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HYPERELASTIC MATERIAL MODELS OF RUBBER AND RUBBER-LIKE MATERIALS

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ABSTRACT
The variety of applications of rubber-like materials in different industrial areas is enormous. In the automotive industry rubber is utilized in tires, as engine and door seals, whereas in aerospace industry rubber rings are used in fuel systems. In other engineering fields, rubbers have been used in conveyor belts, meteorological balloons and vibration isolation bearings or shock and impact absorbers.Rubber-like materials exhibit a highly nonlinear behaviour characterized by hyperelastic deformability and incompressibility or near-incompressibility. Normally, the maximum extensibility of rubber could reach values varying from 500% to 1000% and the typical stress-strain curve in tension is markedly nonlinear so that Hooke's law cannot be applied and it is not possible to assign a definite value to the Young's modulus except in the region of small strains, where the Young's modulus is of the order of 1MPa. It İs too important to know or predict the mechanical behavior of these materials in design for the sake of stres-strain relation, deformation, lifetime and reliability. In this study the hyperelastic material models and Euler hyperelastic relation for tension in case of isotropic, incompressible hyperelastic solid have been presented.
Keywords: rubber-like material, hyperelasticity, mechanical behavior

1. INTRODUCTION
Rubber-like materials are one of the most remarkable materials having a wide range of engineering applications including tires, engine mounts, shock-absorbing bushes, vehicle door seals, adhesive joints, building and bridge bearings, tunnel linings and wind shoes. Beside these applications, rubber-like materials are also used to produce athletic and recreation facilities and to make toys and household products, e.g. sport mats and acoustical underlayment[1,2].
An increase of applications requires a better understanding of the mechanical behaviour of rubber-like materials.There is an enormous difference between rubbers and ordinary hard solids. Unlike metals requring relatively few properties to characterize their behaviour of rubber-like materials can not be described by a simple stress-strain relation, but by the strain energy function.
In the present study, a general review of the hyperelastic models has been presented and then Euler hyperelastic relation for tension in case of isotropic, incompressible hyperelastic solid has been adduced. As a result, a comparison for James's experimental data used subsequently by a number of authors as the basis for comparision of the theory and experiments [3] has been illustrated. It is also possible to find different experimental datas of Treloar in [4,5] .
Rubber-like materials exhibit a highly nonlinear behavior characterised by hyperelastic deformability and reversibility of deformation.Rubber-like materials have a very low shear modulus, G typically being in the range 0,3 to 3 Mpa; it is this feature, together with the very high extensibility, that makes them invaluable for providing controlled compliance in engineered structures. Their bulk Modulus, K, is of the same order as that of liquids; for many engineering purposes it is adequate to take a typical default value of K as 2000 Mpa [6].

2. HYPERELASTIC MODELS

Many attempts have been made to develop a theoretical stress-strain relation that fits experimental results for hyperelastic materials [4, 5, 7].Beside these studies, there have been many papers including finite element simulations compared with hyperelastic relation and models [8, 9, 10, 11, 12]. There are two rather different approaches to the study of rubber elasticity.First, the phenomenological theory which treats the problem from the viewpoint of continuum mechanics constructs a mathematical framework to characterize rubbery behaviour. Second, the statistical or kinetic theory that attempts to derive elastic properties from some idealized model of the structure of vulcanised rubber.

The basic features of stress-strain behaviour have been well modelled by invariant-based and/or stretch-based continuum mechanics theories. For example, Mooney proposed a phenomenological model with two parameters based on the assumption of a linear relation between the stress and strain during simple shear deformation. Later, Treloar published a model based on the statistical theory, the so-called neo-Hookean material model with only one material parameter. However, this was proved to be merely a special case of the Mooney model. In 1950, Rivlin modified the Mooney model to obtain a general expression of the strain energy function expressed in terms of strain invariants. One of the successful models in this class has recently been developed by Yeoh in the form of a third-order polynomial of the first invariant of the right Cauchy-Green tensor. An alternative high-order polynomial model of the first invariant has been proposed by Gent and takes the form of a natural logarithm. In 1972, Ogden proposed a strain energy function expressed in terms of principal stretches, which is a very general method for describing hyperelastic materials. An excellent agreement has been obtained between Ogden’s formula and Treloar’s experimental data for extensions of unfilled natural rubber up to 700% .However, the parameter identification is complicated because of the purely phenomenological character of the Ogden strain energy function [1].

A widely used expression for W is the polynomial form where Cij are constants determined from experiments:

$$W = \sum_{i=0,j=0}^{\infty} C_{ij} (I_1 - 3)^i (I_2 - 3)^j \qquad (1)$$

Neo-Hookeean model:

$$W = C_{10} (I_1 - 3) \text{ where } j = 0, i = 1 \qquad (2)$$

Mooney-Rivlin model:

$$W = C_{10} (I_1 - 3) + C_{01} (I_2 - 3) \text{ where } j = 1, i = 1 \qquad (3)$$

Ogden model:

$$W = \sum_n \frac{\mu_n}{\alpha_n} (\lambda_1^{\alpha_n} + \lambda_2^{\alpha_n} + \lambda_3^{\alpha_n} - 3) \qquad (4)$$

in which ¸ λ_i are the principal stretches, α_n may have any values, positive or negative not necessary integers and μ_n are constants.

Yeoh model[3]:

$$W = C_{10} (I_1 - 3) + C_{20} (I_1 - 3)^2 + C_{30} (I_1 - 3)^3 \qquad (5)$$

Besides these purely phenomenological models, micro-mechanically based idealized network models have also been proposed such as Arruda and Boyce [5] who developed an eight-chain model:

$$W = nk\Theta \left[\frac{1}{2}(I_1 - 3) + \frac{1}{20N}(I_1^2 - 9) + \frac{11}{1050N^2}(I_1^3 - 27) \right] + nk\Theta \left[\frac{19}{7000N^3}(I_1^4 - 81) + \frac{519}{673750N^4}(I_1^5 - 243) \right] \qquad (6)$$

This is the most predictive model of the larger strain behavior under different states of deformation[7].

3. SIMPLE TENSION

In this section, general Euler hyperelastic relation for simple tension is represented [13].Then, a comparison for James's experimental data [3] for extension is illustrated in Figure 1.
Using the coordinate system;

$$x = \lambda_1 X \qquad y = \lambda_2 Y \qquad z = \lambda_2 Z \tag{7}$$

Deformation gradient tensor F:

$$F = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_2 \end{bmatrix} \tag{8}$$

Left Cauchy-Green tensor B:

$$B^{-1} = \begin{bmatrix} \frac{1}{\lambda_1{}^2} & 0 & 0 \\ 0 & \frac{1}{\lambda_2{}^2} & 0 \\ 0 & 0 & \frac{1}{\lambda_2{}^2} \end{bmatrix} \tag{9}$$

Cauchy stress tensor:

$$\sigma = \begin{bmatrix} \frac{P}{\lambda_2{}^2 A_0} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{10}$$

Kirchhoff stress tensor:

$$\tau = \begin{bmatrix} \frac{\lambda_1 P}{A_0} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{11}$$

Stiffness matris:

$$C = \begin{bmatrix} 2G+\lambda & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & 2G+\lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & 2G+\lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & 2G & 0 & 0 \\ 0 & 0 & 0 & 0 & 2G & 0 \\ 0 & 0 & 0 & 0 & 0 & 2G \end{bmatrix} \tag{12}$$

Euler logarithmic strain:

$$e = \ln(V) = \frac{1}{2}\ln(B) \tag{13}$$

Euler hyperelastic relation:

$$\tau = C : e = C : \ln(V) \tag{14.a}$$

$$\sigma_{11} = \frac{1}{\lambda_1}((2G+\lambda)\varepsilon_{11} + \lambda\varepsilon_{22} + \lambda\varepsilon_{22}) \tag{14.b}$$

4. RESULTS

In this study the hyperelastic material models such as Neo-Hookeean, Mooney-Rivlin, Ogden and Yeoh models and Euler hyperelastic relation for tension in case of isotropic, incompressible hyperelastic solid have been presented. By using these basic relations for different elastisite modules a comparison has been done with the experimental datas of James et al. As a result, for 50 % deformation the data of James and present study are compatible.

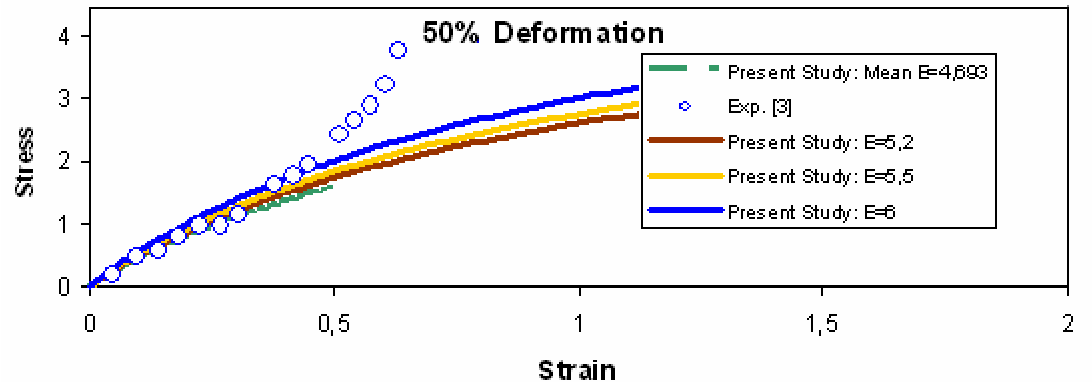

Figure 1 The stress-strain relation for different Elasticity modules in the case of 50% deformation.

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