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A REVIEW ON TRAVELING REPAIRMAN PROBLEM

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ABSTRACT
Suppose G=(V,A) be an undirected graph with n nodes and a source vertex s Є V. Each node has a problem that must be solved by the repairman that has send. We are given the time required by the repairman to travel among nodes. The objective is to find a tour that minimizes the total waiting time of all the problems that are waiting to be solved. Traveling Repairman problem (TRP) is also known as delivery man problem, minimum latency problem and traveling salesman problem with cumulative costs.
In literature, Traveling Repairman problem (TRP) has considered in various types. Previous literature has considered the version of the problem in which the repair time of a customer is assumed to be zero for latency calculations. A complementary version of the Traveling Repairman problem (TRP), in which a latency bound L is given the problem is to find the minimum number of repairmen required to service all the customers such that the latency of no customer is more than L. Another application of Traveling Repairman problem (TRP) is differentiates waiting times. Here the aim is to find a tour that minimizes the total traveling time plus the perceived waiting time sum across every customer.
In this study we summarize the variations of TRP and explain the merits and disadvantages of each one.
Keywords: Approximation Algorithms, Traveling Repairman Problem

1. INTRODUCTION
Given a finite metric on a set of vertices V and a source vertex s € V, the k-traveling repairman (KTR) problem, a generalization of the metric traveling repairman problem (also known as the minimum latency problem, the delivery man problem, and the school bus-driver problem), asks for k tours, each starting at s (depot) and covering all the vertices (customers) such that the sum of the latencies experienced by the customers is minimum. Latency of a customer p is defined to be the distance traveled before visiting p for the first time. The KTR problem is NP-hard [1], even for k = 1. The problem remains NP-hard even for weighted trees.
The KTR problem with k = 1 is known as the minimum latency problem (MLP) in the literature. The first constant factor approximation for MLP was given by Blum et al. [2]. Goemans and Kleinberg [3] improved the ratio for MLP to 3.59α. In the following discussion, let α be the best achievable approximation ratio for the i-MST problem. The current best approximation ratio for the i-MST problem is (2 +€), due to Arora and Karakostas [4], an improvement over the previous best ratio of 3, due to Garg [5]. Archer, Levin and Williamson [6] presented faster algorithms for MLP with a slightly better approximation ratio of 7.18. Recently, Chaudhuri et al. [7] have reduced the ratio by a

factor of 2, to 3.59. They build on Archer, Levin and Williamson's techniques with the key improvement being that they bound the cost of their i-trees by the cost of a minimum cost path visiting i nodes, rather than twice the cost of a minimum cost tree spanning i nodes.

For the KTR problem, Fakcharoenphol, Harrelson and Rao [8], presented a 8.497α-approximation algorithm. Their ratio was recently improved to 2(2 + α) by Chekuri and Kumar [9]. For a multidepot variant of the KTR problem, in which k repairmen start from k different starting locations, Chekuri and Kumar [9] presented a 6α-approximation algorithm. Recently, Chaudhuri et al. [7] have reduced the ratio to 6 for both the KTR problem and its multidepot variant.

2. KTR PROBLEM

Fakcharoenphol [10] considered the k-traveling repairman problem, a generalization of the metric traveling repairman problem, also known as the minimum latency problem, to multiple repairmen. Informally, in the k-traveling repairman problem they are given that there are k repairmen at a common depot s and n customers sitting in some metric space at prescribed distances from each other and the depot. The goal is to find tours on which to send the repairmen that minimize the average time a customer has to wait for a repairman to arrive, while making sure that all customers are served.

Minimizing average wait time is a very natural objective. In addition to the namesake application of scheduling actual repairman or deliverymen, one can also for example imagine attempting to coordinate the paths of many autonomous crawlers traveling over the Internet. As noted by Blum [2], the k-traveling repairmen problem (and its one-repairman special case) can even be viewed as trying to minimize the expected search time to find an object on a metric space, given that it is equally likely to be at any vertex. This is an attractive feature for a search algorithm.

The case where k=1 has been studied in a number of previous papers, and is often called simply the "minimum latency problem" or the "traveling repairman problem". It is NP hard for a general metric and has been recently shown, is even NP hard for weighted trees [11].

2.1. The Generalized KTR Problem

Generalization of the KTR problem (GKTR), is formalized as follows.

GKTR: Given a metric defined on a set of vertices, V, a source vertex s € V and a positive number k. Also given is a non-negative number for each vertex v € {V - s}, denoting the repair time at v. The objective is to find k tours, each starting at s, covering all the vertices such that the sum of the latencies of all the vertices is minimum.

At first, even though it looks like that the GKTR problem can be reduced to the KTR problem in a straight forward manner, taking a deeper look into the problem reveals that such a reduction might not be possible without a compromise in the approximation ratio. One trivial idea would be to incorporate the repair times associated with vertices into edge weights (where the weight of an edge represents the time to traverse that edge), which can be done by boosting the edge weights as follows: for every edge e incident on vertices i and j in the given graph G, increase the weight (or distance) of e by the sum of $r_i/2$ and $r_j/2$, where r_i and r_j are the repair times of i and j respectively. The solution obtained would be a β-approximation for the modified graph G*. However, the obtained solution will not be a β-approximation for the original problem G. This is due to the reason that the lower bounds for the problems defined as G and G* are different, as can be seen from the fact that the latency of a customer v in an optimal solution to G* comprises half of v's repair time, while this is not the case with an optimal solution to G. At first, even though it looks like an optimal solution to G will be off by just a small constant when compared to an optimal solution to G*, in reality, it could be arbitrarily large.

Jothi [5] presented a 3β-approximation algorithm for the GKTR problem, where β is the best achievable approximation ratio (currently 6) for the KTR problem. When the repair times of all the customers are the same, they presented an approximation algorithm with better ratio. Their ratios hold for the respective multidepot variants of the GKTR problem as well.

2.2. The Bounded-Latency Problem

This problem is complementary versions of the KTR problems [5], in which we are given latency bound L and are asked to find the minimum number of repairmen required to service all the customers such that the latency of no customer is more than L. More formally, Jothi [5] defined the bounded-latency problem (BLP) as follows:

BLP: Given a metric defined on a set of vertices, V, a source vertex s € V and a positive number L. The objective is to find k tours, each starting at s, covering all the vertices such that the latency of no customer is more than L and k is minimum.

The bounded-latency problem is very common in real-life as most service providers work only during the day, generally an 8-hour work day. Under these circumstances, the service provider naturally wants to provide service to all its outstanding customers within the work day, by using as small a number of repairmen as possible. For the BLP, Jothi [5] presented a bicriteria approximation algorithm that finds a solution with at most 2/p times the number of repairmen required by an optimal solution, with the latency of no customer exceeding (1 + p)L, p > 0.

3. CONCLUSION

In this paper we have tried to give brief information about the k-traveling repairmen problem. Literature on the KTR problem has considered the version of the problem in which the repair time of a customer is assumed to be zero for latency calculations. In this paper a generalization of the problem in which each customer has an associated repair time has summarized.

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