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BEHAVIOR OF THE ELASTIC SHAFT WITH DES-EQUILIBRATED DISK RELATED TO THE TYPE OF SUPPORT

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ABSTRACT
The oscillation of elastic shaft depends upon its geometry and the type of support and as well as on excitation forces.
In this paper the dynamic model with a single non-central disk is analysed taking into consideration gyroscopic effect.
The behavior of the stiffness coefficients as very important parameters of the assumed dynamic model for different types of the bearings is discussed.
Results of the analysis for adopted elastic shaft - non-central disk model supported in rigid and flexible bearings are graphically presented.
Comparing the results the certain conclusions on model behavior has been made.
Key words: Elastic Shaft, Gyroscopic Effect, Stiffness Coefficients, Rigid and Flexible Bearings

1. INTRODUCTION
Engineering components concerned with the subject of rotordynamics are the rotors of machines, especially turbines and generators known as turbogenerators. During the operation the rotor undergoes bending and torsional oscillation.
The oscillation of the elastic shaft depends upon its geometry and the type of support, as well as on excitation forces.
It is known that the gyroscopic effect influences on the speed, sometimes doubling its number.
For analysis the dynamic model with a single non-central disk is adopted with necessary approximations for two types of support – rigid and flexible bearings. Based on adopted dynamic model the mathematical model was built.
The stiffness coefficients as very important parameters on adopted model behavior are discussed for both cases and graphically are presented and compared.

2. DYNAMIC AND MATHEMATICAL MODEL
The adopted dynamic model of elastic shaft with non-central disk enables analysis of the oscillations under torsion and bending at the same time, as well as elastic line deflection and rotation as a result of disk des-equilibration known as precession and nutation that characterizes the gyroscopic effect.
For the dynamic model (*Figure 1 and Figure 2*) the following approximations has been adopted: non-mass shaft is elastic and disk is rigidly assembled; disk is statically and dynamically des-equilibrated; elastic shaft behaves as spring element; bearings are rigid at model in *Figure 1* and flexible at model

in *Figure 2*; lubricant hydrodynamics of bearings in not taken into consideration; the elastic system is supported on stationery and rigid foundation of machines…

The mathematical model of differential equations for dynamic model supported on rigid bearings [1,2]:

$$
\begin{aligned}
& m\ddot{\xi} + k_1\dot{\xi} + k_2\dot{\xi}^2 + k_3\dot{\xi}^3 + p_{11}\xi + p_{14}\beta^* = me(\ddot{\varphi}\sin\varphi + \dot{\varphi}^2\cos\varphi) \\
& m\ddot{\eta} + k_1\dot{\eta} + k_2\dot{\eta}^2 + k_3\dot{\eta}^3 + p_{22}\eta + p_{23}\beta^* = me(-\ddot{\varphi}\cos\varphi + \dot{\varphi}^2\sin\varphi) \\
& A\ddot{\alpha}^* + J\dot{\varphi}\dot{\beta}^* + J\ddot{\varphi}\beta^* + p_{32}\eta + p_{33}\alpha^* = \delta(A-J)[\ddot{\varphi}\cos(\varphi-\varepsilon) - \dot{\varphi}^2\sin(\varphi-\varepsilon)] \\
& A\ddot{\beta}^* - J\dot{\varphi}\dot{\alpha}^* - J\ddot{\varphi}\alpha^* + p_{41}\xi + p_{44}\beta^* = \delta(A-J)[\ddot{\varphi}\sin(\varphi-\varepsilon) - \dot{\varphi}^2\cos(\varphi-\varepsilon)] \\
& J\ddot{\varphi} + c_1\varphi + c_2\varphi^2 + c_3\varphi^3 + b_1\dot{\varphi} + b_2\dot{\varphi}^2 + b_3\dot{\varphi}^3 = 0
\end{aligned}
\qquad (1)
$$

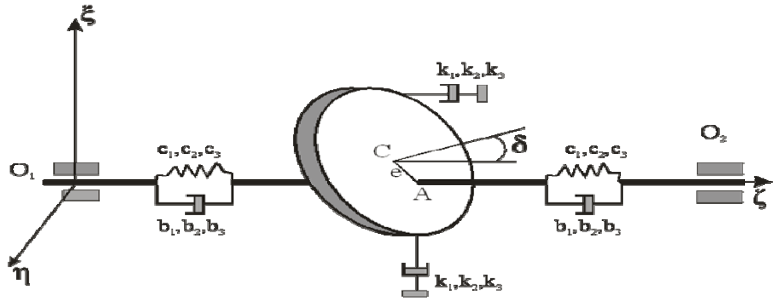

Figure 1. Dynamic model supported on rigid bearings

The mathematical model of differential equations for dynamic model supported on flexible bearings [1,2]:

$$
\begin{aligned}
& m\ddot{\xi} + k_{C1}\dot{\xi} + k_2\dot{\xi}^2 + k_3\dot{\xi}^3 + p_{k11}\xi + p_{k14}\beta^* = me(\ddot{\varphi}\sin\varphi + \dot{\varphi}^2\cos\varphi) \\
& m\ddot{\eta} + k_{C1}\dot{\eta} + k_2\dot{\eta}^2 + k_3\dot{\eta}^3 + p_{k22}\eta + p_{k23}\beta^* = me(-\ddot{\varphi}\cos\varphi + \dot{\varphi}^2\sin\varphi) \\
& A\ddot{\alpha}^* + J\dot{\varphi}\dot{\beta}^* + J\ddot{\varphi}\beta^* + k_{C3}\dot{\alpha}^* + p_{k32}\eta + p_{k33}\alpha^* = \delta(A-J)[\ddot{\varphi}\cos(\varphi-\varepsilon) - \dot{\varphi}^2\sin(\varphi-\varepsilon)] \\
& A\ddot{\beta}^* - J\dot{\varphi}\dot{\alpha}^* - J\ddot{\varphi}\alpha^* + k_{C4}\dot{\beta}^* + p_{k41}\xi + p_{k44}\beta^* = \delta(A-J)[\ddot{\varphi}\sin(\varphi-\varepsilon) - \dot{\varphi}^2\cos(\varphi-\varepsilon)] \\
& J\ddot{\varphi} + c_1\varphi + c_2\varphi^2 + c_3\varphi^3 + b_1\dot{\varphi} + b_2\dot{\varphi}^2 + b_3\dot{\varphi}^3 = 0
\end{aligned}
\qquad (2)
$$

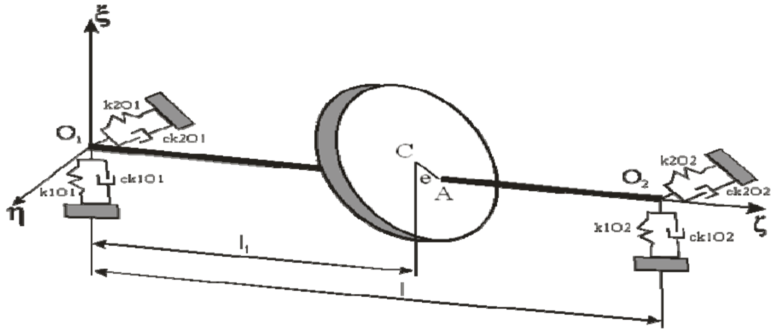

Figure 2. Dynamic model supported on flexible bearings

Where: ***m*** is disk masses; ***A*** is disk's equatorial moment of inertia; ***J*** is disk polar moment of inertia; ***e*** is linear eccentricity (static des-equilibration); ***δ*** is angular eccentricity (dynamic des-equilibration); ***α**** is precession angle; ***β**** is nutation angle; ***φ*** is rotational angle; ***c, b, k*** are stiffness, damping coefficients of shaft or bearings respectively;… [1]

The solution of mathematical model(s) (1) and (2) for adopted dynamic model(s) can be solved by the Fourth Order Runge-Kutta method [1, 2], but in this paper the subject of study is changeability of stiffness coefficient depending on the type of support/bearings.

Therefore, the influence coefficients of displacements at dynamic model of elastic shaft supported to rigid bearings (Figure 1) are calculated by:

$$h_{11}^r = h_{22}^r = \frac{l_1^2 (l-l_1)^2}{3\mathrm{B}l}; h_{23}^r = h_{32}^r = -h_{14}^r = -h_{41}^r = \frac{l_1 (l-l_1)(l-2l_1)}{3\mathrm{B}l}; h_{33}^r = h_{44}^r = \frac{l_1^3 + (l-l_1)^3}{3\mathrm{B}l^2} \quad (3)$$

For elastic bearings (Figure 2) coefficients are:

$$\begin{gathered} h_{11}^e = \frac{(l-l_1)^2}{l^2 k_{1O_1}} + \frac{l_1^2}{l^2 k_{1O_2}}; h_{22}^e = \frac{(l-l_1)^2}{l^2 k_{2O_1}} + \frac{l_1^2}{l^2 k_{2O_2}}; h_{33}^e = \frac{1}{l^2 k_{2O_1}} + \frac{1}{l^2 k_{2O_2}}; h_{44}^e = \frac{1}{l^2 k_{1O_1}} + \frac{1}{l^2 k_{1O_2}} \\ h_{14}^e = h_{41}^e = -\frac{l-l_1}{l^2 k_{1O_1}} + \frac{l_1}{l^2 k_{1O_2}}; h_{23}^e = h_{32}^e = \frac{l-l_1}{l^2 k_{2O_1}} - \frac{l_1}{l^2 k_{2O_2}} \end{gathered} \quad (4)$$

The sum of (3) and (4) gives the total value for the coefficients of displacements at dynamic model of elastic shaft supported to elastic bearings:

$$\begin{gathered} h_{11} = h_{11}^r + h_{11}^e; h_{22} = h_{22}^r + h_{22}^e; h_{33} = h_{33}^r + h_{33}^e; h_{44} = h_{44}^r + h_{44}^e \\ h_{14} = h_{41} = h_{14}^r + h_{14}^e; h_{23} = h_{32} = h_{23}^r + h_{23}^e \end{gathered} \quad (5)$$

For the models from theory of Strength of materials the stiffness coefficients or so called Dual Maxwell Coefficients are given by expressions [1, 2]:

$$\begin{gathered} p_{11} = \frac{h_{44}}{h_{11}h_{44} - h_{14}^2}; p_{22} = \frac{h_{33}}{h_{22}h_{33} - h_{23}^2}; p_{33} = \frac{h_{22}}{h_{22}h_{33} - h_{23}^2}; p_{44} = \frac{h_{11}}{h_{11}h_{44} - h_{14}^2} \\ p_{14} = p_{41} = -\frac{h_{14}}{h_{11}h_{44} - h_{14}^2}; p_{23} = p_{32} = -\frac{h_{23}}{h_{22}h_{33} - h_{23}^2} \end{gathered} \quad (6)$$

3. VARIABILITY ANALYSIS OF THE STIFFNESS COEFFICIENTS

It is very easy to be noticed that the coefficients given by (3), (4), (5) and (6) depend on the position of the disk. For easy discussion of a new non-dimensional variable is adopted:

$$x = \frac{l_1}{l} \quad \text{where } x \in (0,1) \quad (7)$$

So, the stiffness coefficients from (6) are functions of variable $\boldsymbol{x}$ and distance between to bearings $\boldsymbol{l}$ [1].

The variation of the stiffness coefficients from (6) is analysed for two dynamic models:

- For elastic shaft supported on rigid bearings, where at equation (5) all components for elastic bearings are taken equal to zero and stiffness coefficients from (6) will be indicated as in expression:

$$\begin{gathered} h_{11} = h_{11}^r; h_{22} = h_{22}^r; h_{33} = h_{33}^r; h_{44} = h_{44}^r; h_{14} = h_{41} = h_{14}^r; h_{23} = h_{32} = h_{23}^r \\ p_{11}^r = p_{11}; p_{22}^r = p_{22}; p_{33}^r = p_{33}; p_{44}^r = p_{44}; p_{14}^r = p_{14} = p_{41}; p_{23}^r = p_{23} = p_{32} \end{gathered} \quad (8)$$

- For elastic shaft supported on elastic bearings the stiffness coefficients from (6) will be expressed as:

$$p_{11}^e = p_{11}; p_{22}^e = p_{22}; p_{33}^e = p_{33}; p_{44}^e = p_{44}; p_{14}^e = p_{14} = p_{41}; p_{23}^e = p_{23} = p_{32} \quad (9)$$

And the new non-dimensional coefficient as a quotient of stiffness coefficients for rigid and elastic supports is adopted:

$$r_{p_{11}} = \frac{p_{11}^r}{p_{11}^e}; r_{p_{22}} = \frac{p_{22}^r}{p_{22}^e}; r_{p_{33}} = \frac{p_{33}^r}{p_{33}^e}; r_{p_{44}} = \frac{p_{44}^r}{p_{44}^e}; r_{p_{14}} = r_{p_{41}} = \frac{p_{14}^r}{p_{14}^e}; r_{p_{23}} = r_{p_{32}} = \frac{p_{23}^r}{p_{23}^e} \quad (10)$$

The results of analysis for the mathematical model(s) of adopted dynamic model(s)graphically presented in *Figure 3* has been achieved for following values:

$$\begin{gathered} m = 2000[kg]; J = 700[kgm^2]; A = 0.7J; l = 2[m]; B = EI = 5\cdot 10^6[Nm^2] \\ k_{1O_1} = 3.3\cdot 10^6[N/m]; k_{1O_2} = 1.65\cdot 10^6[N/m]; k_{2O_1} = 8\cdot 10^5[N/m]; k_{2O_2} = 4\cdot 10^5[N/m] \end{gathered} \quad (11)$$

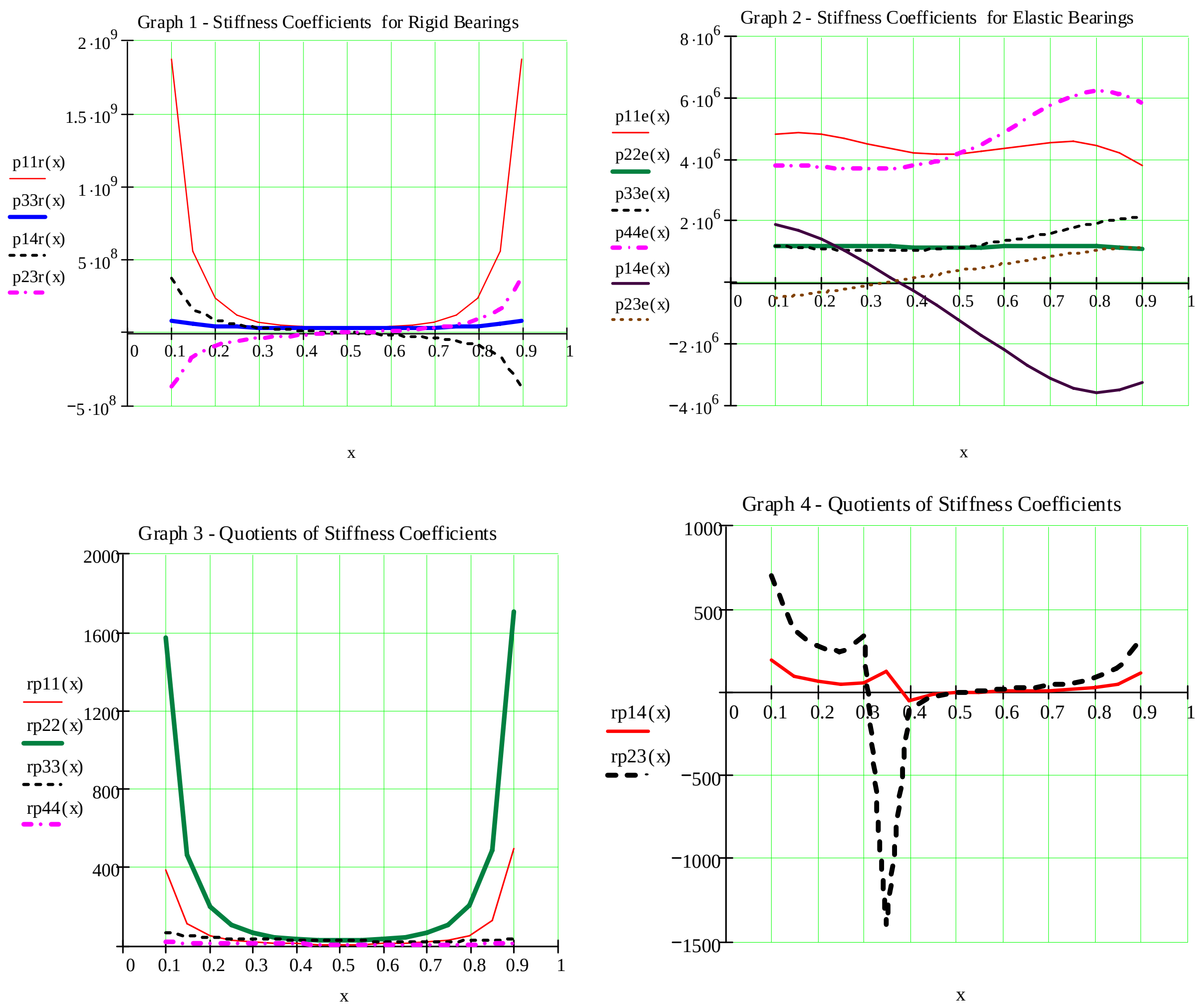

Figure 3. Stiffness Coefficients and their Quotients for Rigid and Elastic Bearings

4. CONCLUSIONS

Referring to graphically presented results in Figure 3 can be noticed that:

- Stiffness coefficients for rigid bearings (Graph 1 at Figure 3) have several times bigger values comparing to those for elastic bearings (Graph 2 at Figure 3);
- The curves for quotients in Graph 3 at Figure 3 are similar with the curves for corresponding stiffness coefficients in Graph 1 at Figure 3 based on above conclusion.

Therefore, generally it can be concluded that while stiffness coefficients for rigid bearings are related only to the type of material of the elastic shaft, the stiffness coefficients for elastic bearings depend also on the stiffness coefficients of the bearings.

5. REFERENCES

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