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CONSIDERATIONS REGARDING THE GEOMETRICAL MODELING OF PARALLEL MINI-MANIPULATORS WITH TRIANGLE PLATFORM

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ABSTRACT
Parallel kinematic structures have a series of advantages that makes them adequate for the mini-robots and micro-robots construction: actuator positioning on the seating, miniaturisation, stiffness, positioning precision and repeatability, actuators separation from the workspace. For such reason, more and more parallel mechanisms with specified number and type of degrees of freedom have been proposed. In this paper is presented a comparative study of the geometrical models for two guided in three points parallel mechanisms types having three degrees of freedom, so called parallel manipulators of "a" and respectively "b" type. These two mechanisms types differ only through the level on which the passive spherical joints are located. The complications which occur in the geometrical model are illustrated when the spherical joints are not located on the mobile plate level. Some simulation results within a complex simulation system, which has been developed, are presented at the end of the paper.The main contribution of this work is the achievement of a unitary approach regarding some classes of 3 d.o.f. parallel minimanipulators with triangular platform. The obtained results have shown that in the case of 'a" type mechanisms the expressions for the geometrical model and the implicit equations form is more simple than in the case of "b" type mechanisms.
Keywords: parallel robot, mini robot, kinematics, graphical simulation.

1. INTRODUCTION
Parallel kinematic structures have a series of advantages that makes them adequate for the mini-robots and micro-robots construction: actuator positioning on the seating, miniaturisation, stiffness, positioning precision and repeatability, actuators separation from the workspace. For such reason, more and more parallel mechanisms with specified number and type of degrees of freedom have been proposed. In [1] and [2] the 3 DOF spatial parallel symmetrical and guided mechanisms are defined. The structural archetype schemes of the "a" and "b" type guided in three points parallel mechanisms are presented in the figure 1 respectively in the figure 2. The joints surrounded by circles, which are located in the proximity of the fixed base, suggest that these joints are actuated, each of them having 1 DOF. Each kinematic chain, which connects the fixed base with the mobile platform, contains also a passive joint with 1 DOF and a passive spherical joint with 3 DOF. The spherical joints centers are conventionally called guiding points. In the case of the "a" type mechanisms, the guiding points A_i (i=1,2, 3) are moving, each of them on a one degree of freedom curve with respect to the base. In the case of the "b" type mechanisms, the guiding points A_i (i=1, 2, 3) are moving, each of them on a fixed curve jointed with the base respectively on a curve jointed with the mobile plate. The actuating joints with 1 DOF could be "R"-revolute or "P"-prismatic. In order to avoid the mechanism locking, the passive joints are of rotational type.

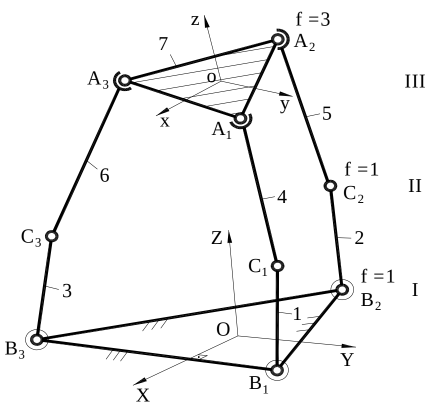

Fig.1. The “a” type parallel mechanisms

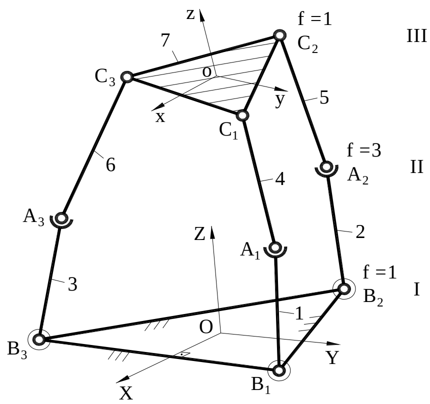

Fig.2. The “b” type parallel mechanisms

2. THE GUIDING CURVES EQUATIONS

To the fixed base is attached the $OXYZ$ reference system and to the mobile platform the $oxzy$ own reference system The mobile plate pose in space is given by the position vector of the platform centre $\overline{P}(X,Y,Z)$ and through the rotational matrix [R]=[$R(\alpha,\beta,\gamma)$].

In the case of “a” type mechanisms the absolutes coordinates of the $A_i(i=1,2,3)$ points depend on the actuated q_i and passive u_i joint coordinates.

$$X_i = X_i(q_i,u_i); Y_i = Y_i(q_i,u_i); Z_i = Z_i(q_i,u_i);\ i = 1,2,3 \tag{1}$$

By eliminating the passive joint coordinates u_i, the Cartesian equations of the described curves by the A_i points in the fixed frame are obtained:

$$F_i(X_i,Y_i,Z_i) = 0\ ;\ G_i(X_i,Y_i,Z_i,q_i) = 0 \tag{2}$$

For the “b” type mechanisms the $A_i(i=1,2,3)$ absolutes coordinates depend only on the actuated joint coordinates q_i while the relative coordinates of the $A_i(i=1,2,3)$ points depend on the passive joint coordinates u_i:

$$X_i = X_i(q_i);\ Y_i = Y_i(q_i);\ Z_i = Z_i(q_i) \tag{3}$$

$$x_i = x_i(u_i);\ y_i = y_i(u_i); z_i = z_i(u_i) \tag{4}$$

By eliminating the joints coordinates q_i from (3) and u_i from (4) the Cartesian equations of the described curves by the A_i points in the fixed frame and in the mobile frame are obtained:

$$F_i(X_i,Y_i,Z_i) = 0;\quad i = 1,2,3\ ;\ G_i(X_i,Y_i,Z_i) = 0;\quad i = 1,2,3 \tag{5}$$

$$f_i(x_i,z_i,y_i) = 0;\quad i = 1,2,3\ ;\quad g_i(x_i,z_i,y_i) = 0;\quad i = 1,2,3 \tag{6}$$

In [5] are given the equations (2), (5) and (6) equations for different guiding cases.

3. THE DIRECT GEOMETRICAL AND THE INVERSE GEOMETRICAL MODELS

The direct geometric model (DGM) assumes to impose the drive joint coordinates $q_i(i=1,2,3)$ and to obtain the mobile platform pose through the *X, Y, Z* coordinates of the mobile plate centre and the α,β,γ orientation angles. The following notations are made: $\overline{P}_i(X_i,Y_i,Z_i)$ - the position vectors of the guided points $A_i(i=1,2,3)$ with respect to the fixed reference frame origin O; $\overline{p}_i(x_i,y_i,z_i)$ - the position vectors of the same points with respect to the mobile platform centre o.

The closure equations for the DGM are represented by the equality of the distances between the guiding points in these two reference systems:

$$\left|\overline{P}_{i+1} - \overline{P}_i\right| = \left|\overline{p}_{i+1} - \overline{p}_i\right|;\ i = 1,2,3;\quad \text{for } i = 3, i+1 = 1 \tag{7}$$

Through numerical solving of the nonlinear equation system (7) are obtained the passive joint coordinates u_i and then the position of the o point and the mobile platform orientation.

The inverse geometrical model (IGM) assumes to impose the mobile platform coordinates and to compute the drive joint coordinates.
As the number of mobility degree for the mechanism is three only three generalized coordinates of the mobile plate are independent and could be imposed, the other three will be deducted. In this mechanisms case the main problem is the defining of three implicit functions, which connect the variables $X, Y, Z, \alpha, \beta, \gamma$. The establishment of theses implicit functions is different for the mechanisms "a" and "b".
In the mechanism case of type "a", the implicit function system is given by the first equations (2) of the fixed surfaces, on which the points A_i are moving. In these equations the variables X_i, Y_i, Z_i are replaced by their expressions with respect to the Cartesian coordinates of the mobile plate:
It yields:

$$H_i = H_i(X, Y, Z, \alpha, \beta, \gamma) = 0. \tag{8}$$

In the mechanism case of type "b" the obtaining of the implicit functions is much more complicated.
The relative coordinates expressions with respect to the absolute ones are replaced in (6):

$$\begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} = [R]^T \begin{bmatrix} X_i - X \\ Y_i - Y \\ Z_i - Z \end{bmatrix} \tag{9}$$

and it yields the equations of the curves which are jointed to the mobile plate in the fixed base reference system,:

$$f_i(X_i, Y_i, Z_i) = 0; \; i = 1,2,3 \; ; \; g_i(X_i, Y_i, Z_i) = 0; \; i = 1,2,3 \tag{10}$$

The implicit functions are obtained through association of the equations (5) and (10) and elimination of the X_i, Y_i, Z_i variables:

$$H_i \equiv H_i(X, Z, \alpha, \beta, \gamma) = 0; \; i = 1,2,3 \tag{11}$$

The determination of the main and secondary coordinates triplet is performed through the calculus of the $C_6^3 = 20$ Jacobians of the functions H_1, H_2, H_3 with respect to three variables from the six Cartesian coordinates *X, Y, Z,* α, β, γ of the mobile plate.
A particular case of the "a" type mechanisms is when the guiding points form an equilateral triangle and they must remain in the planes which contain the OZ axis, symmetrical disposed at 120^0 (the first plane is XOZ), then the main coordinates are Z, α, β and the secondary are X, Z, γ (fig. 3).

$$X = \frac{1}{2} r\left[\left(1 + s^2\alpha\right)c\beta - c^2\alpha\right]; Y = -r\, s\alpha\, s\beta; \gamma = \alpha \tag{12}$$

r is the distance between the guiding points to the mobile plate center.
Generally, the computing of the secondary coordinates of the mobile platform is made through the solving of the implicit equations system.
In the case "a" the actuating joint coordinates q_i result from the equations (2): $G_i(X_i, Y_i, Z_i, q_i) = 0$.
In the case "b" the actuating joint coordinates q_i results from (3): $X_i = X_i(q_i); \; Y_i = Y_i(q_i); \; Z_i = Z_i(q_i)$

4. EXAMPLES

In the figures 3 is represented the kinematical scheme of $3 - P_h RS$ mechanism of "a" type. The main coordinates of the mobile plate are Z, α, β. With the relations (12) are computed the secondary coordinates X, Y, γ.
The drive joint coordinates q_i are:

$$q_i = \sqrt{l^2 - Z_i^2} + \frac{X_i}{c\delta_i} \tag{13}$$

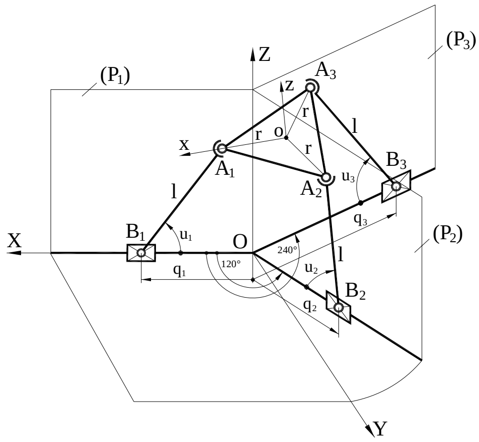

Fig. 3. The $3 - P_h RS$ *parallel mechanism*

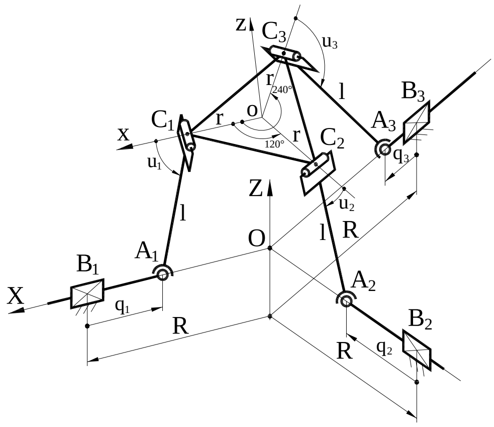

Fig.4. The $3 - P_h SR$ *parallel mechanism*

In the figures 4 the kinematical scheme of a "b" type: $3 - RSR$, $3 - P_h SR$, $3 - P_v SR$ is shown. The evaluations of the main and secondary coordinates are more difficult and the implicit equations form is very complicated.

$$H_i \equiv \left(\left(R - \frac{\overline{P}\cdot[R]\cdot\overline{n}_i}{\overline{\rho}_i\cdot[R]\cdot n_i}\right)\overline{\rho}_i - \overline{P} - [R]\overline{p}_{Ci}\right)^2 - l^2 = 0;\quad i = 1,2,3 \quad q_i = \frac{\overline{P}\,[R]\overline{n}_i}{\rho_i\,[R]\overline{n}_i'} \qquad (14)$$

where

$$\overline{\rho}_i = \begin{bmatrix} c\delta_i \\ s\delta_i \\ 0 \end{bmatrix};\overline{n}_i = \begin{bmatrix} -s\delta_i \\ c\delta_i \\ 0 \end{bmatrix};\ \overline{p}_{Ci} = r\begin{bmatrix} c\delta_i \\ s\delta_i \\ 0 \end{bmatrix}\ \delta_i = 120^0\,(i-1);\ i = 1,2,3$$

5. CONCLUSION

The work contribution is the achievement of a unitary approach regarding some classes of 3 d.o.f. parallel minimanipulators with triangular platform. The geometrical models for two guided in three points parallel mechanisms types with three degrees of freedom are compared. These two parallel mechanisms types differ only through the level on which the passive spherical joints are located. The obtained results have shown that in the case of 'a" type mechanisms the expressions for the geometrical model and the implicit equations form is more simple than in the case of "b" type mechanisms. This leads to the conclusion that it is recommended the building of parallel robots of "a" type.

6. ACKNOWLEDGEMENTS

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7. REFERENCES

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