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COSINE – BASED DECIMATOR

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ABSTRACT
A commonly used decimation filter is the cascaded- integrator-comb (CIC) filter, which consists of two main sections: an integrator and a comb, separated by a down-sampler. This ffilter is attractive in many applications because of its very low complexity which requires no multipliers and storages. The magnitude characteristic of this filter has a low attenuation in the stopband, and a droop in the desired passband, that is dependent upon the decimation factor M and the cascade size K. Additionally the integrator section works at the high input rate. In this paper we present a new decimation filter based on cosine filters. The proposed structure is a multiplier-free and has an improved magnitude response compared with that of the corresponding conventional CIC filter. Additionally there is no filtering at the high input rate.
Keywords: decimation, aliasing, CIC filters, multiplier less filter, cosine filter.

1. INTRODUCTION
There is a continuous trend to replace analog circuitry to digital. Advanced developments in Analogue to Digital (A/D)and Digital to Analogue (D/A) conversion techniques, based on delta sigma modulation are best example of the design trend of shifting more and more signal processing tasks from the analog to the digital domain. This approach avoid strong requirements for the analog anti-aliasing filter , resulting in a simpler analog filter design while requiring fast more complex digital structure. The key digital part of this converter is a decimation filter. Hogenauer [1] proposed a structure called cascaded- integrator-comb (CIC) filter, which is a commonly used decimation filter. This decimation filter is attractive in many applications because of its very low complexity. It consists of two main sections: an integrator and a differentiator section separated by a down-sampler as shown in Fig. 1.

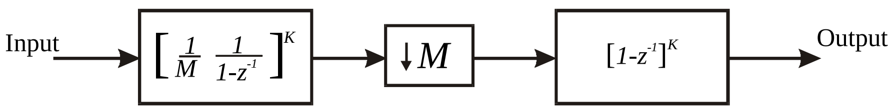

Figure 1. CIC filter

The transfer function of this filter is given as

$$H(z) = \left[\frac{1}{M} \frac{1 - z^{-M}}{1 - z^{-1}} \right]^K , \qquad (1)$$

where M is the decimation ratio and K is the number of cascaded CIC filters.

The magnitude response of the filter can be expressed as

$$H(e^{j\omega}) = \left| \left[\frac{1}{M} \frac{\sin(\omega M/2}{\sin(\omega/2)} \right]^K \right|. \tag{2}$$

The magnitude response has nulls at integer multiples of (1/M) Fs, where F_s is the high-rate sampling frequency. These nulls provide natural alias-rejections introduced by the down-sampling operation of the decimator.

There are two main problems in the application of CIC filters in decimators:

1) While the differentiator section operates at the lower data rate, the integrator section works at the higher input data rate resulting in a larger chip area and a higher power consumption especially when the decimation factor and the filter order are high.

2) The magnitude characteristic has a low attenuation in the stop band and an high droop in the pass band of interest.

Different method have been proposed to improve the characteristic of the CIC filter, while keeping its simplicity [2]-[14]. The use of the non-recursive structure of (1)

$$H(z) = \left[\frac{1}{M}\right]^K \left[1 + z^{-1} + z^{-2} + ... + z^{-(M-1)}\right]^K \tag{3}$$

reduces power consumption and increases the circuit speed, [2]-[3]. More details on a comparison of the performances of the recursive and non-recursive implementation are given in [2].

In this paper we present a new decimation filter based on cosine filters. The proposed structure is a multiplier-free and has an improved magnitude response compared with that of the corresponding conventional CIC filter. The rest of the paper is organised in the following manner. Next section introduces the cosine filters. Section 3 presents the proposed decimation filter along with the efficient structure. The method is illustrated with one example.

2. COSINE FILTERS

For $M = 2$ in Eq. (1) we have

$$G(z) = \frac{1}{2}\left(\frac{1 - z^{-2}}{1 - z^{-1}}\right) = \frac{1}{2}(1 + z^{-1}). \tag{4}$$

The magnitude response of the filter is

$$\left| G(e^{j\omega}) \right| = \left|\cos(\omega/2)\right|. \tag{5}$$

Because of this cosine form this filter is called a cosine filter.

The *N*-expanded filter is obtained by inserting *N*-1 zeros between each sample of the impulse response. In z-domain that means that each delay is replaced by *N* delays

$$G(z^N) = \frac{1}{2}(1 + z^{-N}). \tag{6}$$

The corresponding magnitude response is

$$\left| G(e^{j\omega N}) \right| = \left|\cos(N\omega/2)\right|. \tag{7}$$

The transfer function of the cascade of N_1 expanded cosine filters is given by

$$H_{\cos}(z)=\prod_{N=1}^{N_1} G(z^N)=\prod_{N=1}^{N_1} \frac{1}{2}(1+z^{-N})\,. \qquad (8)$$

The corresponding magnitude response is then

$$\left|H_{\cos}(e^{j\omega})\right|=\left|\prod_{N=1}^{N_1} G(e^{j\omega N})\right|=\left|\prod_{N=1}^{N_1} \cos(N\omega/2)\right|. \qquad (9)$$

3. PROPOSED DECIMATION FILTER

Considering that M is an even number we have from Eq (8)

$$H_{\cos}(z)=\prod_{N=1}^{M/2} \frac{1}{2}(1+z^{-N}) \qquad (10)$$

We propose to split the decimation by M into two stages: $M_1=M/2$ and $M_2=2$.
The proposed filter is given as

$$H_{proposed}(z)=H_0^{k_0}(z)H_1^{k_1}(z^{M_1})\,. \qquad (11)$$

where k_0 and k_1 are the numbers of the cascaded filters $H_0(z)$, and $H_1(z^{M1})$, respectively.

$$H_0(z)=\prod_{N=1}^{M_1-1} \frac{1}{2}(1+z^{-N});\, H_1(z^{M_1})=\frac{1}{2}(1+z^{-M_1})\,. \qquad (12)$$

Using the multirate identity [15] the subfilter $H_1(z^{M1})$ can be moved to lower rate as shown in Figure 2. Using the polyphase decomposition of the filter $H_0(z)$ the polyphase components can also be moved to the lower rate as indicated in Figure 3, where $H_{0p}(z)$ means the polyphase components.

Example 1:

We consider decimation filter for M=8. Therefore M_1=4 and M_2=2. The gain response for the proposed filter using k_0=2 and k_1=4 is given in Figure 4, along with the gain response of the corresponding CIC filter. Note that the frequency characteristic is considerably improved.

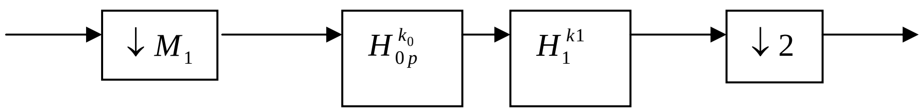

Figure 2. Proposed structure

4. CONCLUSION

New cosine-based decimation filter has been proposed here. The filter is the cascade of expanded cosine filters and thereby is a multiplierless filter. It is suposed that the decimation factor M is even number. The proposed structure is a two-stage structure where the first stage is decimated by $M/2$ and the second stage is decimated by 2. Consequently, the cosine filter expanded by M_1 can be moved to a lower rate. Using the polyphase decompositio,n the cosine filters at high input rate can also be moved to a lower rate. In that way there is no filtering at high input rate. Additionally, the proposed filter exibits an improved magnitude characteristic.

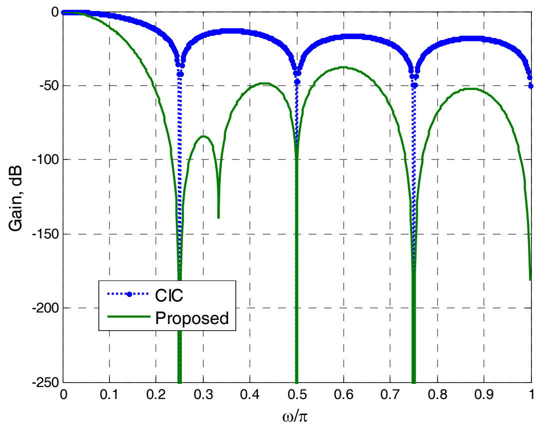

Figure 3. Example 1

5. ACKNOWLEDGEMENT

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