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MATHEMATICAL MODELLING OF THE FLOW OF COMPOSITE MATERIALS WITH SHORT FIBERS

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ABSTRACT
The mathematics of composites materials requires the use of homogenization method .The governing equation of short fibers composite materials flow, treated as viscous compressible, are the Navier-Stokes'partial derivative equations. Apply the homogenization method to these equations, then a variational procedure which permits the use of finite element method. At the end of paper we give a practical application of this approach.

Keywords: Dirichlet-Neumann problem, Cauchy problem, macroscopic problem, microscopic problem, variational problem, temperature field, pressure, velocity field, energy conservation, mass conservation, finite element base, kinetic flow

1.FORMULATION OF MATHEMATICAL MODEL

The functional framework, this mathematical model harmonizes with is that of the Sobolev' spaces. However, the followed applications of continuous and slow plastical deformations at constant pressure of composite materials with short fibers, permits the regularization of the solution and therefore the usual numerical approach.

Suppose that the material occupies in R^3 a bounded domain Ω, with the frontier ∂Ω, sufficiently smooth. Make the following assumptions about the operator of Henky-Nadai's theory, attached to Dirichlet's problem:

1^0.Fix the space $H(\Omega)=L_2(\Omega)$, where $L_2(\Omega)$ is the usual space of real functions at square sumable on $\overline{\Omega}$. Denote by $H_0^2(\Omega)$, the Sobolev space:

$H_0^2(\Omega)=\left\{u \middle| u\in H(\Omega)\cap C^2\left(\overline{\Omega}\right), u|_{\partial\Omega}=0\right\}$

where $C^2\left(\overline{\Omega}\right)$ is the class of C^2 functions on Ω,

$\overline{\Omega}=\Omega\cup\partial\Omega$. Of course, $H_0^2(\Omega)$ is linear and dense in $L_2(\Omega)$.

2^0.Let $P: D(R)\to H^3(\Omega)$, $D(P)\subset H^3(\Omega)$ be an operator generally nonlinear and $f:\Omega\to H^3(\Omega)$.

Consider the homogenized Dirichet's problem:

$$Pu = f, u\in H_0^2(\Omega) \tag{1}$$

$$\Gamma_i u = 0,\ i=1\div s \tag{2}$$

where Γ_i are linear operators restricted at $\partial\Omega$, that is

$u \in M^0 = \{u \in H_0^2(\Omega) | \Gamma_i u = 0,\ i=1 \div s\}$

3^0. There is the Gateaux derivative $(DP)(u) = h$, for all $u, h \in H_1(\Omega)$, linear with h and continuous with u in any two-dimensional hyperplane containing u, where $H_1(\Omega) = I_m(P)$.

4^0. $\langle (DP)(u) \cdot h_1, h_2 \rangle_{H(\Omega)} = \langle DP(u) \cdot h_2, h_1 \rangle_{H(\Omega)}$,

for all $u \in H_1(\Omega)$ and $h_1, h_2 \in M^0$.

5^0. $\langle D(P)(u)h, h \rangle_{H(\Omega)} \rangle 0$, for all $u \in H_1(\Omega), h \in M^0, h \neq 0$.

6^0. $P(0) = 0$

In these assumptios, if there is a solution $u^0 \in M^0$, for the equation (1), then this solutions is unique and realizes the minimum of the functional.

$$F(u) = \phi(u) - \langle f, u \rangle_{H(\Omega)} \tag{3}$$

where

$$\phi(u) = \int_0^1 \langle P(tu), u \rangle_{H(\Omega)} dt \tag{4}$$

and reverse, the minimum of the equation (3) on M^0 is the solution of the equation (3).
Here $\langle .,. \rangle$ denotes the scalar product in the Hilbert space H(Ω). In this paper, the functional F given by (3) is the base of the variational form, used in finite element method, the numerical method used in application.
The problem (1)+(2) with conditions 1^0-6^0 is named the "fundamental Dirichlet problem". In this case the nonlinear operator P is given by the Navier-Stokes'equations, where the coefficients are the homogenized ones, considering as the composite material with short fibers a viscous compressible fluid.
The homogenized coefficients are obtained using an asymptotical development and the macroscopic equation [2].
If $u=(u_1, u_2, u_3)$, denotes the velocities' field of a fluid's particle, p the pressure, ρ, the density, $f=(f_1, f_2, f_3)$, the field of mass' force, υ the cinematik viscosity, in the stationary case, the nonlinear operator P is given by:

$$Pu = \langle u, \nabla u \rangle^*_{R^3} + \frac{1}{\rho} \nabla P - \upsilon \Delta u \tag{5}$$

Observe that from the general expression of Pu it is missing the terms $\frac{\partial u}{\partial t}$ and $\frac{\upsilon}{3}$ grad (div u), because of the stationary case, that is the velocity is independent of time and the fluid is incompressible.

Specify that:

$$\langle u, \nabla u \rangle^*_{R^3} = \left(\sum_{k=1}^3 u_k \cdot \frac{\partial u_1}{\partial x_k}, \sum_{k=1}^3 u_k \cdot \frac{\partial u_2}{\partial x_k}, \sum_{k=1}^3 u_k \cdot \frac{\partial u_3}{\partial x_k} \right)_T \tag{6}$$

and ∇ is the usual "nabla" and Δ is the Laplace's operator applied to the vectorial field u. So, the Navier-Stokes' equations is:

$$Pu=f \tag{7}$$

Analitically, the equation (7) can be given as a system:

$$\begin{cases} u_1 \frac{\partial u_1}{\partial x_1} + u_2 \frac{\partial u_1}{\partial x_2} + u_3 \frac{\partial u_1}{\partial x_3} + \frac{1}{\rho} \frac{\partial p}{\partial x_1} + \upsilon \Delta u_1 = f_1 \\ u_1 \frac{\partial u_2}{\partial x_1} + u_2 \frac{\partial u_2}{\partial x_2} + u_3 \frac{\partial u_2}{\partial x_3} + \frac{1}{\rho} \frac{\partial p}{\partial x_2} + \upsilon \Delta u_2 = f_2 \\ u_1 \frac{\partial u_3}{\partial x_1} + u_2 \frac{\partial u_3}{\partial x_2} + u_3 \frac{\partial u_3}{\partial x_3} + \frac{1}{\rho} \frac{\partial p}{\partial x_3} + \upsilon \Delta u_3 = f_3 \end{cases} \tag{8}$$

The scalar product is that from the space L_2 , (Ω), that is:

$$\langle u, v\rangle^{*}_{H(\Omega)} = \sum_{i=1}^{3} \iiint_{\Omega} u_i(x) v_i(x) dx \text{ , } x=(x_1,x_2,x_3), \text{ } u=(u_1,u_2,u_3)_T \text{ , } v=(v_1,v_2,v_3)_T \quad (9)$$

If $H_n(\Omega)$ is the finite-dimensional space, we search the finite element solution, then $u_n \in H_n(\Omega)$ has the form:

$$u_n(x) = \sum_{k=1}^{n} u^k \cdot \varphi_k(x) \quad (10)$$

where $\{\varphi_k\}$ is the finite element global base attached to a discretization of the composite material domain $\Omega \subset R^3$, and $u^k = (u_1^k, u_2^k, u_3^k)_T$ is the vector of the velocities in knots attached to the global knot "k" of the same discretization.

The functional used to obtain the finite element discret variational form we get replacing in (4) u by u_n, and using (9), we have:

$$\Phi_n(u) = \int_0^1 \langle P(tu_n), u_n\rangle^{*}_{H(\Omega)} dt \quad (11)$$

and

$$F_n(u) = \Phi_n(u) - \langle f, u_n\rangle^{*}_{H(\Omega)} \quad (12)$$

Finally, we obtain the finite element variational equation vanishing the partial derivatives of F_n in respect with all of velocities in knots.

Of course, a finite element procedure of local type can be apllied vanishing the partial derivatives of the functional Φ with respect to the nodal velocities.

2.APPLICATION :SIMULATION OF THE DISCONTINUOUS COMPOSITE FLUID FLOW

The analyses of the melt plastic material in injection molded can be of the type:

- Full flow
- Filling only
- Runner balance
- Molding window
- Gate location

In this paper we used only the first and the second option analysis. In order to analyze the flow of plastic material during it's injection in the mold it's necessary to cover several stages;(fig 1)

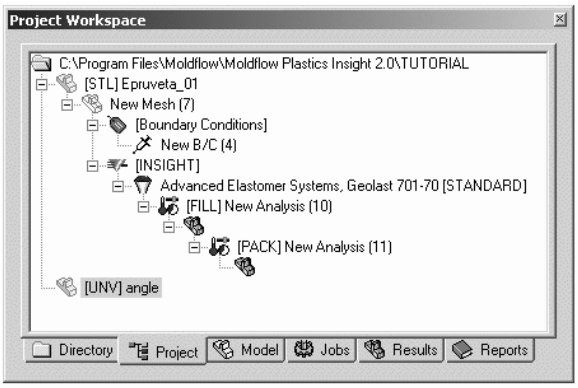

Figure1. Program files / moldflow plastics

1. The import of 3D models of the piece(test tube)
2. The discretization of geometrical model (New Mesh)
3. The selection of the boundary conditions

4. Stualysis Preparation Wizard

In this simulating we'll obtain the following results:

1. Fill pressure
2. The flow front temperature

The fill pressure (fig.2) presents the pressure distribution during the material flow. The pressure must be zero at the end of any flow trajectories and at the end of flow. Usually, the maximum pressure of injection is approximate 200 Mpa.

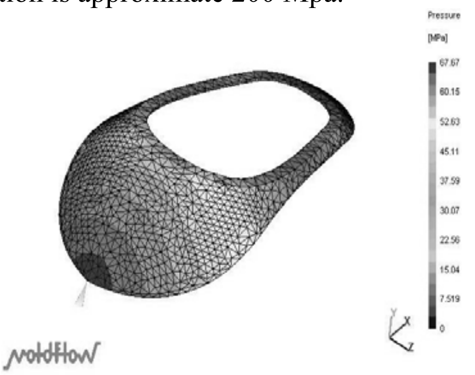

Figure 2. The fill pressure diagram

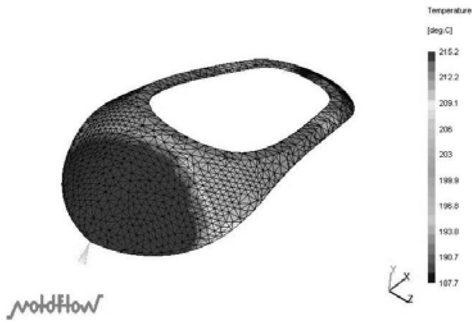

Figure 3. The flow fronts temperature diagram

The diagram of the flow fronts temperature(fig.3) presents the temperature distribution when the flow front reaches a specified point. The graphe can be obtaind at the end of the analysis or at a specified time during the analysis. Ts recomanded a small variation of the flow fronts temperature from the first point of the cavity until the last filled point.

3.CONCLUSIONS

This paper develops a mathematical modelling of injection-molded parts with complex geometry using the homogenization method. For the numerical procedure we use the finite element method. The paper concludes with a numerical example.

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