11th International Research/Expert Conference
"Trends in the Development of Machinery and Associated Technology"
TMT 2007, Hammamet, Tunisia, 05-09 September, 2007.

SCREW MOTION OF SOME BASIC KINEMATIC BLOCKS THROUGH DUAL-VECTOR ALGEBRA

Dr. Sc. Kastriot A. Buza
Faculty of Education
Prishtina
Kosova

Dr. Sc. Agim Anxhaku
Faculty of Mechanical Engineering
Tirana
Albania

Dr. Sc. Ismajl Gojani, Dr.Sc. Arbnor Pajaziti
Faculty of Mechanical Engineering
Prishtina
Kosova

ABSTRACT
In this paper the methodology of dual-vector algebra is used to present the screw motion for design of mechanisms achieving desired output motion based on input design requirements.
The general model of screw motion given through dual-vector algebra includes dual number, direction vector and position vector of the motion complying topological synthesis for mechanisms design.
Motion characteristic code fulfils functional synthesis for mechanisms design.Screw motion and motion characteristic code for a number of basic kinematics blocks are also given in the paper.
Key words: Screw motion, dual-vector, dual-number, basic kinematics blocks, input motion, output motion, motion characteristic code.

1. INTRODUCTION

The mechanism building blocks with single kinematic degree of freedom – one input and one output motion are discussed in this paper. The mathematical model of building blocks contains (a) the spatial orientation of the building block input with regard to its output and (b) types of motions as translation, rotation, helicoidal etc. A general way of presentation for a building block with one degree of freedom is through a model with one input and output motion. In some cases the input and the output can interchanged depending on the application but in other cases such an interchange is not practical – building block with worm-gear and wheel [2,4].
To design the motion characteristics of the basic kinematic building blocks the screw kinematic block is used.
Generally, the screw motion is represented by two methods: one is through screw coordinates, known as Plucker's coordinates [6] and the other one is through dual-vector.
In this paper the method of dual-vector algebra is applied.
The dual-vector algebra has some advantages comparing with Plucker's coordinate representation, because it contains information for the type of motion, its direction and its position [1, 4, 6, 7].

2. METODOLOGY OF DUAL - VECTOR ALGEBRA

It is known that motion of any solid body in three-dimensional space can be represented through screw kinematic couples [1, 2, 3, 4, 5, 6, 7].
Screw can be represented by displacement and line. Line contains the information about position and direction of the motion. Displacement represents the transformation between rotational and translational motion. The motion transformation can be divided into a dual-number and dual-vector.
Vector $\vec{l}$, which is bounded to a line with its moment from origin O is $\vec{r} \times \vec{l}$, can be represented by

two vectors.

Plucker's coordinates are expressed by vectors $\vec{l}$ *and* $\vec{l}_0 = \vec{r} \times \vec{l}$.

For further analysis of the motion and definition of the screw motion the necessary transformations are needed in such a way that the line will be represented as a dual-vector [7] given below:

$$\hat{\mathrm{L}} = \vec{\mathrm{l}} + \varepsilon\, \vec{\mathrm{l}}_0 \qquad (1)$$

where ε is an algebraic infinitive small unit satisfying the condition $\varepsilon^2 = 0$

Similarly, dual-angle is defined as:

$$\hat{A} = \alpha + \varepsilon\, a\,, \qquad (2)$$

where *a* is the magnitude of vector $\vec{a}$ (Fig.1). Generally, the dual-angle refers the screw displacement and the quantity that results from multiplication of the pitch from equation (2) and magnitude is simply called displacement.

After needed algebraic operations with lines (Fig.1) given by equations:

$$\hat{\mathrm{L}}_1 \cdot \hat{\mathrm{L}}_2 = \cos \hat{A} \qquad (3)$$

$$\hat{\mathrm{L}}_1 \times \hat{\mathrm{L}}_2 = \sin \hat{A}\, \hat{\mathrm{L}} \qquad (4)$$

The result is mathematical expression representing not a line but screw (Fig.1) [7].

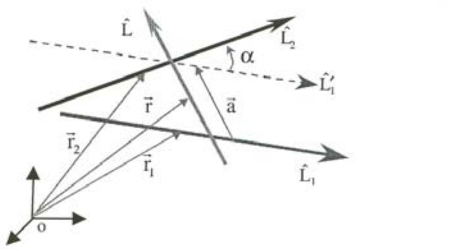


Figure1 Definition of screw and line vector

Screw is a geometric entity that is composed by a displacement and a line. By dual-number representation of the motion that rotates a line unit about unit line vector $\hat{L}$ by α and translates along $\hat{L}$ by *a* at the same time can be represented by screw ($\hat{S}$) as [7]:

$$\hat{\mathrm{S}} = \alpha \cdot \vec{l} + \varepsilon \cdot [a \cdot \vec{l} + \alpha \cdot (\vec{r} \times \vec{l})]$$

$$\hat{\mathrm{S}} = \alpha \cdot \vec{l} + \varepsilon \cdot a \cdot \vec{l} + \varepsilon \cdot \alpha \cdot (\vec{r} \times \vec{l}) + \varepsilon^2 \cdot a \cdot (\vec{r} \times \vec{l})$$

$$\hat{\mathrm{S}} = (\alpha + \varepsilon \cdot a) \cdot [\vec{l} + \varepsilon \cdot (\vec{r} \times \vec{l})]$$

$$\hat{\mathrm{S}} = \hat{A}\hat{\mathrm{L}}. \qquad (5)$$

Therefore, general form of screw motion through dual-vector algebra [1] can be expressed as follows:

$$\hat{S} = (\alpha + \varepsilon a)\{\vec{l} + \varepsilon \vec{l}_0\}$$

$$\hat{S} = (\alpha + \varepsilon a)\{\vec{l} + \varepsilon(\vec{r} \times \vec{l})\}$$

$$\hat{S} = (\alpha + \varepsilon a)\left\{ (l_x \vec{i} \quad l_y \vec{j} \quad l_z \vec{k})^T + \varepsilon \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ r_x & r_y & r_z \\ l_x & l_y & l_z \end{vmatrix}^T \right\}$$

$$\hat{S} = (\alpha + \varepsilon a)\left\{ (l_x \vec{i} \quad l_y \vec{j} \quad l_z \vec{k})^T + \varepsilon\left[(r_y l_z - r_z l_y)\vec{i} \quad (r_z l_x - r_x l_z)\vec{j} \quad (r_x l_y - r_y l_x)\vec{k} \right]^T \right\}$$

$$\hat{S} = (\alpha + \varepsilon a)\left[\begin{Bmatrix} l_x \\ l_y \\ l_z \end{Bmatrix} + \varepsilon \begin{Bmatrix} r_y l_z - r_z l_y \\ r_z l_x - r_x l_z \\ r_x l_y - r_y l_x \end{Bmatrix} \right] \qquad (6)$$

where:

$\vec{l} = l_x\vec{i} + l_y\vec{j} + l_z\vec{k}$ represents the vector of motion direction;

$\vec{l}_0 = l_{0x}\vec{i} + l_{0y}\vec{j} + l_{0z}\vec{k}$ represents the vector of motion position;

$\left|\vec{l}\right| = \sqrt{l_x^2 + l_y^2 + l_z^2} = 1$, l_x, l_y, l_z can be from (-1) to (1).

$\vec{l}\cdot\vec{l}_0 = 0$ represents scalar dot product of the vectors of direction and position,

for $\vec{l}\cdot\vec{l}_0 = \left|\vec{l}\right|\cdot\left|\vec{l}_0\right|\cdot\cos(90^0) = 0$; $\vec{l}\cdot\vec{l}_0 = \left|\vec{l}\right|\cdot\left|\vec{l}_0\right|\cdot\cos(270^0) = 0$ then $\vec{l}\perp\vec{l}_0$.

3. SCREW MOTION FOR SOME BASIC KINEMATIC BLOCKS

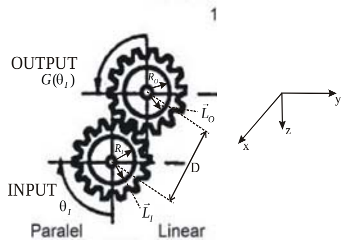
Analyzing screw motion for simple order kinematic couples and their synthesis, the high order kinematic couples can be realized.

Therefore, for a number of basic kinematic blocks are given: screw motion input and output, type of the block, spatial orientation and motion characteristic code (MCC) [1].

<u>Kinematic block – Cylindrical gears</u>

Input motion (0000):
$$\hat{S}_I = (\theta_I + \varepsilon\cdot 0)\left[\begin{Bmatrix}0\\0\\1\end{Bmatrix}+\varepsilon\begin{Bmatrix}0\\0\\0\end{Bmatrix}\right].$$

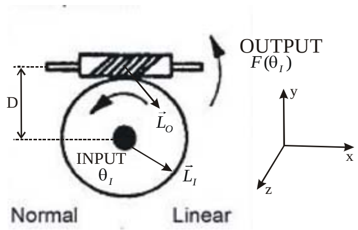
Output motion (0000):
$$\hat{S}_O = (G(\theta_I) + \varepsilon\cdot 0)\left[\begin{Bmatrix}0\\0\\-1\end{Bmatrix}+\varepsilon\begin{Bmatrix}0\\D\\0\end{Bmatrix}\right].$$



<u>Kinematic block – Worm-gear</u>

Input motion (0000):
$$\hat{S}_I = (\theta_I + \varepsilon\cdot 0)\left[\begin{Bmatrix}0\\0\\1\end{Bmatrix}+\varepsilon\begin{Bmatrix}0\\0\\0\end{Bmatrix}\right].$$

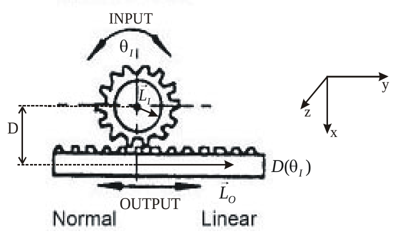
Output motion (0000):
$$\hat{S}_O = (F(\theta_I) + \varepsilon)\left[\begin{Bmatrix}1\\0\\0\end{Bmatrix}+\varepsilon\begin{Bmatrix}0\\0\\-D\end{Bmatrix}\right].$$



<u>Kinematic block – Rack and pinion</u>

Input motion (0000):
$$\hat{S}_I = (\theta_I + \varepsilon\cdot 0)\left[\begin{Bmatrix}0\\0\\1\end{Bmatrix}+\varepsilon\begin{Bmatrix}0\\0\\0\end{Bmatrix}\right].$$

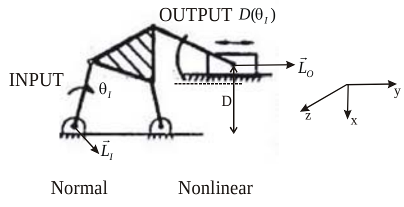
Output motion (1010):
$$\hat{S}_O = (0 + \varepsilon\cdot D(\theta_I))\left[\begin{Bmatrix}0\\1\\0\end{Bmatrix}+\varepsilon\begin{Bmatrix}0\\0\\D\end{Bmatrix}\right].$$



<u>Kinematic block– Six-bar Mechanism</u>

Input motion (0000):
$$\hat{S}_I = (\theta_I + \varepsilon\cdot 0)\left[\begin{Bmatrix}0\\0\\1\end{Bmatrix}+\varepsilon\begin{Bmatrix}0\\0\\0\end{Bmatrix}\right].$$

Output motion (1011):
$$\hat{S}_O = (0 + \varepsilon\cdot D(\theta_I))\left[\begin{Bmatrix}0\\1\\0\end{Bmatrix}+\varepsilon\begin{Bmatrix}0\\0\\D\end{Bmatrix}\right].$$



4. CONCLUSIONS

Based on analysis of the synthesis methodology of the mechanisms using dual-vector algebra, representation of input-output of the screw motion and motion characteristic code for four basic kinematic blocks, can be concluded that:

- Through dual-vector algebra the topological synthesis for mechanisms design can be used;
- Through Motion Characteristic Code (MCC) the functional synthesis at mechanisms design can be done;
- The determination of the screw motion at input and output using dual-vector for basic kinematic blocks presents a good base for synthesis of the high order mechanisms;
- The methodology of dual-vector algebra enables realization of a high number of alternate solutions on mechanisms design.

5. REFERENCES

[1] Buza K.: Kontribut sintezës së mekanizmave duke shfrytëzuar algjebrën e vektorit-dual, Disertacioni i doktoraturës (Doctorate thesis), Prishtina, 2006

[2] Buza K.: Disajnimi automatik i mekanizmave me ndihmen e programeve aplikative, Punim magjistrature (Master work), Prishtina, 2003

[3] Buza K., Pirraj B., Gojani I., Pajaziti A., Anxhaku A.: General Matrix Model at the Second Level of Automatic Synthesis Procedre of the Mechanisms, Proceedings of the 10th International Research/Expert Conference "Trends in the Development of Machinery and Associated Technology", TMT 2006, Barcelona –Lloret de Mar, Spain, 11-15 September, 2006, pp. 785-788.

[4] Moon Y.-M., Kota S.: Automated synthesis of mechanisms using dual-vector algebra, Mechanism and Machine Theory 37, pp.143-166, Ann Arbor, 2002

[5] Waldron K. J., Kinzel G. L.: Kinematics, Dynamics and Design of Machinery, Second Edition, John Wiley and Sons, USA, 2004.

[6] R.Featherstone, *Plücker Basis Vectors*, Department of Information Engineering, The Australian National University.

[7] Y.-M.Moon, *Reconfigurable Machine Tool Design: Theory and Application*, Ph.D. Dissertation, The University of Michigan, Ann Arbor, Michigan, 2000.