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MATHEMATICAL MODEL FORMULATION IN OPTIMAL PROGRAM PLANNING OF INDIVIDUAL AND LEAN PRODUCTION

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ABSTRACT
In order to solve planning problems in economy many disciplines have been developed in the framework of scientific approaches: operational research, mathematical programming, simulation etc. These scientific disciplines have developed scientific methods which provide qualitative and quantitative research of a real state that could be used for building models and methods of optimal planning and making decisions.
Subject analyses in the field of mathematical model formulation in optimal production program planning are conducted on a real industrial complex system where individual and lean production has being implemented in the process of special tool developments for electronic devices and their components.
Key words: mathematical model, restrictions, criteria functions.

1. INTRODUCTION
Planning programs for special tool developments could not be observed isolated from the planning problem of the whole production program in a production and complex industrial system because the production program for special tool developments is closely related to and in function of other industrial products which present the basic operation. According to this, the production planning program belongs to strategic planning which is a foundation of the strategy for development, engineering or enlarging production structure, production system status in surroundings and system life strategy.
The optimal production program for special tools as a part of a strategic plan needs to answer the questions of the interest for the strategy for production system maintaining and growing. The most important of these are: (1) income and profit growing by tool developments; (2) rational and optimal use of available capacities in production structure for tool developments; (3) how many individual special tools need to be developed; (4) in which way they need to be developed; (5) where they need to be produced – which devices they need to be developed on; (7) how much time will be spent on their developments; (8) how much specific work places will be burden – excess and lack of capacities. The built optimization production program method for special tools needs to enable the simulation of determined plan too, that is, the analysis of profit and capacity use depending on expansion of tool production program related to the estimated optimal program.

2. MATHEMATICAL MODEL FORMULATION
Planning production program for special tools and the choice of the optimal program variety is the real problem which is: (a) noticeable, for it is necessary in the configuration of intruded changes which are presented in the production system itself and its environment with an intention to maintain the production system within the limits of designed aim function; (b) measurable, for effects of planning implementation decrease the system entropy and lack of its implementation increases it; (c) there is an interaction between the defined planning problem cause and the effect which characterizes it by its solving.

2.1. Basic elements of the optimal planning production program model for special tools

1) Model objective: The program optimization for special tool developments, that is, determination of optimal values p_j-q_j has for its aim the profit maximization by tool developments and greater production capacity use.
2) Time horizon model : This model element means period which the model or planning refers to. Planning production program belongs to the business function of the process managing which does strategic planning and it is used for determination of future influence of production systems or their future objectives..
3) The other aspect of time model is its dynamics: It means that models could be dynamic and static. In reference to previous presumptions that in production system for special tool developments there was neither labor fluctuation nor available capacities, thus, the period of time for planning is one year; it could be accepted that the model according to the time dependence belongs to the group of static models.
4) Form of the model: Solving the problem of optimal program planning for special tool developments belongs to the structural form of the model because it reflects the production program structure which provides the best available capacity use and maximal profit realization.
5) Model sensibility: Every parameter in a criterion function may change its value during the period of time so it oscillates around a value. These parameter value changes cause changes in model solutions.
6) Model links: The model of optimal planning program for special tool developments is linked to the model of production program planning in a complex production system, on one side, and basic and operative planning in production structure for special tool developments, on the other side.

2.2. The mathematical problem formulation of the optimization production program for special tools

The mathematical model formulating process of multi criteria planning consists of several steps [1]:

1) Definition of variables p_j [number]

On the basis of previous analysis of the observed problem variables in aim functions and restrictions of special tool group types p_j, where j = 1,2,3…,7.

So, the following tools are:

p_1 – tools for piercing and cutting, p_2 – tools for folding, p_3 – tools form thermoplastic masses, p_4 – tools for thermo stable masses, p_5 – tools for casting under the pressure, p_6 – tools for gravitational casting, p_7 – tools for holding in the process of production and installation..

Choosing the values of these variables we influence on capacity use and greater income realization.

2) Definition of the aim function f_k

The observed aim determines k=1,2,3,…5 aim functions. They are:

$f_1(p)$ – criterion function which reflects the income or profit of manufactured j product, $f_2(p)$ – criterion function which reflects technological complexity of j product of special tools, $f_3(p)$ – criterion function which reflects product mass, $f_4(p)$ – criterion function which reflects the total number of products, $f_5(p)$ – criterion function which reflects the reduced product mass.

3) Formulation of restriction b_i [hours/years]

Analyzing relations between chosen variables and aims which want to be achieved the following restrictions are defined:

– **Restrictions conditioned by production system itself**

Those are available, that is, effective capacities of production equipment used for special tool developments for observed plan period – b_i i= 1.2…..12. Those restrictions present the sort of production equipment expressed in [hours/years]. In a concrete case we have: b_1- effective capacities of planes, b_2 - effective capacity of milling cutter, b_3 - effective capacity of rough milling cutter, b_4 - effective capacity of circular grinding, b_5 - effective capacity of flat grinding b_6 - effective capacity of geometrical grinding, b_7 - effective capacity of coordinate piercing and grinding, b_9 - effective capacity of erosive cutting, b_{10} - effective capacity of gravure machine, b_{11}- effective capacity of heat treatment, b_{12} - effective capacity of manual machine tool treatment.

– **Restriction conditioned by production equipment use degree**

Overtime work is not allowed, unless on the machines for erosive piercing and erosive cutting (b_8 and b_9). After the last step in the restriction production structure there is need for introduction of the next step and expansion of machine capacities b_8 and b_9 for 1%. New effective capacities sum: b_8 =5771, b_9=2883.

– **Restrictions conditioned by production program structure**

The structure of production program is, of course, the matter of a decision maker's preference or a production system manager. The manager's preference often influence on the optimal solution provided by the optimization method. In the observed production system the case of preference will be gradually observed for particular special tools in relation with total production structure of special tools and their influence on the optimal solution. The progress will be observed through several steps. The zero step presents no restriction in the sense of product structure. In other steps a gradual structure product restriction will be introduced.

The restriction is introduced for the product which is most profitable in a previous step. The restriction percent in observed process is adopted on the base of decision maker's preference. During the last step the production program structure needs to have: $p_1 \leq 25\%$; $p_2 \leq 10\%$; $p_3 \leq 25\%$; $p_4 \leq 20\%$; $p_5 \leq 5\%$; $p_6 \leq 5\%$; $p_7 \leq 10\%$,

in reference to total number of special tools $\sum_j p_j$.

After this step there could be analyzed the variant where the upper limits of products p_1, p_2, p_3, p_4 and p_7 increase for 2%, and p_5 and p_6 for 0,5%, so the product restriction structure sums: $p_1 \leq 27\%$; $p_2 \leq 12\%$; $p_3 \leq 27\%$; $p_4 \leq 22\%$; $p_5 \leq 5{,}5\%$; $p_6 \leq 5{,}5\%$; $p_7 \leq 12\%$,

4) Formulation of parameters in restriction inequalities a_{ij} [h]

Parameters in restriction inequalities presents times required for i production equipment type for j special tool manufacturing, expressed in work hours –a_{ij}. In our case i=1….12, while j=1…..7. Data in a developed mathematical model are obtained by research in real industrial system, and taken from literature [1].

5) Definition of parameters c_{kj} in aim functions

The parameter c_{kj} in the first criterion function presents technological complexity of the representatives of special tool groups and are presented in the table 4.4.

The parameter c_{kj} in the second criterion function presents the income or profit from produced j special tools expressed in value units [v.j.] The parameter c_{kj} in the third criterion function presents the tool representative's mass and is presented in the [1].

The parameter c_{kj} in the fourth criterion function presents a unit because it reflects the total number of tools. The parameter c_{kj} in the fifth criterion function presents the tool representative's mass reduced to the product p_4.

6) Mathematical model formulation

Based on researched information and adopted presumptions the specific mathematical problem model of multi criteria linear programming with several aim functions is formed which we maximize during the restrictions of production equipment effective capacities and restrictions of production equipment use degree. The restrictions related to the production program structure will be used in the process of post optimal analysis.

Thus, the mathematical model of the problem is defined with the model of multi criteria linear programming [2] as following:

$$\max\left\{f_k(p) = \sum_{j=1}^{7} c_{kj} p_j,\ k = 1,2,3,\cdot\cdot\cdot,5\right\}$$

With restriction:

$$p = \begin{cases} \sum_{j=1}^{7} a_{ij} p_j \leq b_i, i = 1,2,...,12, \\ p_j \geq 0, \qquad j = 1,2,...,7 \end{cases}$$

or in a developed form [1]:

$$\text{Max } f_1(p) = 529{,}71\, p_1 + 141{,}95\, p_2 + 606{,}97\, p_3 + 965{,}77\, p_4 + 1458{,}55\, p_5 + + 213{,}14\, p_6 + 30{,}75\, p_7,$$

$$\text{Max } f_2(p) = 331{,}32\, p_1 + 88{,}74\, p_2 + 379{,}60\, p_3 + 604{,}57\, p_4 + 912{,}16\, p_5 + + 1333{,}33\, p_6 + 19{,}25\, p_7,$$

$$\text{Max } f_3(p) = 80\, p_1 + 35\, p_2 + 98\, p_3 + 180\, p_4 + 200\, p_5 + 62\, p_6 + 6\, p_7,$$

$$\text{Max } f_4(p) = p_1 + p_2 + p_3 + p_4 + p_5 + p_6 + p_7,$$

$$\text{Max } f_5(p) = 0{,}45\, p_1 + 0{,}19\, p_2 + 0{,}54\, p_3 + p_4 + 1{,}11\, p_5 + 0{,}34\, p_6 + 0{,}03\, p_7.$$

With restriction:

26,83 p_1	+ 7,18 p_2	+ 30,74 p_3	+ 48,97 p_4	+ 73,88 p_5	+ 10,80 p_6	+ 1,56 p_7	≤ 5714
51,68 p_1	+ 13,84 p_2	+ 59,22 p_3	+ 94,31 p_4	+ 142,30 p_5	+ 20,80 p_6	+ 3,00 p_7	≤ 11428
27,83 p_1	+ 7,46 p_2	+ 31,89 p_3	+ 50,78 p_4	+ 76,62 p_5	+ 11,20 p_6	+ 1,62 p_7	≤ 5714
16,24 p_1	+ 4,34 p_2	+ 18,60 p_3	+ 29,52 p_4	+ 44,70 p_5	+ 6,53 p_6	+ 0,95 p_7	≤ 2855
17,83 p_1	+ 7,45 p_2	+ 31,89 p_3	+ 50,79 p_4	+ 76,62 p_5	+ 11,20 p_6	+ 1,62 p_7	≤ 5714
44,40 p_1	+ 11,89 p_2	+ 50,87 p_3	+ 81,01 p_4	+ 122,23 p_5	+ 17,87 p_6	+ 2,58 p_7	≤ 8571
8,62 p_1	+ 2,31 p_2	+ 9,87 p_3	+ 15,72 p_4	+ 23,72 p_5	+ 3,4 p_6	+ 0,50 p_7	≤ 2855
34,46 p_1	+ 9,23 p_2	+ 39,48 p_3	+ 62,88 p_4	+ 94,86 p_5	+ 13,87 p_6	+ 2,00 p_7	≤ 5714
15,90 p_1	+ 4,26 p_2	+ 18,22 p_3	+ 29,02 p_4	+ 43,78 p_5	+ 6,40 p_6	+ 0,92 p_7	≤ 2855
5,96 p_1	+ 1,60 p_2	+ 6,83 p_3	+ 10,88 p_4	+ 16,42 p_5	+ 2,40 p_6	+ 0,35 p_7	≤ 2855
6,96 p_1	+ 1,86 p_2	+ 7,97 p_3	+ 12,70 p_4	+ 19,16 p_5	+ 2,79 p_6	+ 0,40 p_7	≤ 2855
64,61 p_1	+ 17,30 p_2	+ 74,02 p_3	+117,89 p_4	+ 177,87 p_5	+ 26,00 p_6	+ 3,75 p_7	≤ 11424

3. CONCLUSION

The subject researching in the field of mathematical model formulation in optimal production program planning requires a complex analysis of the production programs for special tools. For the model formulation it is necessary a previous analysis of dependence on program structure/amount (mass, value, profit), *abc* analysis of given relations and the choice of product representatives.

The mathematical model provides a sufficient foundation for the use of one of operational researching optimization methods like: lexicographic methods, aim programming method, STEP method or other methods. These methods are successfully used by software like GAMS, WinQSB and other. Thus, the mathematical method of multi criteria optimization in observed problem gives an opportunity to decision maker to choose the optimal, close to optimal or aimed final results: product type and number of products that will enable achieving better effects defined by aim functions in an industrial system.

4. REFERENCES

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