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ROBUST STABILITY OF SYSTEMS WITH COMPLICATED UNCERTAINTY STRUCTURE

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ABSTRACT
The parametric uncertainty may occur in modelling and description of real systems as a consequence of inaccurate measurement, identification, due to ambient conditions, etc. The structure of systems with parametric uncertainty is known but parameters can vary within given or supposed intervals. The fundamental factor for choosing of appropriate robust stability analysis tool is the uncertainty structure. This contribution is focused on description of systems with complicated (multilinear, polynomial, general) parametric uncertainty and on demonstration of possible robust stability testing for this class of systems with special emphasis on utilization of The Polynomial Toolbox for MATLAB.
Keywords: robust stability, uncertainty, value set, zero exclusion condition, Polynomial Toolbox

1. INTRODUCTION
Most of the industrial processes can be modelled as linear time-invariant systems, in spite of the fact, that their real behaviour is oftentimes different, much more complicated. The motivation is evident – owing to this the transfer functions can be used for description of such systems and subsequently also the control theory of linear systems, which is very well-developed, can be applied. However, an effort to create the model simple enough almost always leads to the origin of uncertainty. Their emergence often consists in neglect of "less important properties", especially from the realms of fast dynamic effects, nonlinearities or time-variant behaviours of the process. Nevertheless, the presence of uncertainty can not be excluded even if the processes are in essence linear, because the physical parameters are never known exactly, possible they can vary according to operating conditions. Ergo, the principal problem is if the controller designed for nominal system will keep some properties of the feedback control loop also for system really controlled, which falls into certain neigbourhood – in other words, if the controller will keep these properties not only for one nominal system, but for the whole family of systems.
The uncertainty in constructed mathematical model and thus the size of neigbourhood which should the controller cope with can be taken into consideration and described in the two main ways – as parametric or nonparametric uncertainty. The former, nonparametric description of uncertainty lies in restriction of area of possible appearance of frequency characteristic. It is associated with unmodelled dynamics, truncation of high frequency modes, nonlinearities, randomness in the systems, etc. The latter, parametric approach then represents known structure but uncertain knowledge of actual physical parameters of the controlled system. Their possible values are usually bounded by intervals.
This contribution deals with robust stability analysis for systems with complicated uncertainty structure. In such cases, the analytical methods are either quite complex or even do not exist. On that account, the graphical test is utilized in this work. The investigation of robust stability is accomplished via the value set concept and the zero exclusion condition, here practically with the assistance of The Polynomial Toolbox for MATLAB.

2. STRUCTURES OF UNCERTAINTY

From the control theory point of view, the very frequent subject of investigation is the uncertain characteristic polynomial of the closed-loop control system. It can be described by:

$$p(s,q)=\sum_{i=0}^{n}\rho_i(q)s^i \tag{1}$$

In robustness problems, the vector of uncertain parameters q is often supposed to be confined by the uncertainty bounding set Q, which is usually given a priory, e.g. directly by user requirements.

The most important problem consists in ensurance of stability and hence the control engineers are very often interested in a robust stability. It is familiarly known, that the polynomial $p(s)$ is stable if all its roots have negative real part. The family of polynomials

$$P=\{p(\cdot,q):q\in Q\} \tag{2}$$

is robustly stable, if $p(\cdot,q)$ is stable for all $q\in Q$, i.e. all roots of $p(\cdot,q)$ must be located in the left complex half plane for all $q\in Q$.

The uncertainty enters into the polynomial (2) through the coefficient functions $\rho_i(q)$. Nevertheless, the very significant is the way how the uncertain parameters enter into the coefficients of this polynomial. In accordance with this, several basic structures of uncertainty with increasing generality are distinguished:

- independent (interval) uncertainty structure
- affine linear uncertainty structure
- multilinear uncertainty structure
- nonlinear uncertainty structure (polynomial, general)

Moreover, the single parameter uncertainty is considered as a special case.

This paper deals with multilinear and nonlinear uncertainty structures and assumes Q to be a box.

3. TOOLS FOR ROBUST STABILITY ANALYSIS

Most of the analytical techniques are highly specialized and their applicability is restricted to particular type of uncertainty structure. Among the most known methods it belongs Bialas eigenvalue criterion, the Kharitonov theorem, the edge theorem, the mapping theorem, etc. On the contrary, there is a very universal tool – the combination of the value set concept and the zero exclusion condition.

Assume a family of polynomials $P=\{p(\cdot,q):q\in Q\}$. The value set at frequency $\omega\in\mathbf{R}$ is given by:

$$p(j\omega,Q)=\{p(j\omega,q):q\in Q\} \tag{3}$$

In other words, $p(j\omega,Q)$ is the image of Q under $p(j\omega,\cdot)$. For example, substitute s for $j\omega$ in a family $P=\{p(s,q):q\in Q\}$, fix ω and let the vector of uncertain parameters q range over the set Q.

The zero exclusion condition for Hurwitz stability of family of continuous-time polynomials $P=\{p(\cdot,q):q\in Q\}$ says: Suppose invariant degree of polynomials in the family, pathwise connected uncertainty bounding set Q, continuous coefficient functions $a_i(q)$ for $i=0,1,2,\ldots,n$ and at least one stable member $p(s,q^0)$. Then the family P is robustly stable if and only if the complex plane origin is excluded from the value set $p(j\omega,Q)$ at all frequencies $\omega\geq 0$, that is P is robustly stable if and only if:

$$0\notin p(j\omega,Q)\quad\forall\omega\geq 0 \tag{4}$$

Lots of generalizations, remarks and examples can be found e.g. in [1], [3] or [4].

4. SIMULATION EXPERIMENTS IN THE POLYNOMIAL TOOLBOX

The visualization of the value set for systems with complicated uncertainty structure can be conveniently done in The Polynomial Toolbox for MATLAB [7], [9] through the commands “vset” and “vsetplot”.

4.1. Example 1 – multilinear uncertainty

First, consider the family of polynomials from [2] described by:

$$p(s,q)=s^4+(q_1+q_2+2.56)s^3+(q_1q_2+2.06q_1+1.561q_2+2.871)s^2+ \\ +(1.06q_1q_2+4.841q_1+1.561q_2+3.164)s+(4.032q_1q_2+3.773q_1+1.985q_2+1.853) \tag{5}$$

with uncertainty bounds $0 \le q_1 \le 1$ and $0 \le q_2 \le 3$. The figure 1 shows value sets of (5) for frequencies $\omega = \langle 0;2 \rangle$ with step 0.05. As can be seen, the origin of the complex plane is included in these value sets and consequently the polynomial (5) is not robustly stable.

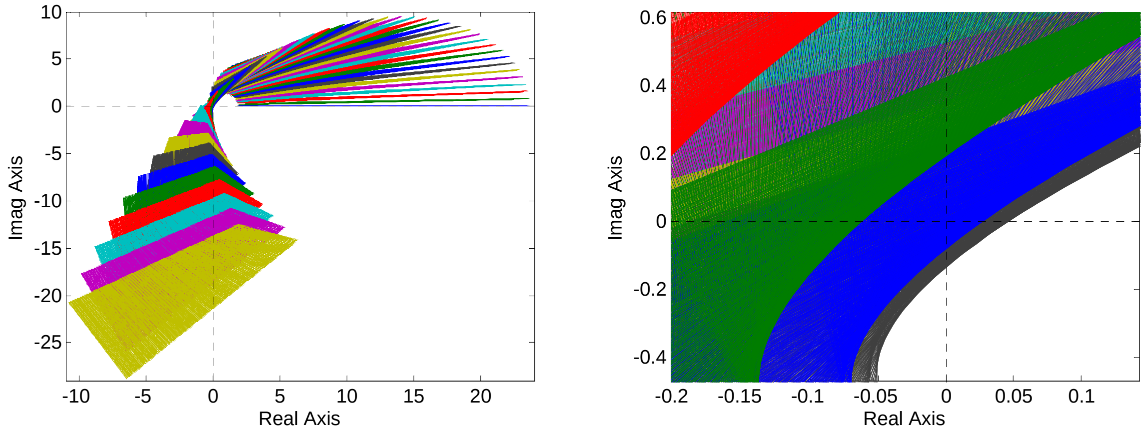

Figure 1. The value sets of family (5) – full view and detail

4.2. Example 2 – polynomial uncertainty

Next, the polynomial family is given by:

$$p(s,q)=s^3+(q_1q_2+2)s^2+(q_1^3-q_2^3-q_1q_2+q_2+10)s+(q_1^3+q_2^3+q_1q_2+q_2+5) \tag{6}$$

and $q_1, q_2 \in \langle -1;1 \rangle$. The value sets for $\omega = \langle 0;5 \rangle$ and step size 0.25 are depicted in figure 2. Now, the origin is excluded from the value sets, moreover, the family (6) has a stable member, thus it represents the robustly stable case.

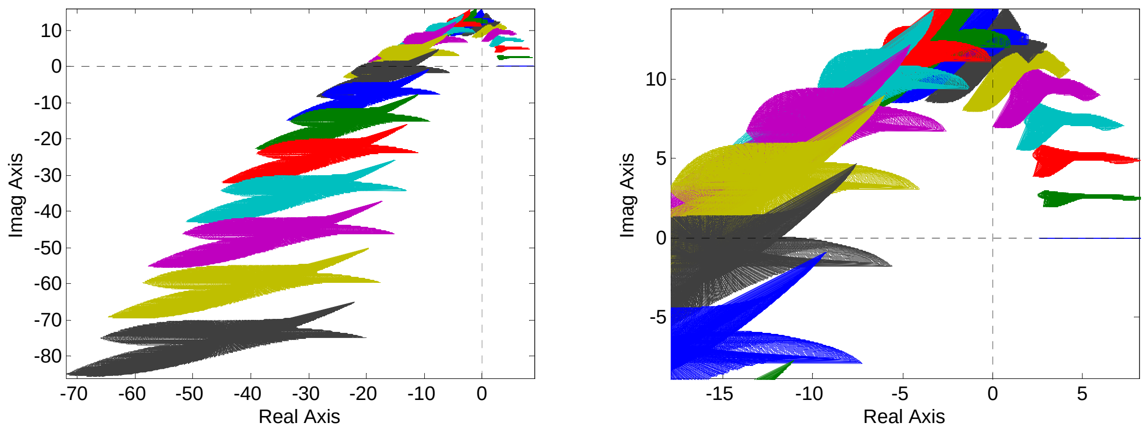

Figure 2. The value sets of family (6) – full view and detail

4.3. Example 3 – general uncertainty

Finally, suppose the family:

$$p(s,q)=s^3+[\cos(q_1q_2)]s^2+\left[5\sqrt{|q_1|}-3\sin q_2-\cos(q_1q_2)+4\right]s+ \\ +\left[-4\sqrt{|q_1|}+\sin q_2+\cos(q_1q_2)+0.1\right] \tag{7}$$

again with $q_1, q_2 \in \langle -1;1 \rangle$. The value sets of this event are shown in figure 3 ($\omega = \langle 0;4 \rangle$ and step 0.2). Due to the position of the zero point inside of the value sets the investigated family is concluded to be robustly unstable.

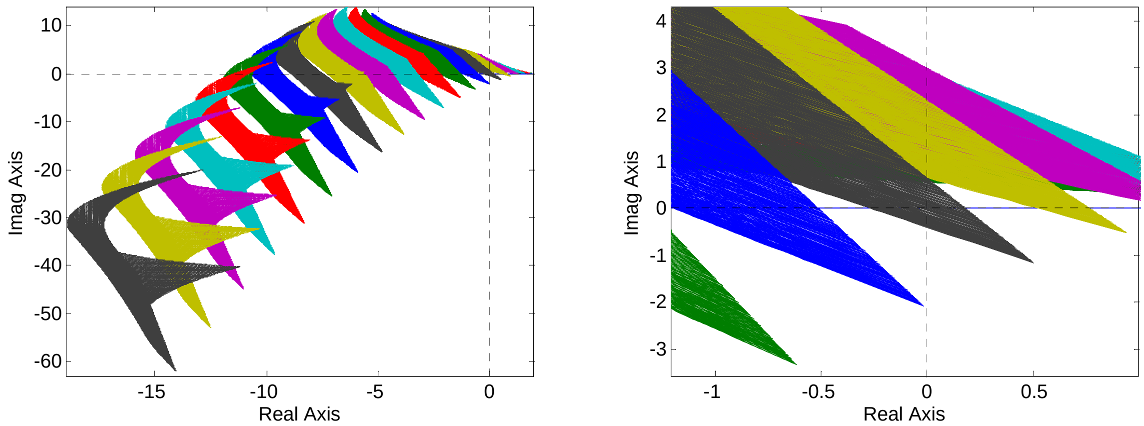

Figure 3. The value sets of family (7) – full view and detail

5. CONCLUSIONS

The contribution has been focused on problems connected with robust stability of systems with more complicated uncertainty structure. Instead of very complex or even not existing analytic tools the highly universal value set concept and the zero exclusion condition have been applied. The practical tests of robust stability based on these key instruments can be handy done in The Polynomial Toolbox for MATLAB. Its possible utilization has been demonstrated on three examples for polynomials with multilinear, polynomial and general uncertainty structure, respectively.

6. ACKNOWLEDGEMENT

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