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MATHEMATICAL MODELLING AND SIMULATION OF PROCESSES WITH MOIST AIR IN A REFRIGERATED ROOM

Matko Bupic
University of Dubrovnik, Department of Mechanical Engineering
Cira Carica 4, HR-20000 Dubrovnik
Croatia (Hrvatska)

Branimir Pavkovic
University of Rijeka, Faculty of Engineering
Vukovarska 58, HR-51000 Rijeka
Croatia (Hrvatska)

ABSTRACT

Knowledge of the psychrometric properties of moist air is essential in the design of air conditioning, cold storage, and drying processes. The presented paper describes the development of mathematical model suitable for calculating psychometric properties and for simulation of dynamic behaviour of moist air in refrigerated room.

Moist air, circulating around refrigerated product within cold chamber, is continuously in direct contact with surface of the goods, chamber walls, chamber outfit, fans, defrosting coils and evaporator surface. Simultaneous heat and mass exchange occurs between moist air and all elements that air flows by. During the flow within the cold chamber, moist air is simultaneously exposed to all typical changes of state, such as cooling, heating, dehumidification, moistening and mixing with outside air stream caused by door opening and air leakage.

Developed mathematical model, describing the dynamics of air temperature and humidity change caused by those multiple influential factors, is based on mass and energy balances. Such model provides calculation of psychometric properties and simulation of dynamic behaviour of unsaturated and saturated moist air between temperature ranges of above and below 0°C (including 0°C).

Keywords: dynamic mathematical model, moist air, refrigerated room

1. INTRODUCTION

Prior to setting a dynamic mathematical model of moist air in cold chamber, it is necessary to define its properties, as well as mutual relations of those properties based onto physical laws of the moist air theory [1, 2]. The water vapour saturation pressure $p_{g\,s,A}$, Pa, at given temperature of the moist air T_A, (K), can be calculated by the application of Hyland-Wexler's equation [1]:

$$p_{g\,s,A}(T_A) = exp\left(\frac{C_1}{T_A} + C_2 + C_3 T_A + C_4 T_A^2 + C_5 T_A^3 + C_6 T_A^4 + C_7\, ln\, T_A\right), \tag{1}$$

where constants C_i are given in *Table 1*.

To calculate the specific enthalpy of moist air h_A, Jkg^{-1}, the following equation is used:

$$h_A = 1005\,\vartheta_A + x_{g\,A}(2500357 + 1830\,\vartheta_A) + x_{l,A}\, c_l\, \vartheta_A - x_{i,A}(334000 - c_i\, \vartheta_A), \tag{2}$$

where: $x_{g\,A}$, kgkg^{-1}, is the vapour content in saturated moist air; $x_{l,A}$, kgkg^{-1}, is liquid water content; $x_{i,A}$, kgkg^{-1}, is ice content. For the specific heat capacity of liquid water and ice, c_l and c_i, Jkg^{-1}K^{-1}, real (local) values depending on the current air temperature ϑ_A, °C will be introduced. The constants are taken from available literature [3, 4, 5] and corrected. Thus, the results of the equation (2) do not derogate more than the value of 0,05 % from the results obtained by Mollier's equation [6].

Table 1. Coefficients of the Hyland-Wexler Equation [1]

	173,15 K < T_A ≤ 273,15 K	273,15 K < T_A < 473,15 K
C_1	– 5,674 359 0 E+03	– 5,800 220 6 E+03
C_2	6,392 524 7 E+00	1,391 499 3 E+00
C_3	– 9,677 843 0 E–03	– 4,864 023 9 E–02
C_4	6,221 570 1 E–07	4,176 476 8 E–05
C_5	2,074 782 5 E–09	– 1,445 209 3 E–08
C_6	– 9,484 024 0 E–13	0,000 000 0 E+00
C_7	4,163 501 9 E+00	6,545 967 3 E+00

2. MODEL DESCRIPTION AND PRESUMPTIONS

Moist air, which is a mixture of dry air ($m_{a,A}$, kg) and moisture ($m_{w,A}$, kg), is circulating around refrigerated product within cold chamber, thus guaranteeing the uniform distribution of temperature within the chamber. Refrigerated moist air (A) is continuously in direct contact with surface of the refrigerated product (P), chamber walls (W), chamber structures and its outfit (S), fans (F), defrosting coils (D) and most of all with evaporator surface (V), as shown in the structural model on figure 1. Consequently, the change of heat and mass between moist air and system elements in contact with it occurs. The moist air in cold chamber is continuously exposed to all characteristic changes of state: refrigerating, cooling, drying, moistening and air streams mixing due to the fact of chamber leakage and door opening.

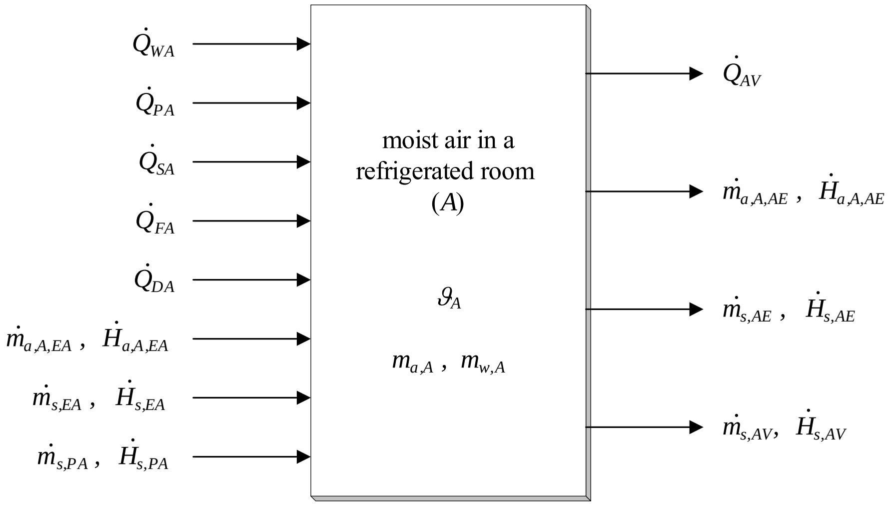

Figure 1. Structural Model of Refrigerated Moist Air [7]

The dynamic mathematical model of refrigerated moist air has been developed, describing the dynamics of air temperature ϑ_A (t), and value of moisture content x_A (t) and relative humidity φ_A (t), caused by those multiple influential factors represented in the structural model in figure 1. During the model setting, following presumptions and simplifications are involved [7]:

- air temperature ϑ_A is equally distributed within the whole space of cold chamber,
- dry air and superheated water vapour are treated as ideal gases,
- change state of moist air occurs at constant volume,
- partial pressure of dry air $p_{a,A}$ in the mixture is changed according to the law of isochor,
- dry air leakage mass flow into the environment $\dot{m}_{a,A,AE}$, is equal to dry air mass flow penetrating into chamber $\dot{m}_{a,A,EA}$, thus resulting in constant dry air mass within the chamber $m_{a,A}$,
- mass flow of moisture which is in a form of liquid water or ice within the moist air is disregarded,
- specific heat capacities of dry air $c_{p\,a,A}$ *and* $c_{v\,a,A}$, and superheated water vapour $c_{p\,s}$ and $c_{v\,s}$ have constant values.

3. EQUATIONS OF ENERGY AND MASS CONSERVATION

Mathematical model is based on mass and energy balances. The equation of energy conservation for refrigerated moist air within cold chamber may be written in the form of:

$$\frac{dU_A(t)}{dt} = \dot{Q}_{A,rez}(t) + \dot{H}_{a,A,rez}(t) + \dot{H}_{s,A,rez}(t), \tag{3}$$

where: U_A, J, is internal energy of moist air; $\dot{Q}_{A,rez}$, Js^{-1}, is a sum of all of heat flows, to which moist air is exposed; $\dot{H}_{a,A,rez}$, Js^{-1}, is a sum of all of enthalpic flows for dry air; and $\dot{H}_{s,A,rez}$, Js^{-1}, is a sum of all of enthalpic flows for water vapour within the mixture of moist air.

The left side of the equation (3) is representing the change of internal energy which is being stored in moist air, and with the presumption on constant mass of the dry air, it may be expressed as follows:

$$\frac{dU_A(t)}{dt} = m_{a,A}\frac{du_{a,A}(t)}{dt} + m_{w,A}(t)\frac{du_{w,A}(t)}{dt} + u_{w,A}(t)\frac{dm_{w,A}(t)}{dt}, \tag{4}$$

where: $u_{a,A} = c_{va,A}\,\vartheta_A$, Jkg^{-1}, is specific internal energy of dry air; and $u_{w,A} = u_{s,A} = \rho_{ls} + c_{vs}\,\vartheta_A$, Jkg^{-1}, is specific internal energy of humidity in the form of superheated vapour within non-saturated moist air. Introduction of $u_{a,A}$ and $u_{w,A}$ into the equation (4) is resulting with:

$$\frac{dU_A(t)}{dt} = m_{a,A}\,c_{va,A}\frac{d\vartheta_A(t)}{dt} + m_{s,A}(t)\,c_{vs}\frac{d\vartheta_A(t)}{dt} + \left[\rho_{ls} + c_{vs}\,\vartheta_A(t)\right]\frac{dm_{s,A}(t)}{dt}. \tag{5}$$

$\dot{Q}_{A,rez}(t)$ is the sum of all heat flows towards and from the moist air within cold room:

$$\dot{Q}_{A,rez}(t) = \dot{Q}_{WA}(t) + \dot{Q}_{PA}(t) + \dot{Q}_{SA}(t) + \dot{Q}_{FA} + \dot{Q}_{DA} - \dot{Q}_{AV}(t). \tag{6}$$

Mass balance of superheated water vapour within non-saturated moist air gives the expression:

$$\frac{dm_{s,A}(t)}{dt} = \dot{m}_{s,PA}(t) + \dot{m}_{s,EA}(t) - \dot{m}_{s,AE}(t) - \dot{m}_{s,AV}(t). \tag{7}$$

Mass flows of dry air and water vapour are connected with their enthalpic flows. The resultant of all of the enthalpic flows of dry air into the moist air is:

$$\dot{H}_{a,A,rez}(t) = \dot{m}_{a,A,EA}(t)\,h_{a,A,E} - \dot{m}_{a,A,AE}(t)\,h_{a,A,A}, \tag{8}$$

while, the resultant of all of the enthalpic flows of water vapour within the moist air is:

$$\dot{H}_{s,A,rez}(t) = \dot{m}_{s,PA}(t)\,h_{s,P} + \dot{m}_{s,EA}(t)\,h_{s,E} - \dot{m}_{s,AE}(t)\,h_{s,A} - \dot{m}_{s,AV}(t)\,h_{s,A}. \tag{9}$$

Rearrangement of previous equations [7], results in equations which are describing the dynamics of the temperature $\vartheta_A(t)$ of refrigerated non-saturated moist air within the refrigerated room:

$$\frac{d\vartheta_A(t)}{dt} = \frac{\dot{Q}_{WA}(t) + \dot{Q}_{PA}(t) + \dot{Q}_{SA}(t) + \dot{Q}_{FA} + \dot{Q}_{DA} - \dot{Q}_{AV}(t) + \dot{m}_{a,A,EA}(t)c_{pa,A}\left[\vartheta_E(t) - \vartheta_A(t)\right]}{m_{a,A}\,c_{va,A} + m_{s,A}(t)\,c_{vs}} + \\ + \frac{\dot{m}_{s,PA}(t)\,h_{s,P} + \dot{m}_{s,EA}(t)\,h_{s,E} - \left[\dot{m}_{s,AE}(t) + \dot{m}_{s,AV}(t)\right]h_{s,A} - \dot{m}_{s,A}(t)\left[\rho_{ls} + c_{vs}\,\vartheta_A(t)\right]}{m_{a,A}\,c_{va,A} + m_{s,A}(t)\,c_{vs}}, \tag{10}$$

and expressions for the current value of relative humidity $\varphi_A(t)$ and moisture content $x_A(t)$:

$$\varphi_A(t) = \frac{m_{s,A}(t)\,R_s\,T_A(t)}{V_A\,p_{gs,A}(\vartheta_A)}, \tag{11}$$

$$x_A(t) = 0{,}62198\frac{\varphi_A(t)\,p_{gs,A}(\vartheta_A)}{p_A(t) - \varphi_A(t)\,p_{gs,A}(\vartheta_A)}. \tag{12}$$

4. MODIFICATION OF THE MODEL FOR THE PHASE CHANGES PHENOMENA

When the moisture content x_A exceeds moisture content of saturated air $x_{g,A}$ ($\boldsymbol{x_A \geq x_{g,A}}$), the condensation and/or desublimation of water vapour as well as the phenomenon of liquid droplets and/or ice crystals occurs, depending on value of the air temperature $\vartheta_A(t)$. When the air temperature is $\boldsymbol{\vartheta_{A.}(t) > 0}$ **°C**, the saturated moist air contains the moisture in the form of superheated water vapour and liquid droplets. In that case equation (10) is replaced with (13), which is describing dynamics of the temperature of the saturated moist air with the surplus of humidity $m_{l,A}(t)$ in the form of fog:

$$\frac{d\vartheta_A(t)}{dt} = \frac{\dot{Q}_{WA}(t) + \dot{Q}_{PA}(t) + \dot{Q}_{SA}(t) + \dot{Q}_{FA} + \dot{Q}_{DA} - \dot{Q}_{AV}(t) + \dot{m}_{a,A,EA}(t)\,c_{pa,A}\left[\vartheta_E(t) - \vartheta_A(t)\right] + \dot{m}_{s,PA}(t)\,h_{s,P}}{m_{a,A}\,c_{va,A} + m_{gs,A}(\vartheta_A)\,c_{vs} + m_{l,A}(t)\,c_l} +$$
$$+ \frac{\dot{m}_{s,EA}(t)\,h_{s,E} - \left[\dot{m}_{s,AE}(t) + \dot{m}_{s,AV}(t)\right]h_{s,A} - \dot{m}_{gs,A}(\vartheta_A)\left[\rho_{ls} + c_{vs}\,\vartheta_A(t)\right] - \dot{m}_{l,A}(t)\,c_l\,\vartheta_A(t)}{m_{a,A}\,c_{va,A} + m_{gs,A}(\vartheta_A)\,c_{vs} + m_{l,A}(t)\,c_l}. \tag{13}$$

If air temperature is $\boldsymbol{\vartheta_A(t) < 0}$ **°C**, the humidity in the saturated moist air is in the form of superheated water vapour and small ice crystals, and equation (10) assumes the form of equation (14), describing dynamics of temperature of the saturated moist air with the surplus of humidity $m_{i,A}(t)$ in the form of white ice or snow:

$$\frac{d\vartheta_Z(t)}{dt} = \frac{\dot{Q}_{WA}(t) + \dot{Q}_{PA}(t) + \dot{Q}_{SA}(t) + \dot{Q}_{FA} + \dot{Q}_{DA} - \dot{Q}_{AV}(t) + \dot{m}_{a,A,EA}(t)\,c_{pa,A}\left[\vartheta_E(t) - \vartheta_A(t)\right] + \dot{m}_{s,PA}(t)\,h_{s,P}}{m_{a,A}\,c_{va,A} + m_{gs,A}(\vartheta_A)\,c_{vs} + m_{i,A}(t)\,c_i} +$$
$$+ \frac{\dot{m}_{s,EA}(t)\,h_{s,E} - \left[\dot{m}_{s,AE}(t) + \dot{m}_{s,AV}(t)\right]h_{s,A} - \dot{m}_{gs,A}(\vartheta_A)\left[\rho_{ls} + c_{vs}\,\vartheta_A(t)\right] - \dot{m}_{i,A}(t)\,c_i\,\vartheta_A(t)}{m_{a,A}\,c_{va,A} + m_{gs,A}(\vartheta_A)\,c_{vs} + m_{i,A}(t)\,c_i}. \tag{14}$$

Finally, if the temperature of saturated moist air is $\boldsymbol{\vartheta_A(t) = 0}$ **°C**, the humidity can appear in all three aggregate forms – as superheated vapour, liquid water and ice. Equation (15), describing the dynamics of liquid water within the saturated moist air, is derived by modification of the equation (10):

$$\frac{dm_{l,A}(t)}{dt} = \frac{1}{r_{il}}\Big[\dot{Q}_{WA}(t) + \dot{Q}_{PA}(t) + \dot{Q}_{SA}(t) + \dot{Q}_{FA} + \dot{Q}_{DA} - \dot{Q}_{AV}(t) + \dot{m}_{a,A,EA}(t)\,c_{pa,A}\,\vartheta_E(t) +$$
$$+ \dot{m}_{s,PA}(t)\,h_{s,P} + \dot{m}_{s,EA}(t)\,h_{s,E} - \dot{m}_{s,AE}(t)\,h_{s,A} - \dot{m}_{s,AV}(t)\,h_{s,A} + 334000\,\dot{m}_{w,A}(t)\Big]. \tag{15}$$

5. CONCLUSIONS

In the presented paper, the mathematical model for calculation of all possible states of refrigerated moist air within the cold chamber has been developed. It can be used for simulation of dynamic behaviour and those refrigerating systems where, due to door opening, moisture content $x_A(t)$ of air in cold chamber occasionally over exceeds saturation limit $x_{g\,A}$. Developed model can enable calculations of psychometric properties and simulation of dynamic behaviour of non-saturated and saturated moist air, at temperatures over and under 0 °C. The method of *System Dynamics*, known as Forrester's Dynamics, was used for development of dynamic numerical model, and *Powersim* program was used for simulation. The model is verified and confirmed by comparison of simulation results with experimental results.

6. REFERENCES

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