYLINDRICAL, CONICAL WORM SURFACES AND HOBS FOR MACHINING OF WORM GEARS AND PINIONS

The mathematical model published in this paper solves the problems as follows:

a) Restricted angular displacement of work table (δ_1=max 5°).
b) Absence of controlling of work table in the direction perpendicular for main spindle.
c) Pitch problems of conical worms by solving with centre and translation of origin (δ_1>5°).

Elimination all the problems of a), b) and c) at the same time by a new thread grinding machine, and a new general mathematic model which describes that.

Figure 1 contains the general model, where:

a_1, c	are the *y, x* coordinates of the origin (O_2) the tool coordinate system in the K_0 coordinate system;
φ_1	is the angular displacement of the helicoid (parameter for angular displacement, for meshing and for movement);
φ_2	is the angular displacement of the tool (milling cutter or grinding wheel);
i_{21}	$i_{21}=\varphi_2 / \varphi_1$ gearing ratio;
γ	is the inclination of the tool equal to the lead angle on the reference cylinder of the helicoid;
α	is the inclination of the tool to the profile of the helicoid measured in characteristic section (eg grinding of involute worm using plane flank surface tool);
p	is the lead parameter.

The coordinate systems are as follows:

$K_0(x_0,y_0,z_0)$	stationary coordinate system joined to manufacturing machine tool;
$K_{1F}(x_{1F},y_{1F},z_{1F})$	coordinate system following a thread path, joined to worm surface;
$K_2(x_2,y_2,z_2)$	stationary coordinate system of tool;
$K_{2F}(x_{2F},y_{2F},z_{2F})$	rotating coordinate system of tool;
$K_k(x_k,y_k,z_k)$	secondary coordinate system;
$K_S(\xi,\eta,\zeta)$	coordinate system of generator curve motion (when lathe turning it is stationary).

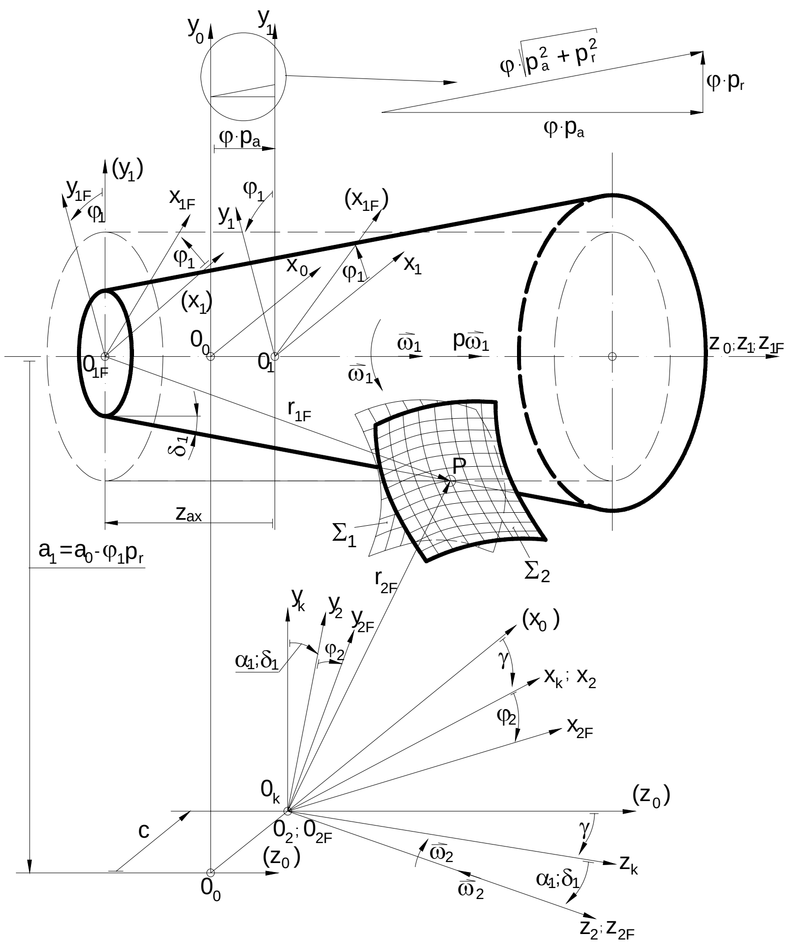

Figure 1. Connection between coordinate systems to investigate process grinding

Applied co-ordinate systems and denominations: p_a – screw parameter in axial direction, p_r – screw parameter in radial direction, c – distance of tool lift out (in case of convolute and evolvent worm).

Conditions of movement are interpreted on the right-hand thread surfaces and on the right side of thread profiles. The statements are valid for left-hand thread surfaces.

During our investigations [3, 4] the kinematics of generation [5] was handled so that the helicoid surface followed a thread path and the tool surface performed a rotary motion on the left side of the thread profiles; the lead of thread and generator curve together with signs should be taken into consideration.

It is necessary to determine generally valid rules for generation of the cylindrical thread surface when discussing geometric problems of manufacturing in general.

The position vector $\vec{r}_g$ of the generating curve in the coordinate system $K_S(\xi,\eta,\zeta)$ is given as a function. This generating curve can be the edge of tool (eg in lathe turning) or the contact curve (eg in grinding).

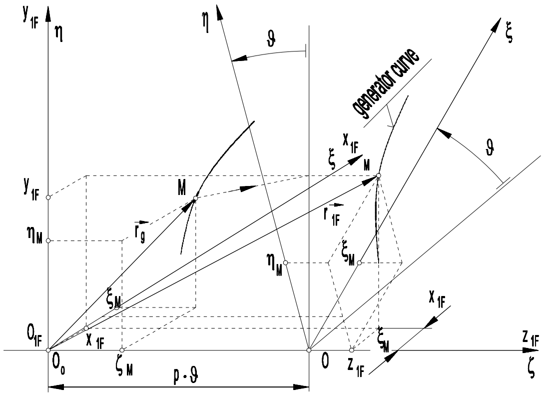

Figure 2. The generating curve of the thread surface in K_{1F} coordinate system

To formulate the equation of the generating curve, from the practical point of view let the parameter η be chosen as an independent variable.

In this way the parametric vector function of the generating curve is found to be:

$$\vec{r}_g=\xi(\eta)\vec{i}+\eta\vec{j}+\zeta(\eta)\vec{k} \qquad (1)$$

The generating curve parametric equation $\vec{r}_g$ is carried by the generating curve parametric equation $K_S(\xi,\eta,\zeta)$ the coordinate system is forced on the thread path along axis ζ with parameter p, so the generating curve will describe a thread surface in coordinate system $K_{1F}(x_{1F}, y_{1F}, z_{1F})$ which, before this movement was performed, coincided with the K_S coordinate system (see Figure 2).

The thread surface described by the generating curve $\vec{r}_g$ can be determined in $K_{1F}(x_{1F}, y_{1F}, z_{1F})$ coordinate system as:

$$\vec{r}_{1F}=\underline{M}_{1F,S}\cdot\vec{r}_g \qquad (2)$$

It can be seen from the structure of transformation matrix $\underline{M}_{1F,S}$ and the general equation of the worm surface (4) that the generating curve $\vec{r}_g$ and worm parameter *p* determine basically the worm surface.

The generator curve $\vec{r}_g$ has a decisive role in the case of tool surface generation too. During generation of the tool surface, the generator curve can be the meridian curve or the contact curve. In this case the $\vec{r}_{gsz}$ curve is interpreted in the $K_{20}(x_{20}, y_{20}, z_{20})$ coordinate system using y_{20} as a parameter, so its form is:

$$\vec{r}_{gsz}=x_{20}(y_{20})\vec{i}+y_{20}\vec{j}+z_{20}(y_{20})\vec{k} \qquad (5)$$

Rotating $\vec{r}_{gsz}$ generating curve with the $K_{20}(x_{20}, y_{20}, z_{20})$ coordinate system round the z_{20} axis the $\vec{r}_{gsz}$ curve will describe the tool surface in the $K_{2F}(x_{2F}, y_{2F}, z_{2F})$ coordinate system (see Figure 3.)

The tool surface determined this way could be written as:

$$\vec{r}_{2F}=\underline{M}_{2F,20}\cdot\vec{r}_{gsz} \qquad (6)$$

where:

$\vec{r}_{2F}$ is the position vector of an oblique point fitted on tool surface,

$\underline{M}_{2F,20}$ is the transformation matrix between coordinate systems K_{2F} and K_{20}.

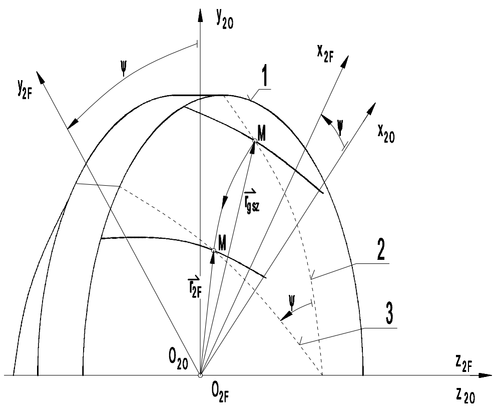

Figure 3. The surface wrapped by tool generating curve ($\vec{r}_{gsz}$) in the coordinate system K_{2F}

2. APPLICABILITY OF THE MODEL

The model applicable for examination of mating of cylindrical and conical helicoids surfaces – occurring in the practice –, having constant pitch, by cylindrical shaped cutting tools(e.g. grinding wheel)[1, 2, 3].

By the help of the model the contact lines can be determined in two way: **direct task:** can be originated from $\vec{r}_{1F}$ (workpiece) surface, **indirect (inverse task:** the surface of $\vec{r}_{2F}$ (the cutting tool) is given. The determined contact line, as a ruling curve, can be used for determination of 2. tool surface, and 1. workpiece surface.

The surface of the workpiece 1. can have a cylindrical or a conical base surface having an arbitrary generating surface (with thread cross sectional area).

It si advantageous to give surface of revolution for the cutting tool surface, but other type of surface can be imagined as well, e.g. a single point cutting tool with φ_2=const [3].

3. SUMMARY

The model suitable for examination of mating of conical and cylindrical helicoids surfaces having constant pitch -occurring in practice- with conventional single point cutting tools e.g. turning tools. That is suitable for examination of machining by cylindrical type cutting tool. It can be extended for examination of manufacturing of globoids and analyzing of mating of driving pairs as well.

4. ACKNOWLEDGEMENTS

This research was sponsored by Hungarian National Scientific Research Fund. (Project number: **OTKA K 63367.**

5. REFERENCES

[1] I. Dudás, Zs. Balajti: Modelling and developoment for describing the bearing pattern of the spiroid driving, Proceedings of the Sixth IASTED Internetional Conference on ”Robotics and Applications, 2005. október 31. – november 02. 2005., Cambridge, USA, pp.:203-208

[2] Zs. Balajti, K. Bányai: A possibility method for the solve of 3d evaluation with 3 ccd cameras. Production Process and Systems. A Publication of the University of Miskolc, Miskolc University Press 2004, pp.: 237-242., HU ISSN 1215-0851

[3] I. Dudás: The Theory and Practice of Worm Gear Drives. Prenton Press, London, 2000.

[4] I. Dudás: Theory of production of helicoid surfaces Akademica doktoral dissertacion, Budapest, 1991. (In Hungarian)

[5] Litvin, F. L.: Gear geometry and applied theory. Englewood Cliffs, Prentice Hall, NJ., 1994.

[6] I. Dudás, Gy. Varga, K. Bányai: Bearing Pattern Localization of Worm Gearing VDI-Gesellschaft Entwicklung Konstruktion Vertrieb, International Conference on Gears, Dresden, 22-24. 04. 1996., pp.427-441.

[7] Zs. Balajti, I. Dudás: Mathematical Model For Analysing Of Helicoid Surfaces Having The Same Axis. India, 2006. International Journal of Mathematical Sciences, under publication.