11h International Research/Expert Conference
"Trends in the Development of Machinery and Associated Technology"
TMT 2007, Hammamet, Tunisia, 05-09 September 2007.

DYNAMIC CONDITIONS OF CONTACT OF ELEMENTS OF MECHANISM WITH ROTATIONAL CAM AND ROTATIONAL BAR

Avdo Voloder, Maida Čohodar
Faculty of Mechanical Engineering
Sarajevo

Seniha Karić
Faculty of Mechanical Engineering
Tuzla
Bosnia and Herzegovina

ABSTRACT
Mechanism with rotational cam and rotational bar has important application in technique. In target of regular work of this mechanism, contact between plate bar and cam must be provided, in other words, force between cam and bar must exist. For satisfaction this conditions, we often use elastic elements (springs). This paper presents analysis of influential parameters on this conditions.
Key words: mechanism, rotational cam, rotational bar, conditions of contact

1. FORMULATION OF DYNAMIC MODEL
Figure 1. presents mechanism with rotational cam and rotational bar, where angular velocity of cam ($\vec{\omega}$) has same course like angular velocity of bar ($\vec{\omega}_1$). Let we introduce next signs: $\vec{N}, \vec{T}$ - perpendicular force and force of friction between cam and bar; τ, n - tangent and normal axis in point of contact of these two elements of mechanism; φ - angle of rotation of cam; γ - beginning angle between bar and axis which goes through centres of rotation cam and bar (O, B); ψ - working angle of rotation of bar; $\vec{v}$ - velocity of pick of bar.

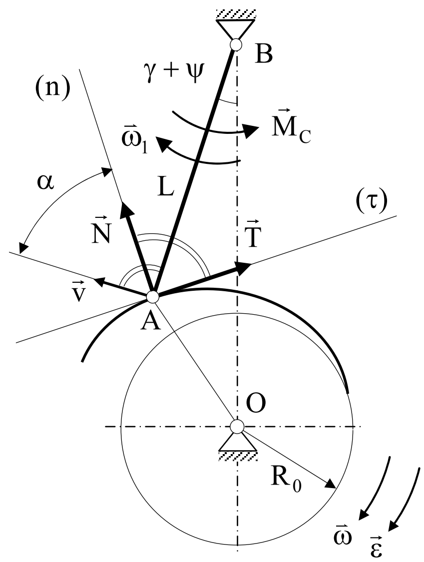

Figure 1. Mechanism with rotational cam and rotational bar with same course of angular velocity of cam and angular velocity of bar

We put dynamic equation of rotation of bar [1]

$$J_B \frac{d^2\psi}{dt^2} = \sum M_B \,, \qquad \ldots(1)$$

where: J_B - inertia moment of bar by point B; t- time.

On other side, we can write

$$\frac{d^2\psi}{dt^2} = \frac{d^2\psi}{d\varphi^2}\omega^2 + \frac{d\psi}{d\varphi}\varepsilon = \psi'' \cdot \omega^2 + \psi' \cdot \varepsilon \,, \qquad \ldots(2)$$

where: ε - angular acceleration of cam.

We can write

$$J_B\left(\psi'' \cdot \omega^2 + \psi' \cdot \varepsilon\right) = N\cos\alpha \cdot L - T\sin\alpha \cdot L - M_C \,, \qquad \ldots(3)$$

where: L - length of bar; M_C - moment of spring which is need for contact between cam and bar, for point B as pole.

Moment of spring has value

$$M_C = M_{C0} + c\psi \,, \qquad \ldots(4)$$

where: M_{C0} - beginning moment of spring, c - stiffness of spring, α - pressure angle of mechanism.

Pressure angle is defined as angle between normal on cam in point of tangent between cam and beam and velocity of pick of bar of mechanism. Absolute value of pressure angle of cams must be in definite borders, in target prevention of blockade of such mechanisms. On other side, with increase of absolute value of pressure angle we have decrease level of utility of mechanism [2]. Maximal permissible pressure angle is directly connected with characteristics of friction between beam and cam of mechanism. Its value can be until 50^0 [3].

When

$$T = \mu N \qquad \ldots(5)$$

where: μ - coefficient of friction between cam and bar, it is following

$$J_B\left(\psi'' \cdot \omega^2 + \psi' \cdot \varepsilon\right) = NL \cdot (\cos\alpha - \mu\sin\alpha) - (M_{C0} + c\psi). \qquad \ldots(6)$$

According last term we obtain perpendicular force N

$$N = \frac{J_B\left(\psi'' \cdot \omega^2 + \psi' \cdot \varepsilon\right) + M_{C0} + c\psi}{L \cdot (\cos\alpha - \mu\sin\alpha)} . \qquad \ldots(7)$$

Condition that contact between cam and bar exists, is: $N \rangle 0$.

From term (7) and last condition we obtain

$$\frac{J_B\left(\psi'' \cdot \omega^2 + \psi' \cdot \varepsilon\right) + M_{C0} + c\psi}{L \cdot (\cos\alpha - \mu\sin\alpha)} \rangle 0 . \qquad \ldots(8)$$

With regard to maximal permissible pressure angle is $\alpha_{max} = 50^0$, and: $\mu \approx 0{,}1$ (for still at still), it is following

$$\cos\alpha - \mu\sin\alpha \rangle 0 \,, \qquad \ldots(9)$$

from (8) next term following

$$J_B\left(\psi''\cdot\omega^2+\psi'\cdot\varepsilon\right)+M_{C0}+c\psi\rangle 0\,. \qquad \ldots(10)$$

For mechanism with rotational cam and rotational bar, with opposite course of angular velocity of these elements (Fig. 2), we obtain same term like (10).

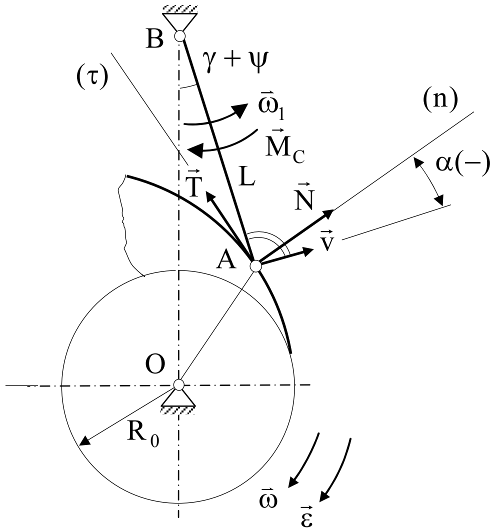

Figure 2. Mechanism with rotational cam and rotational bar, with opposite course of angular velocity of cam and angular velocity of bar

Last term presents fundamental condition of contact between cam and bar for mechanism with rotational cam and rotational bar.

2. EXAMPLE

For mechanism with rotational cam and rotational bar, low of change of angle rotation of cam is: $\psi=\frac{\pi}{12}\cdot\left(1-\cos 2\varphi\right)$, in interval: $0\le\varphi_p\le\pi$. It is need calculate beginning moment of spring M_{C0}, necessary for contact of cam and bar, if next values are known: inertia moments of bar: $J_B=0{,}006$ kgm^2; number of revolutions of cam in minute: $n=700$ 0/min (= const.); stiffness of spring: $c=20$ Nm/rad.

Solution:

Second derivation of angle of rotation of bar by angle of rotation of cam is

$$\psi''=\frac{\pi}{3}\cdot\cos 2\varphi\,.$$

Figure 3. presents angle of rotation of bar and its second by angle of rotation of cam.

Angular velocity of bar is

$$\omega=\frac{\pi n}{30}=\frac{\pi\cdot 700}{30}=73{,}3038 \quad s^{-1}.$$

According term (10) and when $\omega=\text{const.}$ we obtain

$$\left(M_{C0}\right)_{min}=\left(-J_B\psi''\cdot\omega^2-c\psi\right)_{max}\,.$$

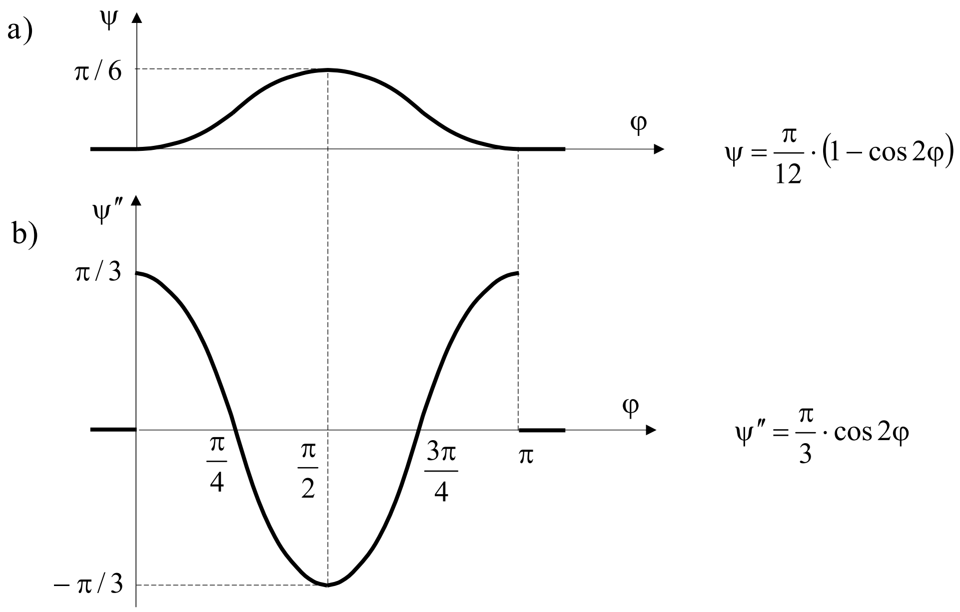

Figure 3. Diagram of angle of rotation of cam (a) and its second derivation by angle of rotation of cam in example (b)

We observe function

$$F(\varphi) = -J_B \psi'' \cdot \omega^2 - c\psi ,$$

which for our example has form

$$F(\varphi) = -0{,}006 \cdot \frac{\pi}{3} \cos 2\varphi \cdot 73{,}3038^2 - 20 \cdot \frac{\pi}{12}(1 - \cos 2\varphi),$$

$$F(\varphi) = -28{,}5264 \cdot \cos 2\varphi - 5{,}2359 .$$

Maximal value of function $F(\varphi)$ is for: $\varphi = \frac{\pi}{2}$ and it is

$$F(\varphi)_{max} = 23{,}2905 \quad \text{Nm.}$$

Consequently, minimal value of beginning moment of spring M_{C0}, necessary for contact of cam and bar, is

$$(M_{0C})_{min} = 23{,}2905 \quad \text{Nm.}$$

3. CONCLUSION

In target of determination of conditions of contact between cam and bar of mechanism with rotational bar and rotational bar, equation which describe these conditions is derived. These conditions are depend of: lows of motion of cam, first and second derivation of angle rotation of bar by angle rotation of cam and of stiffness of spring which is need for contact between cam and bar. In target illustration this problem, one example is shown.

4. REFERENCES

[1] Beer, F.P., Johnston, E.R.: Engineers (Static and Dynamic). McGraw-Hill Book company, 1997.
[2] Voloder A.: Teorija mehanizama, Univerzitetski udžbenik, Mašinski fakultet Sarajevo, 2005.
[3] Artobolovskij, I.I. & Edeljštein, B.V. Sbornik zadač po teorii mehanizmov i mašin, Nauka, Moskva, 1973.,
[4] Norton,R. L.: Design of machinery, McGraw-Hill, Inc. Company, New York , 1992.,
[5] Eckhardt H.D.: Kinematic Design of Machines and Mechanisms, McGraw-Hill, New York, 1998.