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OPTIMISATION OF OPERATION OF THE HEAT SOURCE WITH COMBINED PRODUCTION OF HEAT AND POWER

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ABSTRACT
The paper deals with the access to solving optimisation of operation of heat source with combined production of heat and power by methods of linear programming and non-linear programming.
Key words : heat, CPM – combined production of heat and power, linear , non-linear, optimisation.

1. INTRODUCTION

It will be shown the access to solving optimisation of operation of an energy source with combined production of heat and power by methods of linear programming and non-linear programming. In generating plants of heat and electric energy, which have larger number of co-operating production units, there is a problem of economical distribution of load between these production units. The task of economical distribution of load is one of the basic tasks of optimum control. Its basic principle is usually the equality of the relative increase of cots (consumption of fuel) on the delivered heat output. The basic presumption of economical production is knowledge of energy economical characteristics of separate production equipments. In our specific case this applies to consumption characteristics of boilers, which generally have non-linear course.

From the mathematical point of view, the aim of optimum control will be to achieve extreme value of the objective function. In our case i.e .at minimising the production cots decreasing the costs of fuel, we shall look for the minimum of the objective.

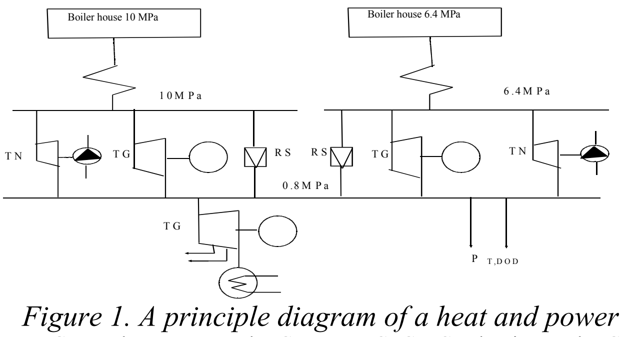

Figure 1. A principle diagram of a heat and power plant

Legend: TFP – Turbo-Feed-Pump, BPTG – Back-Pressure-Turbo-Generator, CTG – Condensing-Turbo-Generator, RS – Reducing Station, $P_{T,DOD}$ – Heat output supply

The total immediate electric output P and the heat output supplied to the heat network $P_{T,DOD}$ can be determined. There are two methods as the solutions to form mathematical models: linear and non-linear model.

2. MATHEMATICAL LINEAR MODEL

The way to form the mathematical linear model for optimisation in use of linear programming was designed in the publication [1]. In the main, the linear model (balance equations of steam pipe-lines) was created, the limiting non-linear conditions of dependent variables and objective function are linear. Independent variable is supplied heat output in the heat network $P_{T,DOD}$. In order to form the mathematical linear model, it is necessary to perform the following requirements:

- the consumption parameters of production units must have a convex monotonic continuance.
- it is necessary to approximate the convex curves of the consumption parameters by linear sections.

The way of linearization of the consumption parameters characteristics is illustrated on the Fig. 2a.
In concrete case, it is able to approximate the consumption parameters of boiler by means of two linear sections. Then, according to the Fig.2a, we can describe the heat output of the boiler:

$$P_{T,K} = \underline{P}_{T,K} + \sum_{s=1}^{2} \Delta P_{T,K,s} \quad . \tag{1}$$

and the boiler heat consumption in fuel:

$$P_{PAL} = \underline{P}_{PAL} + \sum_{s=1}^{2} b_{K,s} \Delta P_{T,K,s} \tag{2}$$

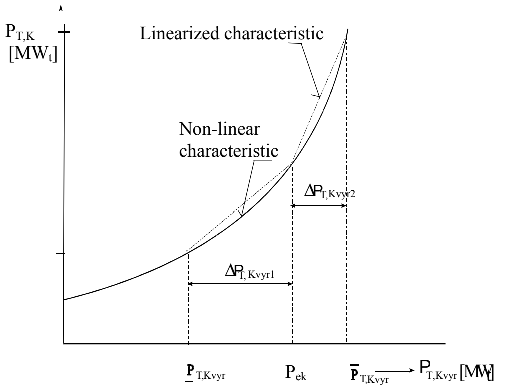

Fig. 2a. – Economic characteristic of steam boiler and its linearization

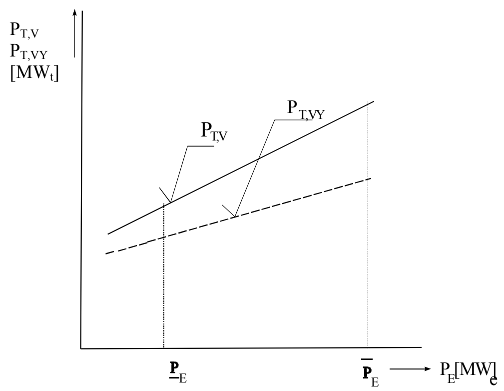

Fig. 2b. – Economic characteristic of back pressure turbine

Where $b_{K,s}$ is the consumption relative increment of boiler: $b_{K,s} = \dfrac{\Delta P_{PAL}}{\Delta P_{T,K,s}}$ [-] (3)

(In this case within linear programming $b_{K,s}$ is constant).

The further variables sign: $\underline{P}_{T,K}$ the minimal heat output boilers
$\Delta P_{T,K,s}$ the increments of output of boilers
s the number of linearized section.

Consumption characteristics for back-pressure turbines have the linear course (Fig. 2b.) and therefore it is possible to approach it by one linear section: $P_{T,V} = \underline{P}_{T,V} + b_{T,V}.\Delta P$ (4)

that is the heat output on the input of turbine.

We can describe by analogy way the heat output on the exit of turbine: $P_{T,VY} = \underline{P}_{T,VY} + b_{T,VY}.\Delta P$ (5)

And than applies

$$b_{T,V} = \frac{\Delta P_{T,V}}{\Delta P} = const. \,, \quad b_{T,VY} = \frac{\Delta P_{T,VY}}{\Delta P} = const. \quad \text{in the range } \overline{P} - \underline{P} \,, \tag{6) (7}$$

where $b_{T,V}$ is relative increment of heat consumption on the input of turbine,
$b_{T,VY}$ is relative increment of heat consumption on the exit of turbine and in mathematic model is used to definition of heat balance equation of steam pipe-line 0,8 MPa (eq. 10).

The mathematic model of determine scheme (see Fig. 1.):
It is described by heat balance equations for steam pipe-lines marked 10 MPa, 6,4 MPa, 0,8 MPa., by sum total equation of produce power energy and by definition of objective function *E*.
The following balance equations were created:
- for steam pipe-line 10MPa we can apply:

$$\sum_{i=1}^{n}\left(\underline{P}_{T,K}^{10,i}+\sum_{s=1}^{k}\Delta P_{T,K,s}^{10,i}\right)=\left[\sum_{j=1}^{m}\left(\underline{P}_{T,V}^{10,j}+b_{T,V}^{10,j}.\Delta P^{10,j}\right)+P_{T,N,V}^{10}+P_{T,R}^{10/0.8}\right].k_{q}^{10} \tag{8}$$

- for steam pipe-line 6.4MPa we can apply:

$$\sum_{i=1}^{n}\left(\underline{P}_{T,K}^{6,4,i}+\sum_{s=1}^{k}\Delta P_{T,K}^{6.4,i}\right)=\left[\sum_{j=1}^{m}\left(\underline{P}_{T,V}^{6.4,j}+b_{T,V}^{6.4,j}.\Delta P^{6.4,j}\right)+P_{T,N,V}^{6.4}+P_{T,R}^{6.4/0.8}\right].k_{q}^{6.4} \tag{9}$$

and also for steam pipe-line 0.8 MPa we can apply :

$$\sum_{j=1}^{m}\left(\underline{P}_{T,T,VY}^{10,j}+b_{T,VY}^{10,j}.\Delta P^{10,j}\right)+\sum_{j=1}^{m}\left(\underline{P}_{T,T,VY}^{6.4,j}+b_{T,VY}^{6.4,j}.\Delta P^{6.4,j}\right)+P_{T,N,VY}^{10}+P_{T,N,VY}^{6.4} \tag{10}$$
$$+P_{T,R}^{10/0.8}+P_{T,R}^{6.4/0.8}=\left[\left(\underline{P}_{T,T,V}^{0.8,j}+b_{T,el,s}^{0.8,j}.\Delta P^{0.8,j}\right)+P_{T,DOD}\right]k_{q}^{0.8}$$

The total output of power would be determined by the following relation :

$$\sum P=\sum_{j=1}^{m,10}\left(\underline{P}^{10,j}+\Delta P^{10,j}\right)+\sum_{j=1}^{m,6.4}\left(\underline{P}^{6.4,j}+\Delta P^{6.4,j}\right)+\sum_{j=1}^{m,0.8}\left(\underline{P}^{0.8,j}+\Delta P^{0.8,j}\right) \tag{11}$$

The solutions should respond the following inequalities or conditions:

$$0\le\Delta P^{6.4}\le\Delta\overline{P}^{6.4}\qquad 0\le\Delta P_{T,K,s}^{10,i}\le\Delta\overline{P}_{T,K,s}^{10,i}\qquad 0\le\Delta P^{10}\le\Delta\overline{P}^{10}\qquad 0\le\Delta P_{T,K,s}^{6.4,i}\le\Delta\overline{P}_{T,K,s}^{6.4,i}\qquad 0\le\Delta P_{T,R}^{6.4/0.8}\le\Delta\overline{P}_{T,R}^{6.4/0.8}\qquad 0\le\Delta P_{T,R}^{10/0.8}\le\Delta\overline{P}_{T,R}^{10/0.8} \tag{12}$$

$P_{T,el}$ is a constant in determined schema. It presents the own heat consumption of each the turbine. Commonly is possible to use a constant in determined schema.

The variables in mathematical model, contain not only the increments of boilers output $\Delta P_{T,K}$, but also the increments of electric output of individual turbines ΔP. The sum total of these variables, where each variable is multiplied with the appropriate price coefficient c_j (in case of $\Delta P^{i}_{T,K}$, the coefficients are expressed by the heat consumption relative increments $b^{i}_{K,s}$), defines the objective function E.

The objective function form:

$$E=\sum_{i=1}^{n,10}\sum_{s=1}^{k}b_{K,s}^{10,i}.\Delta P_{T,KVyr,s}^{10,i}+\sum_{i=1}^{n,6.4}\sum_{s=1}^{k}b_{K,s}^{6.4,i}.\Delta P_{T,KVyr,s}^{6.4,i} \tag{13}$$

Meaning of the variables in the equations:

The symbol $\underline{P}$ signs the minimum value and in opposite, the symbol $\overline{P}$ signs the maximum one of the appropriate quantity

$P_{T,Kvyr}=\dot{m}(i_{pK}-i_{nv})$	[MW_t] the boilers output (performance)
$P_{T,T,V}^{10}$, $P_{T,T,V}^{6.4}$	[MW_t] the heat outputs (performance) of turbine input
$P_{T,T,VY}^{10}$, $P_{T,T,VY}^{6.4}$	[MW_t] the heat outputs (performance) of turbine exit
$P_{T,N,V}^{10}$, $P_{T,N,V}^{6.4}$	[MW_t] the heat outputs of input of the turbo-feed pump
$P_{T,N,VY}^{10}$, $P_{T,N,VY}^{6.4}$	[MW_t] the heat outputs of exit of the turbo-feed pump
$P_{T,R}^{10/0.8}$, $P_{T,R}^{6.4/0.8}$	[MW_t] heat outputs of the reduction stations
$P_{T,DOD}$	[MW_t] the heat outputs supplied to the heating network
k_q^{10} , $k_q^{6.4}$, $k_q^{0.8}$	[-] the coefficients of heat consumption or losses in each phase of technology
$b_{T,el}$	[MWt/MWe] coefficient of own heat consumption of power production of each condensing turbine

The mathematical linear model includes the equations No. (8), (9), (10), (11), (12), (13).

3. MATHEMATICAL NON-LINEAR MODEL

Mathematical non-linear model is the main item of this article. All of signs presented for linear model are applicable for the non-linear model too.

a. the derivation of consumption characteristic:

Generally, steam-boiler consumption characteristics have non-linear continuances (Fig. 2a.). These continuances can be replaced by non-linear approximation, for example by the index form:

$$y=a.e^{b.x} \tag{14}$$

Their heating consumption proportionate increment are expressed by derivation :

$$y'=a.b.e^{b.x} \tag{15}$$

b. Creation of a non-linear mathematical model *b1. balance equations :*

- for steam pipe-line 10MPa we can apply:

$$\sum_{i=1}^{n}\left(\underline{P}_{T,K}^{10,i}+\int dP_{T,K}^{10,i}\right)=\left[\sum_{j=1}^{m}\left(\underline{P}_{T,K}^{10,j}+b_{T,V}^{10,j}.\int dP^{10,j}\right)+P_{T,N,V}^{10}+P_{T,R}^{10/0.8}\right].k_q^{10} \tag{16}$$

- for steam pipe-line 6,4 MPa we can apply:

$$\sum_{i=1}^{n}\left(\underline{P}_{T,K}^{6.4,i}+\int dP_{T,K}^{6.4,i}\right)=\left[\sum_{j=1}^{m}\left(\underline{P}_{T,K}^{6.4,j}+b_{T,V}^{6.4,j}.\int dP^{6.4,j}\right)+P_{T,N,V}^{6.4}+P_{T,R}^{6.4/0.8}\right].k_q^{6.4} \tag{17}$$

and also for steam pipe-line 0.8 MPa we can apply:

$$\begin{aligned}&\sum_{j=1}^{m}\left(\underline{P}_{T,T,VY}^{10,j}+b_{T,VY}^{10,j}.\int dP^{10,j}\right)+\sum_{j=1}^{m}\left(\underline{P}_{T,T,VY}^{6.4,j}+b_{T,VY}^{6.4,j}.\int dP^{6.4,j}\right)+P_{T,N,VY}^{10}+P_{T,N,VY}^{6.4}+P_{T,R}^{10/0.8}+P_{T,R}^{6.4/0.8}\\&=\left[\left(\underline{P}_{T,T,V}^{0.8,j}+b_{T,el,s}^{0.8,j}.\int dP^{0.8,j}\right)+P_{T,el}+P_{DOD}\right]k_q^{0.8}\end{aligned} \tag{18}$$

The total electric output would be determined by the following relation :

$$\sum P=\sum_{j=1}^{m,10}\left(\underline{P}^{10,j}+\int dP^{10,j}\right)+\sum_{j=1}^{m,6.4}\left(\underline{P}^{6.4,j}+\int dP^{6.4,j}\right)+\sum_{j=1}^{m,0.8}\left(\underline{P}^{0.8,j}+\int dP^{0.8,j}\right) \tag{19}$$

b2. objective function :

The variables in mathematical model, contain not only the increments of boiler performance $\Delta P^{i}_{T,K}$, but also the increments of power outpot of individual turbines ΔP^{i}. The total of these variables, where each variable is multiplied with the appropriate price coefficient c_j (in case of $\Delta P^{i}_{T,K}$, the coefficients are expressed by the heat consumption proportionate increments $b_{K,s}$), defines the objective function E.

The objective function form:

$$E=\sum_{i=1}^{n,10} b_{k,s}^{10,i}.\int dP_{T,Kvyr}^{10,i}{}_{,s}+\sum_{i=1}^{n,6.4} b_{k,s}^{6.4,i}.\int dP_{T,Kvyr}^{6.4,i}{}_{,s} \tag{20}$$

In this case, $b_{K,s}$ will be the non-linear functions (see equation 15)

Limiting non-negative conditions are following:

$$0\le dP^{6.4}\le \Delta\overline{P}^{6.4} \quad 0\le dP_{T,K,s}^{10,i}\le \Delta\overline{P}_{T,K,s}^{10,i} \quad 0\le dP^{10}\le \Delta\overline{P}^{10} \quad 0\le dP_{T,K,s}^{6.4,i}\le \Delta\overline{P}_{T,K,s}^{6.4,i} \quad 0\le dP_{T,R}^{6.4/0.8}\le \Delta\overline{P}_{T,R}^{6.4/0.8} \quad 0\le dP_{T,R}^{10/0.8}\le \Delta\overline{P}_{T,R}^{10/0.8}$$

4. CONCLUSION

This paper has explained two methods to form mathematical model of a heat and power plant which produce heat and electric energy in order to optimisation its operation.

Certainly, the differences between both models consist in reduction of the base variables number, the consumption characteristic approximation of non-linear equations and forming of the other objective function.

In the case of the linear model, the objective function is linear (the consumption proportionate increments of boiler $b_{K,s}$ are constant) and it is non-linear in the second one (the consumption proportionate increments of boiler $b_{K,s}$ depend on index function). In order we would gain the values for the general coefficients in the above mentioned mathematical models, it is necessary to process the whole rank of bases and consumption parameters of the production units. The obtained results would be used as background for operative planning and own operation control of production of heat and power energy in real time too.

5. REFERENCES

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