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SIMPLE APPROACH TO CONTROL OF MIMO CONTROL LOOPS

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ABSTRACT
This paper describes one of possible approaches to control of MIMO control loops with utilisation of binding members and correction members. Binding members are used here for ensuring autonomy of control loop. Binding members are determined from so called main controllers, which are main diagonal elements of the transfer matrix of controller $\boldsymbol{G}_R$ *. The design of main controllers is carried out by any SISO method of synthesis. Correction members of transfer matrix of controller* $\boldsymbol{G}_{KC}$ *serve for ensuring invariance of control loop and they are determined by using analogy of SISO branched control loops with measuring of dominant disturbance variable. Simulation verification was carried out for three-variable loop of a steam turbine.*
Keywords: MIMO control loop, control algorithm, autonomy, invariance

1. INTRODUCTION
At large numbers of controlled object (air-conditioning plants, distillation columns, steam boilers, turbines, etc.) several variables have to be controlled at the same time. In this case there is not larger member of independent SISO (single-input/single-output) control loop. These control loops are complex with several controlled variables where separate variables are not mutually independent. Mutual coupling of controlled variables is usually given by simultaneous action of each of input (manipulated and disturbance) variables of controlled plant to all controlled variables. These control loops are called MIMO (multi-input/multi-output) control loops and they are a complex of mutually influencing simpler control loops [2].

2. MIMO CONTROL LOOP
We will consider MIMO control loop with measurement of disturbance (see Figure 1). $\boldsymbol{G}_S(s)$, $\boldsymbol{G}_{SV}(s)$, $\boldsymbol{G}_R(s)$ a $\boldsymbol{G}_{KC}(s)$ are transfer matrixes of a controlled plant, disturbance variables, controller and correction members. Signal $\boldsymbol{Y}(s)$ [n×1] is a vector of controlled variables, $\boldsymbol{U}(s)$ [n×1] is a vector of manipulated variables and $\boldsymbol{V}(s)$ [m×1] is a vector of disturbance variables.

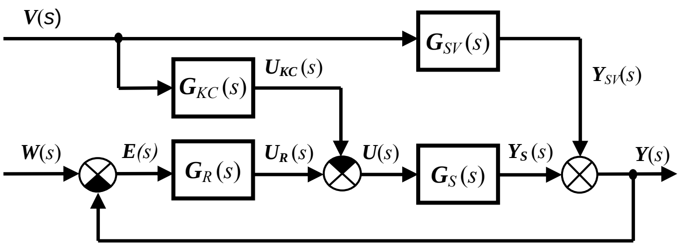

Figure 1. MIMO control loop with measurement of disturbance

Transfer matrixes of controlled plant and of disturbance variables are considered in the following forms

$$\boldsymbol{G}_S(s)=\frac{\boldsymbol{Y}_S(s)}{\boldsymbol{U}(s)}=\begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1n} \\ S_{21} & S_{22} & \cdots & S_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ S_{n1} & S_{n2} & \cdots & S_{nn} \end{bmatrix} \qquad \boldsymbol{G}_{SV}(s)=\frac{\boldsymbol{Y}_{SV}(s)}{\boldsymbol{V}(s)}=\begin{bmatrix} S_{V11} & S_{V12} & \cdots & S_{V1m} \\ S_{V21} & S_{V22} & \cdots & S_{V2m} \\ \vdots & \vdots & \cdots & \vdots \\ S_{Vn1} & S_{Vn2} & \cdots & S_{Vnm} \end{bmatrix} \tag{1}$$

Transfer matrixes of controller and of correction member are considered in these forms

$$\boldsymbol{G}_R(s)=\frac{\boldsymbol{U}_R(s)}{\boldsymbol{E}(s)}=\begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1n} \\ R_{21} & R_{22} & \cdots & R_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ R_{n1} & R_{n2} & \cdots & R_{nn} \end{bmatrix} \qquad \boldsymbol{G}_{KC}(s)=\frac{\boldsymbol{U}_{KC}(s)}{\boldsymbol{V}(s)}=\begin{bmatrix} KC_{11} & 0 & \cdots & 0 \\ 0 & KC_{22} & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & KC_{nn} \end{bmatrix} \tag{2}$$

2.1. Autonomy and invariance of control loop

At synthesis of MIMO control loop, beside stability and quality of control, it is often required for control loop to be autonomous and invariant. In order to determine the conditions for autonomy and invariance we start from a command transfer matrix $\boldsymbol{G_W}(s)$ and disturbance transfer matrix $\boldsymbol{G_V}(s)$ [2], therefore

$$\boldsymbol{G}_{\mathrm{W}}(\mathrm{s})=[\boldsymbol{I}+\boldsymbol{G}_{\mathrm{S}}(\mathrm{s})\boldsymbol{G}_{\mathrm{R}}(\mathrm{s})]^{-1}\boldsymbol{G}_{\mathrm{S}}(\mathrm{s})\boldsymbol{G}_{\mathrm{R}}(\mathrm{s}) \tag{3}$$

$$\boldsymbol{G}_{\mathrm{V}}(\mathrm{s})=[\boldsymbol{I}+\boldsymbol{G}_{\mathrm{S}}(\mathrm{s})\boldsymbol{G}_{\mathrm{R}}(\mathrm{s})]^{-1}[\boldsymbol{G}_{\mathrm{SV}}(\mathrm{s})-\boldsymbol{G}_{S}(\mathrm{s})\boldsymbol{G}_{\mathrm{KC}}(\mathrm{s})] \tag{4}$$

Autonomy

It results from the equation (3) that the control loop is autonomous if it is ensured that the matrix $\boldsymbol{G_S}(s)\ \boldsymbol{G_R}(s)$ is diagonal. On the base of this condition it is possible to derive the following relation

$$\frac{R_{kl}}{R_{ml}}=\frac{s_{lk}}{s_{lm}} \qquad k,l,m=<1,\ldots,n>, s_{lm}\neq 0 \tag{5}$$

s_{ij} - algebraic supplements of separate elements of a transfer matrix of controlled plant $\boldsymbol{G_S}(s)$

R_{ij} - separate members (binding members) of a transfer matrix of controller $\boldsymbol{G_R}(s)$

Diagonal (main) controllers R_{11}, R_{22}, R_{33} etc. are usually known already from the first design of conception of control. The design of main controllers is carried out by any SISO method of synthesis. The above mentioned relation (5) is therefore used for calculation of all remaining members of matrix controller $\boldsymbol{G_R}(s)$, i.e. for calculation of transfers of binding members.

Invariance

For ensuring absolute invariance it is necessary that the disturbance transfer matrix $\boldsymbol{G_V}(s)$ (4) is zero. This is possible if the following relation is valid

$$\boldsymbol{G}_{\mathrm{KC}}(\mathrm{s})=\boldsymbol{G}_{S}^{-1}(\mathrm{s})\boldsymbol{G}_{\mathrm{SV}}(\mathrm{s}) \tag{6}$$

At design of correction members, the task of which is to eliminate the influence of disturbance variable on control loop, internal couplings are omitted at MIMO control loop and thus n SISO branched control loop with measuring of a disturbance variable are gained. Connection of all these SISO control loops is the same and they differ only in separate transfers of controlled plants, controllers, correction members and disturbance variables [2]. Common connection of these control loops is presented on the following Figure 2.

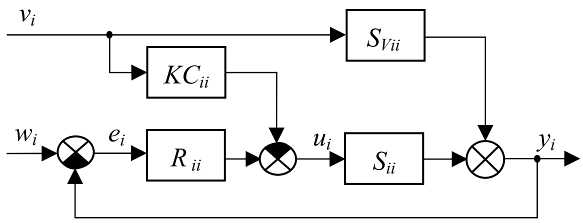

Figure 2. Block diagram of SISO control loop with measuring of disturbance variable v_i

Correction members *KC,* which serve for ensuring invariance of control loop, are determined on the base of the condition (6). The invariance of the control loop is ensured, according to the above mentioned method, by using analogy of SISO branched control loops with measuring of disturbance variable *v*. Transfer of correction members *KC* are gained by using the equation (6) in the following form

$$KC_{ii}=\frac{S_{Vii}}{S_{ii}} \qquad i=<1,\ldots,n>, S_{ii}\neq 0 \tag{7}$$

S_{Vii}- separate members of transfer matrix of disturbance variables $\boldsymbol{G_{SV}}$(s)

S_{ii} - separate members of transfer matrix of controlled plant $\boldsymbol{G_S}$(s)

2.2. MIMO control loops synthesis

In practice the possible approximate solution of MIMO control loop is applied from analysis of MIMO control loop and really used control schemes in particular technological equipments [2]. One of the possible methods of solution of MIMO control loops synthesis is described in the following part of this paper. Generally it is possible to divide this solution into three parts

- design of **main (diagonal) controllers** by any synthesis method of SISO control loops, i.e. design of parameters of main controllers for n SISO control loops (R_1, R_2, …, R_n),
- ensuring autonomy of control loop via **binding members** of transfer matrix of controller $\boldsymbol{G_R}$(s),
- ensuring invariance control loop by means of correction members *KC* by using n SISO control loops with measuring of disturbance variables.

3. SIMULATION EXAMPLE

3.1. MIMO controlled plant

Steam turbine is a typical example of MIMO controlled plant. In this case is considered the turbine with two controlled withdrawals which drives electric generator supplying determined part of electric network (it means the turbine operates without phasing into power network). Here, the turbine represent three-variable control loop. The scheme of three-variable control loop of steam turbine is presented on the Figure 3.

Denominations on Figure 3 mean: Δy_{VT}, Δy_{ST}, Δy_{NT} - change of opening position of control valves of high-pressure, medium-pressure and low-pressure part of turbine, $\Delta m'_{01}$, $\Delta m'_{02}$ - change of mass flow of withdrawn steam, Δp_{01}, Δp_{02} - change of steam pressure in corresponding withdrawals, $\Delta\omega$ - change of angular speed of turbo-generator, ΔM_G - change of electric load of turbo-generator.
Controlled variables are $\Delta\omega$, Δp_{01}, Δp_{02}, disturbance variables ΔM_G, $\Delta m'_{01}$, $\Delta m'_{02}$ and manipulated variables are Δy_{VT}, Δy_{ST}, Δy_{NT}.

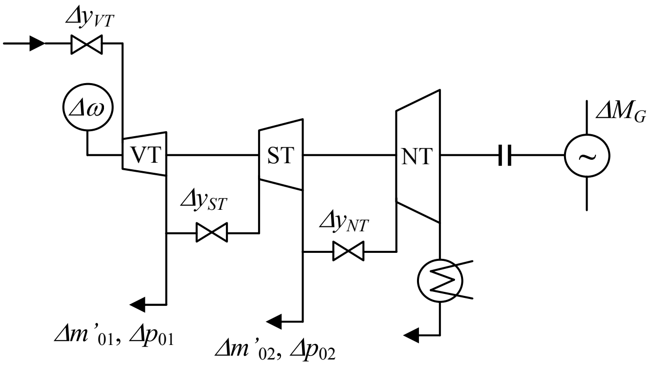

Figure 3. Three-variable control loop of steam turbine

3.2. Mathematical model of the controlled plant

Resulting differential equations for creating mathematical model of the plant were gained already after deriving and using linearization from the project OTROKOVICE elaborated by the firm ALSTOM Power [1]. Differential equations were re-write into better-arranged form by introducing relative values (with regard to starting stable state-operational, i.e. to calculated point) at which relation of values can be generally written in the form $\varphi_X = \Delta X/(X)_0$. Then, the Laplace transform of an output (controlled) variable was given by the following relation

$$\boldsymbol{Y}(s)=\boldsymbol{G}_S(s)\boldsymbol{U}(s)+\boldsymbol{G}_{SV}(s)\boldsymbol{V}(s)\Rightarrow\begin{bmatrix}\varphi_\omega(s)\\ \varphi_{p01}(s)\\ \varphi_{p02}(s)\end{bmatrix}=\boldsymbol{G}_S(s)\begin{bmatrix}\varphi_{yVT}(s)\\ \varphi_{yST}(s)\\ \varphi_{yNT}(s)\end{bmatrix}+\boldsymbol{G}_{SV}(s)\begin{bmatrix}\varphi_{M_G}(s)\\ \varphi_{m'01}(s)\\ \varphi_{m'02}(s)\end{bmatrix} \tag{8}$$

where

$$\boldsymbol{G}_S(s)=\begin{bmatrix}\frac{0{,}73s^2+1{,}59s+1{,}11}{12{,}32s^3+21{,}78s^2+10{,}67s+1} & \frac{0{,}455s^2+0{,}740s+0{,}087}{12{,}32s^3+21{,}78s^2+10{,}67s+1} & \frac{0{,}321s^2+0{,}327s+0{,}037}{12{,}32s^3+21{,}78s^2+10{,}67s+1}\\ \frac{1{,}68s+1{,}31}{1{,}505s^2+2{,}475s+1} & \frac{-1{,}246s-0{,}959}{1{,}505s^2+2{,}475s+1} & \frac{-0{,}011}{1{,}505s^2+2{,}475s+1}\\ \frac{1{,}76}{1{,}505s^2+2{,}475s+1} & \frac{1{,}561s+0{,}039}{1{,}505s^2+2{,}475s+1} & \frac{-1{,}118s-0{,}966}{1{,}505s^2+2{,}475s+1}\end{bmatrix} \tag{9}$$

$$\boldsymbol{G}_{SV}(s)=\begin{bmatrix}\frac{-1{,}505s^2-2{,}477s-1}{12{,}32s^3+21{,}78s^2+10{,}67s+1} & \frac{-0{,}092s-0{,}148}{12{,}32s^3+21{,}78s^2+10{,}67s+1} & \frac{-0{,}09s-0{,}079}{12{,}32s^3+21{,}78s^2+10{,}67s+1}\\ 0 & \frac{-0{,}400s-0{,}313}{1{,}505s^2+2{,}475s+1} & \frac{-0{,}005}{1{,}505s^2+2{,}475s+1}\\ 0 & \frac{-0{,}420}{1{,}505s^2+2{,}475s+1} & \frac{-0{,}501s-0{,}432}{1{,}505s^2+2{,}475s+1}\end{bmatrix} \tag{10}$$

3.3. Synthesis of three-variable control loop of a steam turbine

The principal described above (see paragraph 2.2) is used at solution of synthesis of the three-variable control loop. First transfers of main controllers R_{11}, R_{22}, R_{33} are determined for transfer functions S_{11}, S_{22} a S_{33} then autonomy of control loop (5) is being solved and in the end fulfilment of the condition of invariance (approximate invariance) of control loop is ensured by using equation (7). At design of parameters of main controllers the following methods were used: Ziegler Nichols step response method [2], method of desired model (method of dynamics inversion) [5], polynomial method of synthesis - 1DOF (1 degree of freedom) configuration [4]. In the next part of this paper one chosen method for design of parameters of main controllers, i.e. the Ziegler Nichols step response method, is used.

Transfer matrix of controllers $\boldsymbol{G}_R(s)$ with utilization of Ziegler Nichols step response method and transfer matrix of correction members $\boldsymbol{G}_{KC}(s)$ are given by the equation (11).

$$\boldsymbol{G}_R(s)=\begin{bmatrix}\frac{34{,}28s+34{,}18}{s} & \frac{1{,}35s^2+0{,}56s+0{,}05}{s(s+2{,}09)} & \frac{0{,}29s^2+0{,}12s+0{,}01}{s(s+1{,}06)}\\ \frac{46{,}24s+46{,}10}{s} & \frac{-1{,}09s-0{,}32}{s} & \frac{0{,}39s^2+0{,}18s+0{,}02}{s(s+1{,}06)}\\ \frac{64{,}53s+64{,}33}{s} & \frac{-1{,}53s^2-0{,}22s+0{,}07}{s(s+2{,}09)} & \frac{-1{,}22s-0{,}35}{s}\end{bmatrix}\quad \boldsymbol{G}_{KC}(s)=\begin{bmatrix}\frac{-2{,}06s^2-3{,}39s-1{,}37}{s^2+2{,}18s+1{,}52} & 0 & 0\\ 0 & \frac{0{,}32s+0{,}25}{s+0{,}77} & 0\\ 0 & 0 & 0{,}45\end{bmatrix} \tag{11}$$

Simulation results

Simulation of three-variable control loop of a steam turbine with utilization of one chosen SISO synthesis method is presented on the following figure (see Figure 4) [3].

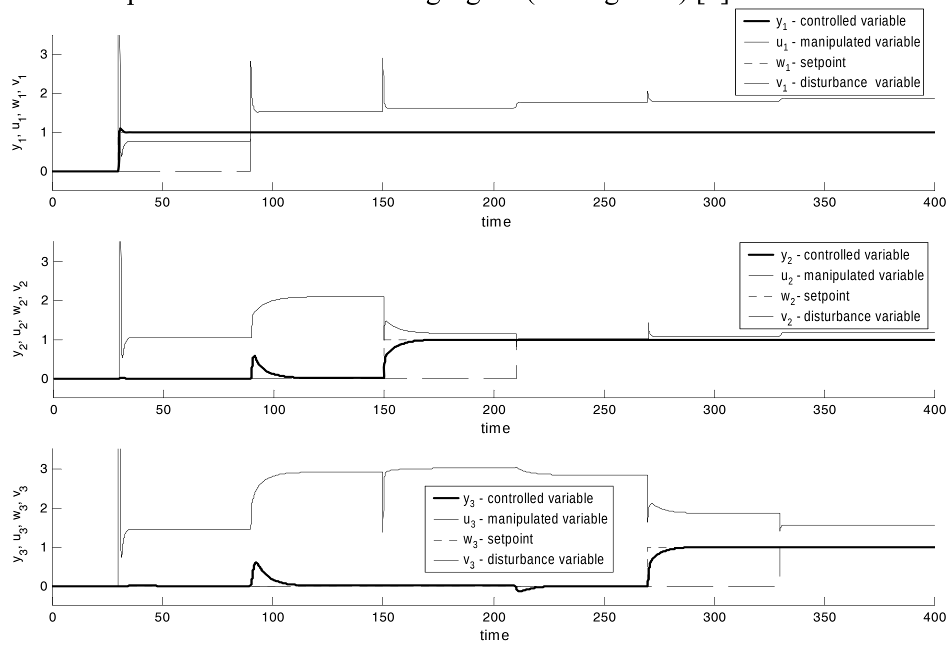

Figure 4. Simulation of control loop with utilization of Ziegler Nichols step response method

Designation of variables on figure corresponds to definite variables existing in the three-variable control loop of steam turbine, i.e. $y_1 \to \varphi_\omega$, $y_2 \to \varphi_{p01}$, $y_3 \to \varphi_{p02}$, $v_1 \to \varphi_{MG}$, $v_2 \to \varphi_{m'01}$, $v_3 \to \varphi_{m'02}$, $u_1 \to \varphi_{yVT}$, $u_2 \to \varphi_{yST}$, $u_3 \to \varphi_{yNT}$

3.4. Evaluation of simulation experiments

It is obvious from the simulation of control process presented above (see see Figure 4) and from other simulation experiments that the condition of autonomy was fulfilled. Fulfilment of this condition was ensured by means of using binding members R_{ij} (aside-from-diagonal elements of transfer matrix of controller $\boldsymbol{G}_R(s)$). It is also obvious from the simulation process control that the control loop is invariant or let us say approximately invariant, i.e. that influence of disturbance variables is eliminated only at steady state. Fulfilment of the condition of invariance was ensured by means of correction members KC_{ii} which are considered for elimination of influence of dominant disturbance variables by means of using analogy of SISO branched control loop with measurement of disturbance *v*.

4. CONCLUSION

It was described **one of possible procedure** to control of MIMO control loop. Simulation verification of chosen procedure of control is presented on three-variable control loop of steam turbine. At first our method deals with setting-up of main (diagonal) controllers, then determination of binding members for ensuring autonomy and in the end calculation of correction members for ensuring invariance.

The designed method shows the combination of classical way to ensure autonomy of control loop via binding members and the use of the method of SISO branched control loops with measurement of dominant disturbance variables to ensure of invariance of control loop.

5. ACKNOWLEDGMENTS

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