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ANALYSIS OF THE OF HELICOIDAL SHELL BENDING ON CYLINDRICAL SHELL

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ABSTRACT
In this paper comparative analyses of analytical, FEM an experimental researches of the bending of helicoidal shell on cylindrical shell are given. According to a differential equation in displacement for bending of a helicoidal shell subjected to uniform pressure and a differential equation for cylindrical shell loaded by uniformly distributed couple along helix, the analytical results are given. The comparable results obtained by FEM software code Pro/MECHANICA are also given in the paper. The original model for experimental research on the structural elements of helicoidal shell shape is used. It is obtained a good accordance between analytical, FEM and experimental results.
Keywords: Shell, Helicoidal, Structure, Plate, Cylindrical, Differential, Strains, Force, Numerical, Finite Element, Mesh

1. INTRODUCTION
At special machines there may be encountered constructional elements in the shape of a thin cylindrical shell on which a helicoidal shell is welded. The load of the helicoidal shell is transmitted to the cylindrical shell through a line of cross section of central surfaces, i. e. through the helicoidal shell. In case the helicoidal shell is under pressure, the resultant of that load on the cylinder is consisted of a continually distributed force in a tangential plane of the cylinder and a continual coupling in tangent ional direction on the helicoidal line on the cylinder. Helicoidal shell bending on the cylindrical shell by applying analytical, FEM and experimental methods are considered in the paper. Parallel results according to displacements and tensions in radial direction for all three types of analysis are given in the end.

2. ANALYTICAL ANALYSES
A helicoidal shell is a part of the surface of conoidal helicoid which is placed on cylinder along its helicoidal line, whereas on its external part, it is limited by a helicoidal line of the same stroke and by a constant diameter, like in Figure 1. - model A. In a special case when the helicoidal line stroke is H=0, the helicoid becomes a circumference, whereas a conoidal helix turns into a circle ring, like in Figure 2. - model B. If stroke $H\rightarrow\infty$, then helicoid becomes a line, this being a special case for rectangle plate placed on a cylindrical shell, like in Figure 3. - model C.
Using the results of the analytical analyses made for the helicoidal shell continually stressed and for the cylindrical shell loaded by a continual coupling along the helicoidal line, a problem of connected shells is considered here. In the paper [6] there is performed a differential equation of displacement for the helicoidal shell in the shape of.

$$\frac{d^4u_3}{dr^4}+\frac{2r}{r^2+k^2}\frac{d^3u_3}{dr^3}-\frac{r^2+k^2(\nu+1)}{\left(r^2+k^2\right)^2}\frac{d^2u_3}{dr^2}+\frac{r\left[r^2+k^2(3\nu+4)\right]}{\left(r^2+k^2\right)^3}\frac{du_3}{dr}+$$

$$+\left[\frac{2k^2(\nu+1)\left(3k^3-8r^2\right)}{\left(r^2+k^2\right)^4}+\frac{24k^2(1-\nu)}{{h_h}^2\left(r^2+k^2\right)^2}\right]u_3+\frac{P}{B_h}=0$$

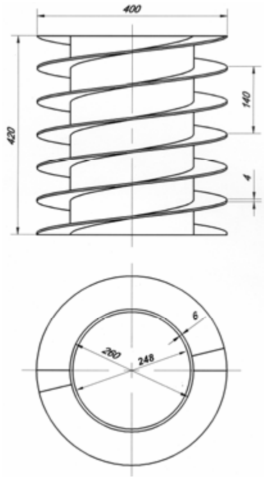
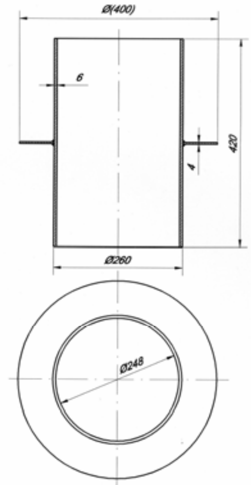


Figure 1.



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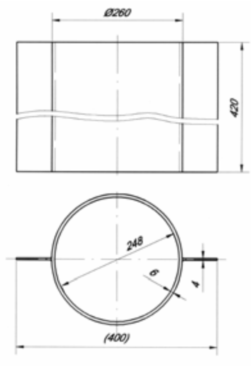
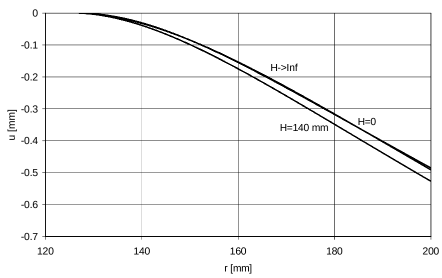


Figure 3.

In differential equation r is a curved coordinate on the helicoidal surface, h_h - thickness of the helicoidal shell, $k=H/2\pi$ - bending of the helicoidal surface, H - height of the helicoidal surface stroke, E - elasticity module, ν - Poison's coefficient, P - load uniformly distributed along the shell surface, B_h - stiffness to shell bending determinated by relation $B_h=Eh_h^3/12(1-\nu^2)$. The programme for numerical solution of differential equation of helicoidal shell bending is made in the programme package MATLAB. In Figures 4-5. are given the displacement and tension dependences in radial direction for the considered models A, B and C.



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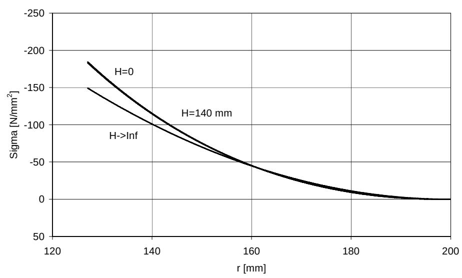


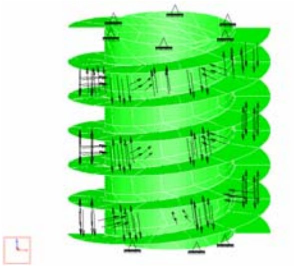
Figure 5.

Concrete scientific and professional contribution to an automatized projecting of the helicoidal shell on cylindrical shell represents a proposed method of determining shell parameters for the models of different geometry, load, material characteristics, and others.

3. FEM ANALYSES

FEM analyses for three models of the helicoidal shell on the cylindrical shell have been made, according to Figures 1-3. using a programme package Pro/MECHANICA. The nets of finite elements of the considered models are given in Figures 6-8. whereas the results of the tension analyses in radial direction and displacement for models A, B and C are given in Figures 9-11.

A very little difference of the results of both analytic and FEM analyses which, if compared to the criteria of maximum stress in radial direction below 2% for all the considered models, confirms a high accuracy of these analyses. It also confirms the accuracy of mathematical models in analytical analyses of helicoidal and cylindrical shells.



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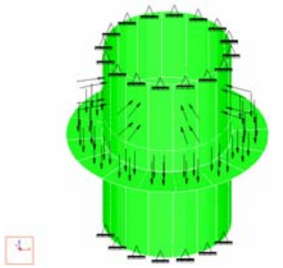

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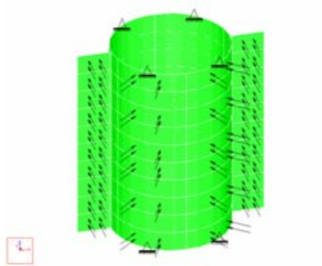

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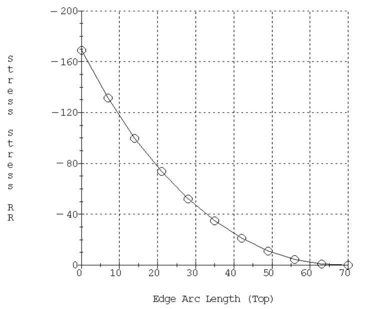

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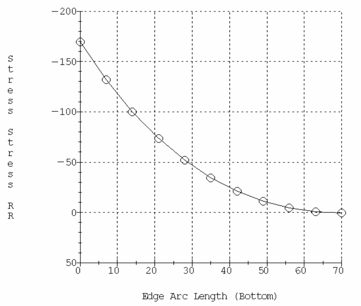

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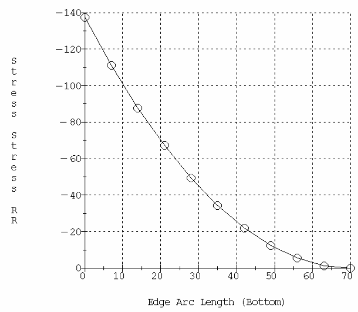
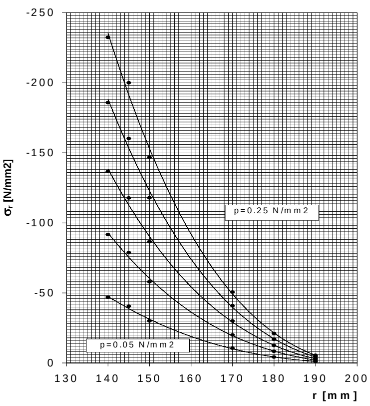


Figure 11.

4. EXPERIMENTAL ANALYSES

The investigation results on original model for experimental construction elements in the shape of helicoidal shell are given in this paper. Deformation and stress analyses are done by tenzometrical investigations placing strain guage on free surfaces of helicoidal shell.



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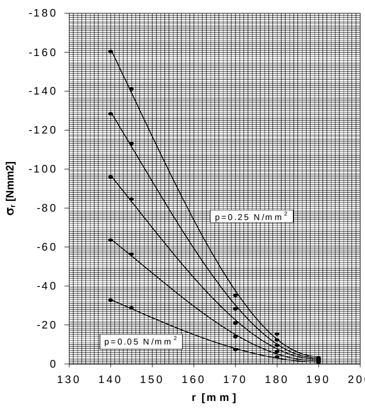

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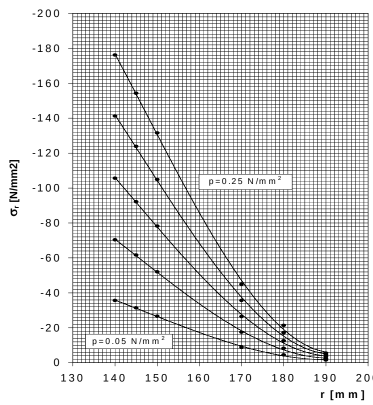

Figure 14.

On these free surfaces in the chambers without pressure, measuring instruments for displacements of points on helicoidal shell are placed. In Figures 12-14. stress diagrams in radial direction in dependence on coordinate for pressure increments 0.05, 0.1, 0.15, 0.2, 0.25 N/mm^2 are derived.

5. PARALLEL RESULTS

Based on tenzometrical analyses in experimental model, the derived diagrams for stress components show an accordance of the results obtained with analitical and FEM analyses. The results of experimental investigations as well as the previous analytical an FEM analyses confirm that stresses in radial direction in the root of helicoidal shell at cylindrical shell are maximum and that the criterion for allowed values of these stresses is relevant for determining geometrical parameters of helicoidal shell. Parallel values to be compared for stress in radial direction and displacement for models A, B and C is given in Table 1. The results of both analytical and FEM analyses for the models considered show an accordance in narrow limits.

Table 1. Parallel values for stresses and displacements for models A, B and C

			Stress in radial direction [N/mm^2]			Displacement in z direction [mm]		
			r=140 [mm]			r=195 [mm]		
Model	H [mm]	h_h [mm]	Analytical analysis	FEM analysis Pro/M	Experimental analysis	Analytical analysis	FEM analysis Pro/M	Experimental analysis
A	140	3.6	-141.7820	-144.8671	-141.7030	-0.6438	-0.6578	-0.40
B[2)]	0	4.15	-108.3903	-110.2363	-95.0440	-0.3347	-0.3499	-0.26
C	∞	4	-100.9882	-101.3476	-103.9010	-0.4474	-0.4734	-0.40

The results of experimental investigations differ below 1% for model A, about 13% for model B and around 3% for model C if compared to the results of analytical analyses according to criteria of stress in radial direction. Besides deviations of single geometrical parameters of shell that really appear at making such construction shapes, an agreement of the results of experimental investigations with the results of both analytical and FEM analyses are obtained.

6. CONCLUSION

An application of analytical analyses with numerical solutions, FEM analysis and experimental method of investigating construction elements of helicoidal shell shape, has enabled a reliable and integral insight into interrelated influences of the mechanical model parameters of this shell and development of the methodology of automatized projecting of these construction elements.

An agreement of analytical, FEM and experimental results has proved that: an application of FEM is a powerful numerical method in static analysis and this complex structure, efficiency of the original method for experimental investigation of construction elements of helicoidal shell exposed to a uniform stress and correctness of introduced suppositions and procedures in analytical investigation of this shape of shell.

The original model for experimental investigation has proved the efficiency of deformation analyses of construction elements of helicoidal shell exposed to a uniform load vertically to surface.

In experimental investigations, the accuracy of results is influenced by the preciseness of making models, at which the thickness of shell as the most influential factor on the obtained values of stress and displacement should be emphasized.

The solutions applied to experimental methods are possible to be also used in investigating helicoidal shells of changeable thickness that may be made on modern CNC machines.

7. REFERENCES

[1] Cohen J. W., The Inadequacy of the Classical Stress-Strain Relations for the Right Helicoidal Shell, Proceedings of the Symposium on the Theory of Thin Elastic Shells, North-Holland, Delft, 1959, pp. 415-433

[2] Mikhlin S.G., Estimate of the Error in the Computation of an Elastic Shell as a Flat Plate, Akademija nauka SSSR, Prikladnaja matematika 16, Moskva, 1952, 399

[3] Savićević S., Vukasojević R., Ćulafić Z., Kalajdžić M., A Model for Experimental Investigation of Construction Elements of Helical Shell-Shape, 26. JUPITER Conference, Belgrade, 2000, pp. 2.47-2.54

[4] Savićević S., Kalajdžić M.: An Automatized Determination of Helical Shell Parameters on Cylindrical Shell, 28. Symposium on Productive Mechanical Engineering of Yugoslavia, Kraljevo, 2000.

[5] Savićević S., Đogović V., Bending of Structural Elements of the Helical Shell Shape, 23rd Yugoslav Congress of Theoretical and Applied Mechanics, 1999.

[6] Savićević S., A Development of Automatized Projection of Construction Elements of Helical Shell Shape, Ph.D. Dissertation, Faculty of Mechanical Engineering, Podgorica, 2001.