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STUDY OF VIBRATIONS GENERATED BY THE PASSAGE OF A TRAIN. A WHEEL-TRACK SYSTEM MODELIZATION.

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ABSTRACT.
The profile irregularities of railway wheels are the main source of vibrations generated by the passage of a train. These vibrations are transmitted to the surroundings, affecting the zones near the railway network as well as the own railway equipment. In this work, an analytical wheel-rail contact model is presented, with the aim of studying the vibration in a track location generated by the irregularities in the wheel profile. Each bogie's wheel is modeled as a one degree of freedom system with its stiffness and viscous damping depending on the primary bogie suspension. On the other hand, two different models are used in the track description; the first one considers the rail as an infinite Euler beam supported on uniformly distributed sleepers and ballast with hysteretic damping, whereas the last one is a modal approximation of the previous model, which consists in a system with two degrees of freedom with viscous damping. The wheel-rail contact force is modeled by means of the nonlinear hertzian contact theory. Using this model, the rail response to the presence of wheel flats and other wheel profile irregularities is obtained and the results corresponding to these situations are also presented.
Keywords: wheel-rail contact, nonlinear hertzian contact theory, wheel profile irregularities.

1. INTRODUCTION.
Generation models of the vibrations produced by wheel-rail contact, allow to study the relationship between the vibration in a rail location and the wheel irregularities profile. Among these irregularities, wheel flats stand out, because they are responsible for high vibration levels, according to the established by [1].
The information obtained by simulation of the system behavior with vibration generation models, is necessary in order to develop wheel flat identification algorithms, based on the vibration measures obtained in one or several points of the track to the passage of a train.
In this paper a global vibration generation model is presented. This model includes the track model, the wheel model and the wheel-rail contact force model. This models will be as simple as possible, trying that the system will be suitably represented without loss of fundamental information.

2. TRACK DYNAMIC MODEL.
In order to study the railway track dynamic behavior, two different models have been developed. The first one is a track continuous model in which the rail is considered as an infinite Euler beam. The sleepers and the ballast are considered as a uniform mass layer distributed along the rail direction. Between the rail and the sleepers, as well as between the sleepers and the ballast, stiffness and hysteretic damping are also uniformly distributed.

Previously results published by [2,3] demonstrate that this model correctly describes the railway dynamic behavior for frequencies up to 500 Hz. For frequencies above this level, vibration modes associated to the rail discrete support on the sleepers, as well as other rail vibration modes, appear. Vibrations produced by the passage of a train and their effect on the surroundings are in the frequency range 10 – 400 Hz, which justifies the consideration of the track continuous model.
The second track model is a two degree of freedom model obtained by means of modal analysis from the track continuous model. Modal parameters: inertia, stiffness and damping, in this case viscous damping is considered, are obtained from the railway vertical receptance analysis, computed using the track continuous model. Receptance is the frequency response function, that relates the railway vertical displacement with the vertical force applied at the same point.
Railway vertical receptance shows two vibration modes under 500 Hz, which are related with the two degrees of freedom of the simplified model. Figure 1 presents a railway vertical receptance comparison, amplitude and phase, obtained with the two described models. Significant differences between both results in the frequency range of interest are not observed.

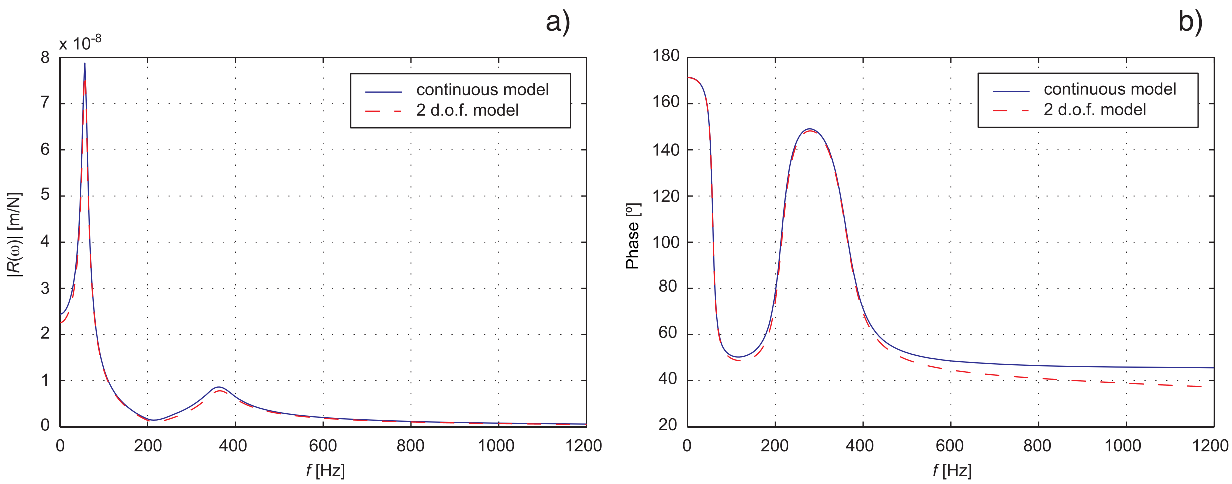

Figure 1. Railway vertical receptance comparison.
a. Amplitude. b. Phase.

The interest in this two degree of freedom model lays on its easier implementation in the equation system that leads to the vibration generation simulation.

3. WHEEL-RAIL INTERACTION DYNAMIC MODEL.

The aim of the dynamic model presented is the contact force temporal history determination. The dynamic model consists of three parts: track model, wheel model and contact force model. The track model is the two degree of freedom model exposed in section 3. The wheel is modeled as a rigid mass with a primary suspension, formed by a spring-damper system with linear behavior. The non suspended mass, bogie and coach, is not affected by the suspension movement. This statement is valid for frequencies over 10 Hz, which is the considered case. The coach weight is equally distributed on the eight wheels of the two bogies of a coach. Through this model, wheel center vertical displacement that takes part in the contact force expression, is determined.
The contact force model is based on the nonlinear hertzian contact theory. Thus the contact force is proportional to the elastic contact deflection to the power 3/2. Hertzian model has been employed for several authors, which have checked its validity, [4]. The matrix expression 1 shows the equations of motion that describe the dynamics of the interaction model,

$$\begin{bmatrix} m_w & 0 & 0 \\ 0 & m_r & 0 \\ 0 & 0 & m_s \end{bmatrix} \begin{Bmatrix} \ddot{y}_w \\ \ddot{y}_r \\ \ddot{y}_s \end{Bmatrix} + \begin{bmatrix} c_w & 0 & 0 \\ 0 & c_r & -c_r \\ 0 & -c_r & c_r + c_s \end{bmatrix} \begin{Bmatrix} \dot{y}_w \\ \dot{y}_r \\ \dot{y}_s \end{Bmatrix} + \begin{bmatrix} k_w & 0 & 0 \\ 0 & k_r & -k_r \\ 0 & -k_r & k_r + k_s \end{bmatrix} \begin{Bmatrix} y_w \\ y_r \\ y_s \end{Bmatrix} + \begin{Bmatrix} -F \\ F \\ 0 \end{Bmatrix} = \begin{Bmatrix} W_1 \\ W_2 \\ W_3 \end{Bmatrix} \quad ...(1)$$

where the subscripts w, r and s identify respectively the inertia, damping and stiffness parameters of the wheel, the rail and the sleepers. W_1, W_2 and W_3 are the static load components and F is the contact force, whose formulation is defined by equation 2, which depends of the nonlinear elasticity constant K_{H}, the vertical rail and wheel displacements, the wheel radius r and the wheel irregularities profile ε.

$$F = \begin{cases} K_{\mathrm{H}}\left(y_{\mathrm{r}} - y_{\mathrm{w}} + r - \varepsilon(x)\right)^{3/2} & y_{\mathrm{r}} - y_{\mathrm{w}} + r - \varepsilon(x) > 0 \\ 0 & y_{\mathrm{r}} - y_{\mathrm{w}} + r - \varepsilon(x) \leq 0 \end{cases} \qquad \ldots(2)$$

4. CONTACT FORCE TIME HISTORY.

In this section the contact force time history obtained by means of the proposed model is presented. Two wheel-rail interaction situations are considered, in the first one the wheel does not have any irregularity in its profile, nevertheless the second situation considers a theoretical wheel flat which fulfills the wheel flat description presented by [5]. Figure 2 shows the contact force time history for both situations when the train moves at 65 km/h.

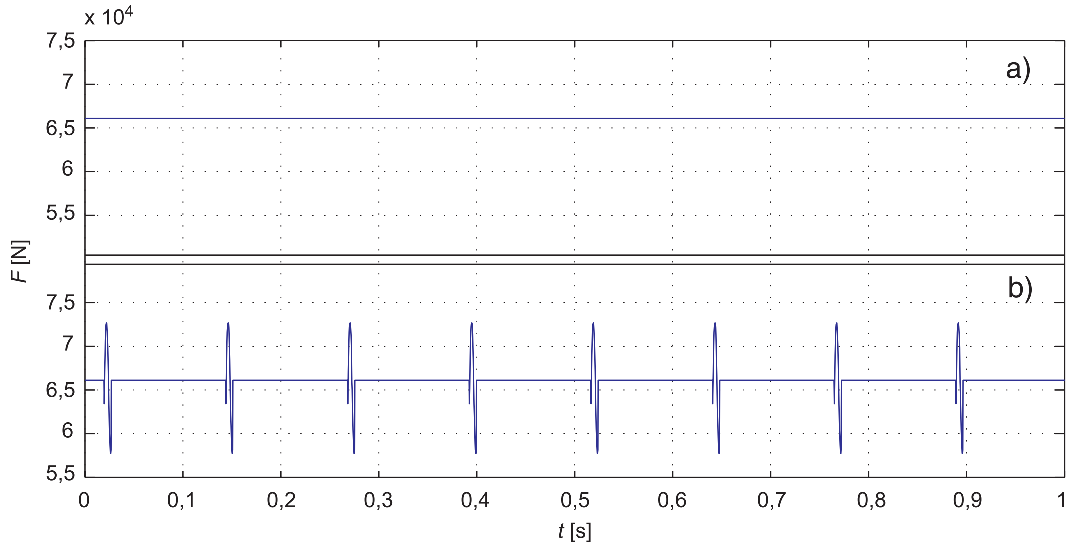

Figure 2. Contact force time history.
a. Wheel without any irregularity. b. Wheel with a wheel flat.

It is observed in the first case, and as expected, that the contact force is constant an equal to the static load related to the coach, bogie and wheel weight. In the second situation appears, added to the static load, a damped pulse vibration which repeats periodically every wheel revolution.

5. TIME HISTORY OF THE RAIL DEFLECTION IN A FIXED POINT.

In this section the temporal evolution of the rail vertical displacement in a fixed point due to the contact force applied on the rail, is presented. This displacement is computed by means the convolution method with variable kernel, proposed by [6]. This methodology is based on the track impulse response function, obtained as FFT^{-1} of the vertical receptance. The impulse response is the convolution kernel and relates the rail deflection in a fixed point with the contact force that moves along the rail. So, the impulse response changes as the distance between the contact force location an the considered fixed point does. Then, the convolution kernel has to be recalculated at every integration step. Figure 3 shows the rail deflection for both situations proposed in the contact force history determination.

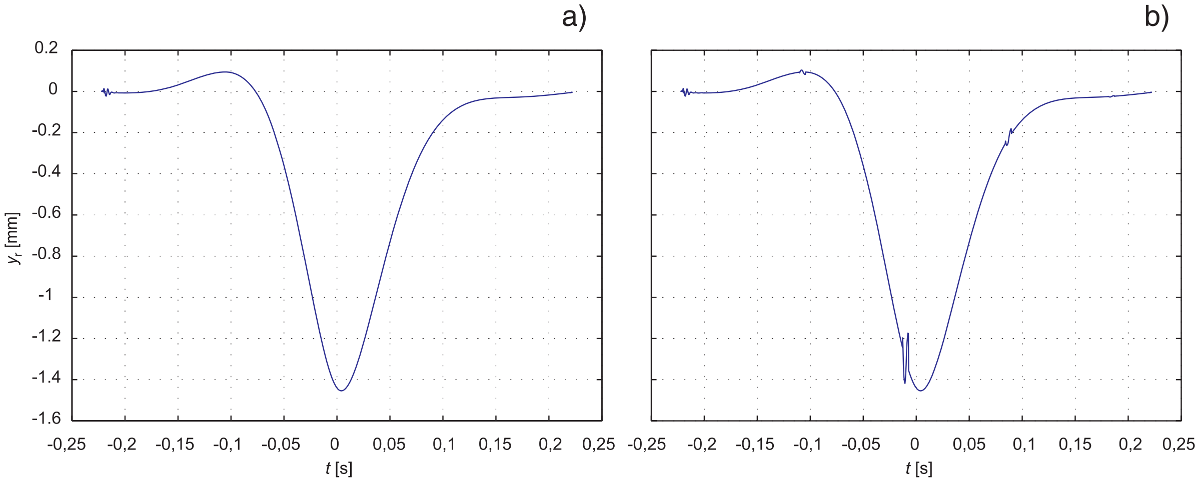

Figure 3. Time history of the rail deflection in a fixed point.
a. Wheel without any irregularity. b. Wheel with a wheel flat.

6. CONCLUSIONS.

The simplified railway model obtained by means of the modal analysis, is appropriate to study the wheel-rail dynamic interaction for the frequency range of interest. The nonlinear hertzian contact is a valid method to determine the wheel-rail contact force temporal evolution.

A wheel-rail global contact model has been developed. This model allows to determine the rail deflection in a fixed point to the passage of a single wheel, with different characteristics in its surface.

The presented model is the base for the study of the vibration generation due to the passage of a train and the development of identification algorithms of wheel profile irregularities.

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