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THEORETICAL METHOD FOR THE DETERMINATION OF VELOCITY IN GENERALIZED GEOMETRIES

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ABSTRACT

A method is presented for the determination of the distributions of velocity, local friction, and heat transfer coefficients in a forced axial turbulent flow with an arbitrary cross section. The method uses as basis the characteristics of the laminar flow. A comparison is made with some experimental results concerning different geometries.

1. INTRODUCTION

A method is proposed for the determination of the velocity distribution of the local heat transfer and friction factor for fully developed turbulent flows of fluids of constant properties through ducts of arbitrary cross section with a prescribed heat flux distribution at the wall. The method can be divided into three parts. The first part concerns the distribution of the velocity near the wall. The velocity profile on a line normal to the wall is represented by equations of the same type as those used for the circular geometry, with the reduced velocity u^+ as a function of the reduced distance n^+ to the wall. Only the friction velocity u^* varies. When the distribution of the local friction velocity is known, the velocity profile in each point is known. This approach is supposed to be valid as long as the local radius curvature of the wall is not too small compared to the average hydraulic diameter. The second part concerns the velocity distribution in the core. In this region it is assumed that the flow can be represented by a gradient-diffusion type of equation.

2. VELOCITY DISTRIBUTION

The flow in a given geometry (Figure 1) can be divided into two parts: a zone near the wall, made up successively of a laminar, buffer, and turbulent layers; and the turbulent core. Steady state conditions are assumed.

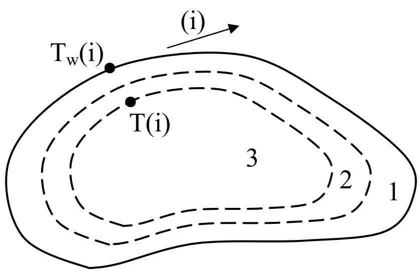

Fig. 1. Generalized geometry
1. Laminar and buffer layers;
2. Turbulent layer near the wall;
3. Turbulent care.

Velocity Profile in the Layer Zone. It is assumed that in the vicinity of the wall, where distances are small compared to the mean hydraulic diameter, the velocity distribution can be represented by function identical in form to those used for the circular geometry. Among different available expressions, we have retained Deissler's formulas.

$$n^+ = \frac{\frac{1}{a}\int_0^{au^+} \frac{e^{-(au^+)^2}}{d(au^+)}}{e^{-(au^+)^2}/2} \text{ with } 0 < n^+ < 2b \quad (1)$$

a = 0.109, $u^+ = 3.80 + 2.78 \ln n^+$ with $n^+ > 26$ (2)

The local characteristic of the function appears through the local friction velocity $u^*(i)$ where (i) is the wall are coordinate: $n^+ = \frac{n_i u^*(i)}{\nu}$ (3) $u^+ = \frac{u_i}{u^*(i)}$ (4)

The local friction velocity $u^*(i)$ is defined by: $u^*(i) = u_t\sqrt{\frac{f_t}{2 \cdot g(i)}}$ (5)

The local shear stress r(i) is: $r(i) = \rho u^{*2}(i)$ (6)

When g(i), which is a dimensionless function of (i) and f_t, which is the average friction factor for the considered geometry, are known, the velocity distribution in the layer zone is fully determined.

The Turbulent Case. In an arbitrary geometry, at greater distances from the wall, there is no reason to believe that the velocity distribution, on a line normal to equal velocity curves, could be represented in all points by a logarithmic function (consider for instance a rectangular geometry where the ratio of the two sides is very). In a general form, the velocity distribution is given by the equation:

$$\frac{1}{\rho}\frac{dp}{dz} = \frac{\partial}{\partial x}\left[(\nu + \varepsilon_{mx})\frac{\partial u}{\partial x}\right] + \frac{\partial}{\partial y}\left[(\nu + \varepsilon_{mx})\frac{\partial u}{\partial y}\right] \quad (7)$$

According to different authors, the anisotropy of the eddy diffusivity of momentum function ε_m is not negligible, especially near the wall. Nevertheless experimental results on the subject are scarce and difficult to inject into general formulation. In our present case, the kinematics' viscosity ν is neglected, and it is supposed that the eddy diffusely of momentum is constant in the turbulent core and equal to:

$$\frac{\varepsilon_m}{\nu} = 0.033 \cdot \mathrm{Re}\sqrt{\frac{f_t}{2}} \quad (8)$$

This value has been derived from Cess's formula after simplification. The equation (7) becomes a Poisson's equation of which the boundary conditions are defined in the next paragraph.

3. TEMPERATURE DISTRIBUTION AND LOCAL HEAT TRANSFER COEFFICIENTS

The heat flux at the wall is defined by the function Φ(i), and the wall temperature distribution is defined by function $T_w(i)$. It is assumed that the Prandtl number is high enough (Pr > 1) so that the temperature drops in the film are predominant compared to the temperature variations in the bulk. The influence of temperature on the fluid characteristic has been neglected. Steady state conditions are assumed. The average fluid temperature in a given cross section is taken as reference.

Temperature Distribution in the turbulent core and the Wall. In a general form the fluid temperature is represented by the equation (10): $\frac{\partial}{\partial x}\left[(\alpha + \varepsilon_{kx})\frac{\partial T}{\partial x}\right] + \frac{\partial}{\partial y}\left[(\alpha + \varepsilon_{kx})\frac{\partial T}{\partial y}\right] = u\frac{dT}{dx}$ (10)

This equation shows similarities with the equation (7). The same remarks made previously on the anisotropy of the eddy diffusivity of momentum ε_m apply here as far as eddy diffusivity of heat ε_h is concerted. The thermal diffusivity α is neglected. It is assumed that ε_h is isotropic and constant. The equation (10) becomes: $\varepsilon_h\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right] = u\frac{dT}{dx}$ (11)

The boundary conditions are different from those of equation (7):

$$\varepsilon_k \cdot \rho \cdot C_p \frac{dT}{dn} = \Phi(i) \text{ at the wall} \quad (12) \qquad \text{with} \iint^{s} T ds = 0 \quad (13)$$

To perform these calculations the limits of the turbulent flow are extrapolated to the wall. The boundary condition (12) means that in effect the transversal diffusion of heat near the wall in the laminar and buffer layers is neglected. It should not be forgotten that as far the wall temperature distribution is concerted, the variations in the bulk temperature will generally represent a small influence which justifies the simplifications, and in most cases the equation (11) could be further simplified by supposing that the turbulent velocity u is constant and equal to the average velocity u_t. The solution of the equation (11) will give the fluid temperature distribution T(i) at the boundary of the turbulent core, identified here with the wall. The value of ε_h is obtained by supposing that the ratio $\frac{\varepsilon_h}{\varepsilon_m}$ is constant. Some authors take this ratio equal to one. Some experimental results seem to indicate that the ratio is greater than one and probably not constant over the whole cross section.

Determination of the Turbulent Local Heat Transfer Coefficient. The local heat transfer coefficient h(i) is defined by the relation: $h(i) = \frac{\Phi(i)}{T_w(i) - T(i)}$ (14)

where T(i) is the local fluid turbulent temperature near the wall. We have chosen that definition in order that the local heat transfer coefficient be expressed solely in terms of local parameters, and to underline that the wall temperature variations are not only depended upon the variations of the heat transfer coefficient but also upon the variations of the temperature in the bulk, which in turn are influenced by the heat flux function Φ(i). It is assumed that the distribution of the local heat transfer coefficient h(i) is identical to the distribution of the friction velocity u^*(i): $\frac{h(i)}{h_0} = \frac{u^*(i)}{u^*(io)}$ (15)

h_o which is the heat transfer coefficient for the circular geometry at the same hydraulic diameter, same Reynolds number, and the same Prandtl number, can be determined, for instance, from the formula of Dittus and Boettler: $Nu_o = 0.023\ Re^{0.8}\ Pr^{0.4}$ (16)

4. APPLICATION OF THE METHOD

A few examples of applications of the method and comparisons with experimental results are given.

Friction Factor and Heat Transfer Coefficient in an Equilateral Triangle. As an example, let us consider an equilateral triangle. Fig. 2, for which the reduced laminar velocity function F_t is:

$$F_t = \frac{30}{a^3} \cdot x[(a-x)^2 - 3y^2] \qquad (17)$$

The laminar form coefficient k_l is determined by equation (13) and found equal to 0.833. By reason of symmetry, it is sufficient to consider the normal derivative on the side corresponding to x = 0 where.

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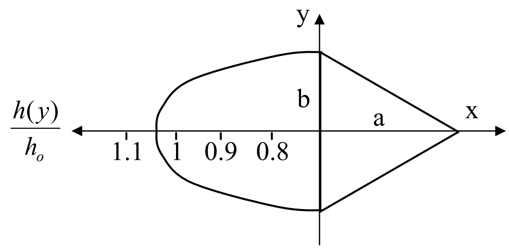
The turbulent form coefficient k_t can then be calculated by equation $\frac{f_t}{f_{ot}} = k_t = \frac{\overline{G}^2}{G^2(io)}$ and is found equal to 0.92. By this method it is found that the turbulent friction factor for the equilateral triangle is 8 percent smaller than the friction factor for the circular geometry with the same hydraulic diameter and Reynold number. The distribution of the local turbulent heat transfer coefficient is calculated by equation (15). See Fig. 2. A similar procedure can be applied to other geometries for witch there exists an analytical solution of the laminar flow, such as the right isosceles triangle, the ellipse, of the symmetrical annular geometry, etc.

In Figure 3 a comparison is made between the calculated velocity distributions and the experimental points obtained by Eifer and Nijsing in the same geometry, at a Reynolds number of 30,000 and at two values of the pitch, for three different angular positions. There seems to be rather satisfactory agreement between the experimental pints and the calculated curves. In these curves, the velocity profile in the turbulent core has been limited of a distance from the wall equal to 10 per cent of the mean hydraulic diameter.



Near the wall the velocity profile is given by equation (2). As the velocity distributions are very near one another in that zone, the choice of the exact boundary between them is no fundamental importance.

Fig. 2. Local heat transfer coefficient in an equilateral triangle (up to 0.95 b)

6. CONCLUSIONS

The comparison of the method with a certain number of experimental results shows a rather satisfactory agreement, witch is the more encouraging because these results concern three different fields of experimentation (velocity distribution, average friction factor, local heat transfer coefficient), but it is clear that comparisons with more experimental results must be made.

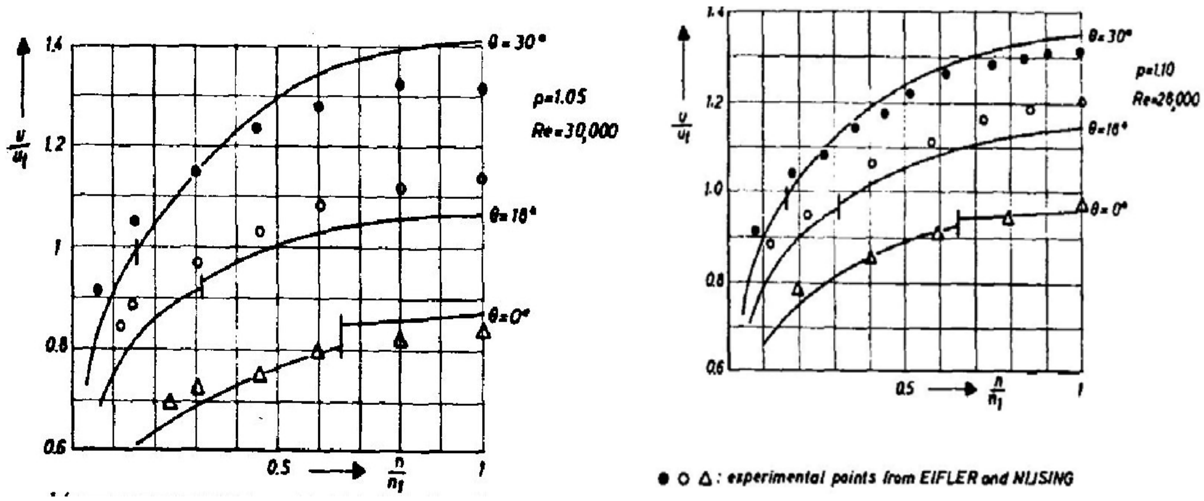


Fig. 3. Calculated and experimental velocity distributions

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