11th International Research/Expert Conference
"Trends in the Development of Machinery and Associated Technology"
TMT 2007, Hammamet, Tunisia, 05-09 September, 2007.

THE ENERGY BALANCE PROCEDURE FOR MODELING NONLINEAR SELF-EXCITATION OSCILLATIONS

Seniha Karić
University of Tuzla, Faculty of Mechanical Engineering
Tuzla

Avdo Voloder
University of Sarajevo, Faculty of Mechanical Engineering
Sarajevo

Dušan Vukojević
University of Zenica, Faculty of Mechanical Engineering
Zenica
Bosnia & Herzegovina

ABSTRACT
Research of the behavior nonlinear mechanical systems enables finding of the possible states of system stability as well as analysis of stability and processes of system transition from one state of motion stability to another. In this paper, a procedure of energy balance is presented, for the purpose of making a mathematical model of self-excitation oscillations in the system with one degree of freedom and analysis of phase portraits, i.e. existence of stable states of motion for different initial conditions. The procedure was applied in modeling of self-excitation oscillations for high-speed milling and is based on determination of non-linear self-excitation force and non-linear coefficient. The mathematical model is made on the basis of experimentally acquired results that are basis for determination of real parameters of the system.
Keywords: self-excitation vibration, energy balance, phase portrait.

1. INTRODUCTION
The analysis of energy balance gives a quality insight into the behaviour of self-excitation oscillations, both harmonic and non-harmonic, as well as an answer to a question of existance of periodical oscillations in systems with one degree of freedom of motion. The cause of self-excitation oscillations is energy source by which energy is brought into system during the oscillations. That additional energy from outside of the system depends on self oscillations. Howevere, amlitudes of self-excitation oscillations don't decrease as in case of free oscillations due to damping, but they rise due to energy brought into the system, so for certain amplitudes an energy balance may be reached including energy of damping and energy of self-excitation. The result of the above mentioned are periodic oscillations and the system is in stable state.

2. THE ENERGY BALANCE PROCEDURE
The equation of motion for self-excited oscillations matches the equation of motion of free non-linear oscillations and can be presented by the following expression

$$m\ddot{q} + f^{dz}(q,\dot{q}) + f^{k}(q) = 0, \qquad \ldots(1)$$

where $f^{k}(q)$ is the force stiffness (conservative forces), and $f^{dz}(q,\dot{q})$ is the non-conservative forces which consists partly from self-excitation force which adds energy to the system $f^{z}(q,\dot{q})$ and disipation force which takes away energy from the system $f^{d}(q,\dot{q})$. It is assumed that these two parts may be separated so that the equation of motion can be presented as

$$m\ddot{q} + f^{z}(q,\dot{q}) + f^{d}(q,\dot{q}) + f^{k}(q) = 0\,, \qquad \ldots(2)$$

where the self-excitation force is equal to zero for initial conditions equal zero. $f^{z}(q=0,\dot{q}=0)=0$.

The energy equation for this system is

$$E^{kin} + E^{pot} = E^{0} - E^{d} + E^{z}\,, \qquad \ldots(3)$$

where E^{kin} is the kinetic energy, E^{pot} is the potential energy, E^{0} is constant energy, E^{d} -the dissipative energy and $E^{z} = \int f^{z}(q,\dot{q})dq$ is the energy brought into system from outside. By using the energy balance a solution for the equation (3) is assumed in the shape of the periodic function, which, for one period, is approximatelly equal to one free non-damping oscillation

$$q(t) = C\cos(\omega t - \alpha), \qquad \ldots(4)$$

In orderr to establish non-periodical oscillations defined by eq. (2) it is necessary that dissipation energy and energy brought from outside are equal. From this condition, boundary amplitudes may be determined C^{b} of possible periodic oscillation, while under certain conditions several solutions might exist.

$$\Delta E^{z}(C) = \Delta E^{d}(C) \qquad \ldots(5)$$

where $\Delta E^{d}(C)$ is the energy loss during one period depending on the amplitude C, while $\Delta E^{z}(C)$ is the energy brought to the system during one period.

3. MATHEMATICAL MODEL

The procedure of energy balance is applied on a model of the self-excitation oscillations caused by dry-friction, which often occur during processing on the tool machines. The mathematic model is created on the basis of the experimental results measured on high-speed the milling machine [1]. The experimental results served for adoption of real parameters of system. The equation (6) where $f(t)$ is excitation force from outside with harmonic form which describes force excitations (as a result of cutting process) before activity of self-excitation force.

$$m\ddot{q} + d\dot{q} + kq = f(t) \qquad \ldots(6)$$

The equation of free self-excitation oscillations which is caused by dry-friction can be written in the following form

$$\ddot{q} + 2\omega D\dot{q} - \frac{1}{m}\nu_{0}\cdot sign(\dot{q}) + \omega^{2}q = 0\,. \qquad \ldots(7)$$

The coefficient ν_{0} is determined from the condition that self-excitation oscillation amplitudes C for excitation force $f(t)=0$ are in compliance with amplitudes in steady-state with present excitation force $f(t)$ without nonlinear member $f^{z}(\dot{q})=0$. The change of dumping energy for viscosity dumping force is calculated from the expression $\Delta E^{d} = \int_{0}^{T} f^{d}(q,\dot{q})\,\dot{q}dt$, while the change of energy from self-excitation is calculated by the following expression $\Delta E^{z}=\int_{0}^{T} f^{z}(q,\dot{q})\dot{q}dt$. Using the previous expressions in equation (5) the value non-linearity coefficient ν_0 is obtained. Equation (5) is solved by numerical method Runge-Kutta fourth order for initial conditions of vector state space $\underline{x}=\begin{bmatrix} q \\ \dot{q} \end{bmatrix}$ different by zero. An appropriate simulation is made in MATLAB. Fig. (1) shows energy diagram and phase portrait for equation (5) with adequate parameters. It is important to say that the oscillations may be established for initial conditions different from zero, which is not the case for free oscillations. The analysis for phase portrait was made for different values of initial conditions, where forming of boundary cycle was obvious. In the first case initial value is smaller than amplitude from boundary cycle and there is obvious tendency to achieve stationary state is making (boundary cycle). For the second initial conditions the value amplitude is bigger than amplitude of boundary cycle and again there is a visible tendency of moving toward boundary cycle. The direction of movement is clock-wise, which points to a stable state.

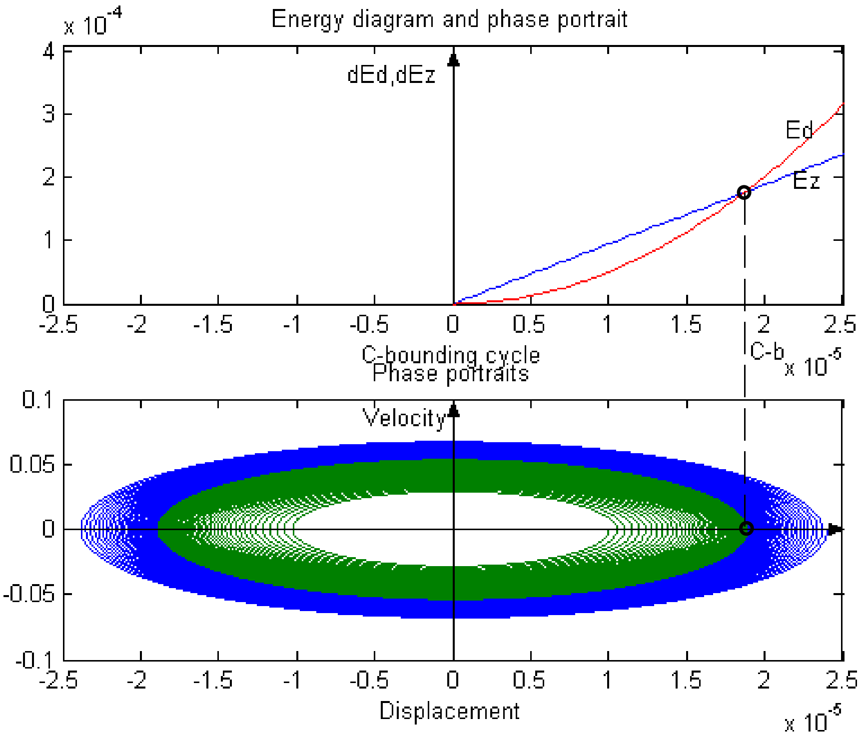

Figure 1 Energy diagram and phase portrait with one boundary cycle

With appearance of self-excitations oscillations (dry friction phenomenon during the contact between tool and workpiece) the motion can be described by equation

$$\ddot{q}+2\omega D\dot{q}-\frac{1}{m}\nu_0\cdot sign(\dot{q})+\omega^2 q=\frac{1}{m}f(t) \qquad \dots(8)$$

The solution of equation (8) can be assumed as a sum of two harmonic functions with participation of excitation frequency and self-excitation frequency

$$q=C\cos(\omega t)+Q\cos(\Omega t-\chi) \qquad \dots(9)$$

where C is periodic motion amplitude before arise of self-excitation oscillations, and Q is amplitude in stacionary state after appereance of self-excitation oscillations. By using energy balance again the nonlinear coefficient is calculated for a certain ratio of amplitude before and after self-excitation oscilllation. Obtained results also point to existence a few of boundary cycles so the motion is also stable after self-excitation Fig (2).

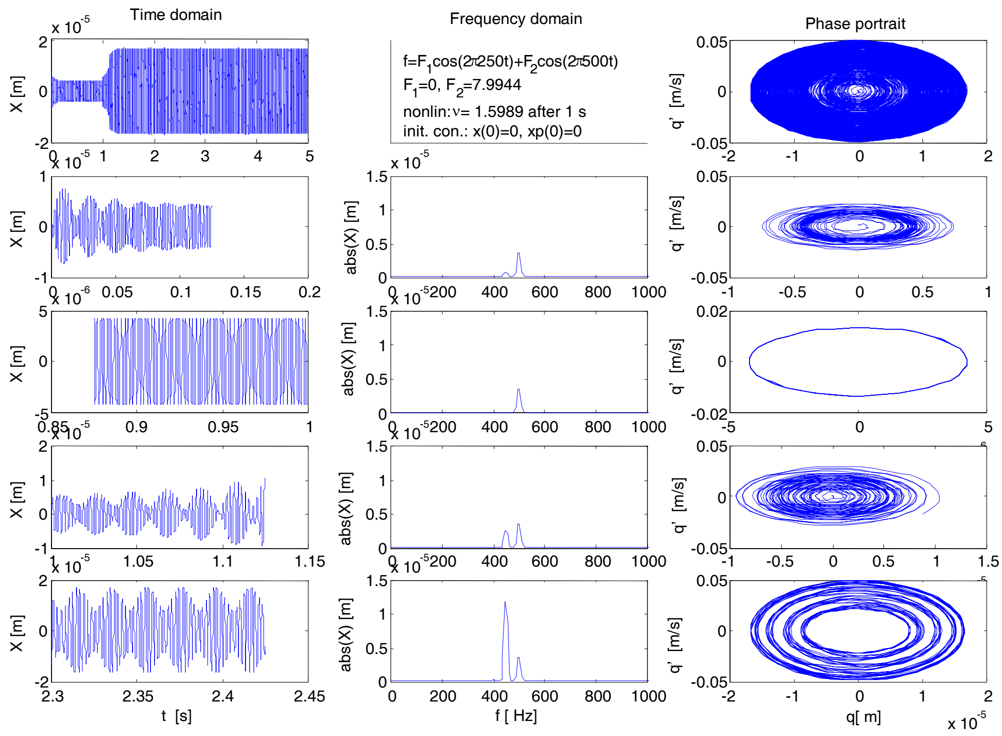

Figure2. Scheme of oscillation for time and frequency domain and adequately phase portraits

4. CONCLUSIONS

The computer application program in solving complex nonlinear equations of oscillatory motion as well as simulations of processes make the system analysis and creating of the mathematical model easier, faster and more accurate at the same time givin enough data for system identification. Method of energy balance is a precise method for determining the parameters of system (where reasons of self-excitations are not well known) as well as the analysises of system stability for existing and the changeable parameters of system. It is also suitable for nonharmonic systems when the analysis of phase portraits gives trustworthy insight into the stability. The created mathematical model has defined the reason of self-excitation occurence (dry friction) with determinated values of parameters. As a result of energy balance application boundary cycles, which acknowledge stability state, are also obtained.

5. REFERENCES

[1] S. Hadžić, Doprinos fenomenološkom pristupu matematskog modeliranja oscilacija mehaničkih sistema, Magistarski rad, Mašinski fakultet u Tuzli, Tuzla, 2004.
[2] H. P. Woelfel, Maschinendynamik, Darmstadt, 2004.
[3] D. Vukojević, E. Ekinović, Teorija oscilacija, Zenica, 2004.
[4] M. Stegić, *Teorija vibracija linearnih diskretnih mehaničkih sistema*, Zagreb, 1996.
[5] S. H. Strogatz, *Nonlinear Dynamics and Chaos*, Perseus Books Publishing, LLC, 1994.
[6] A. Tondl, *On the Interaction between self-excitated and forced vibrations*, Prague, 1976.
[7] V.I. Gulaev, V.A. Baznov, S.L. Popov, *Prikladnie zadači teorii nelinejnih kolebanij mehaničeskih sistem*, Moskva, 1989.