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FINITE ELEMENT ANALYSIS OF PLASTIC YIELDING IN AN ANISOTROPIC PLATE WITH A HOLE USING REFINED PLATE THEORY

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ABSTRACT
An elastic-plastic analysis of anisotropic laminated composite plate with a circular hole is carried out using a 2-dimensional finite element model based on higher order shear deformation theory. The variation in material properties through thickness is defined using discrete layer approach. The generalized Huber-Mises yield criterion for anisotropic material is used to determine the onset of plastic flow. The inclusion of anisotropic parameters of plasticity generalizes the plastic yield function. These anisotropic parameters of plasticity are updated during the hardening history of anisotropic laminated composite plate. The plastic potential, which is an anisotropic yield function, is used to determine an associated flow rule. Finally, an elastic-plastic incremental constitutive relation is obtained using finite element method. By means of an incremental and iterative procedure, the numerical solution of deformation problems in nonlinear anisotropic laminated plate is achieved using higher order shear deformation theory. The spread of plastic zones around the circular holes are discussed. The method yield accurate values of transverse shear stresses as compared to other shear deformation theories and hence the growth of plastic zones.
Keywords: Laminated plates, anisotropic plasticity, elastic-plastic analysis

1. INTRODUCTION
It is common to assume isotropic material behavior while studying the deformation of metallic solids due to applied external loads. Typically, this assumption is extended to both elastic and elastic-plastic responses. However, there are two categories of problems which involve anisotropic behavior. First is the introduction of anisotropy due to a deformation mechanism into an initially isotropic material, most commonly observed in metal forming operations. This phenomenon is usually termed as induced anisotropy. Second is the class of materials which are initially anisotropic or orthrotropic in nature usually termed as composites. In later category, there is a lack of well defined generalized plasticity theory to explain the nonlinear behavior due to material or geometric nonlinearities. Several investigators have developed theories to describe induced anisotropy while fewer attempts have been made to describe anisotropic plasticity in composites. Hill [1] proposed an anisotropic yield criterion based on generalization of Von-Mises criterion. Subsequently, there have been several modifications have been suggested by researchers. Yet, these theories do not properly account for all the effects associated with orientation dependent deformation mechanism.

Extension of these theories to describe inelastic behavior of laminated orthotropic composite plates is also limited. Whang [2] was the first one to consider the elastic-plastic analysis of orthotropic laminated plates based on classical lamination theory. Owen [3] had extended the analysis of plates and shells based on first-order shear deformation theory (FSDT). The analysis for laminated plates was further developed to account for higher-order shear deformation theory (HSDT) by Pervez [4]. Several research had been carried out for elastic-plastic analysis of a laminated composite plate with regular and irregular shaped hole [5-7] using only classical lamination theory and FSDT. The work in this paper is based on HSDT to study the orientation dependent deformation behavior in the vicinity and around a circular hole. The study will help the designer to avoid failure in the areas of geometric discontinuities while designing tailor made structural/machine components having holes or slots. In particular, the study was conducted to design components required in solid expandable tubular (SET) technology, which is an emerging field. SET has already started to have a significant impact in the way oil/gas wells will be drilled and repaired for failure during operational life.

2. THEORETICAL FORMULATION

Consider a plate of thickness 'h' composed of a finite number 'n' of the fiber reinforced orthotropic lamina with fibers oriented at angles Φ_1, Φ_2,...Φ_n. The present formulation is limited to infinitesimal deformations and rate-independent plasticity. The displacement fields based on higher-order shear deformation theory of plate are defined as

$$u(x,y,z) = u_o(x,y) + z\theta_x + z^3\phi_x \ , \quad v(x,y,z) = v_o(x,y) + z\theta_y + z^3\phi_y \ , \quad w(x,y,z) = w_o(x,y) \qquad (1)$$

where, u, v and w are the mid-plane displacements in x, y and z directions, and θ_x and θ_y are the rotations of transverse normals about x and y directions, respectively. The HSDT contains two additional terms involving, ϕ_x and ϕ_y which are the warping functions. These terms yield more accurate stress results as compared to FSDT [8]. Assuming a state of plane stress, the constitutive equations for the composite laminate can be obtained and is well established in the literature. The generalized yield criterion, with no Baushinger effect, can be written as $\bar{\sigma} - \sigma_Y = 0$, where the effective stress, $\bar{\sigma}$, is defined as

$$\bar{\sigma}^2 = a_1\sigma_{11}^2 + a_2\sigma_{22}^2 + a_4\sigma_{11}\sigma_{22} + a_7\sigma_{12}^2 + a_8\sigma_{13}^2 + a_9\sigma_{23}^2$$

(2)

where a's are the anisotropic parameters defining the current state of anisotropy. The initial parameters of anisotropy are determined by six independent yield tests [2]. These parameters are updated during the plastic deformation process as described in [2].

In present work, associated flow rule is assumed. Hence the plastic flow develops along the normal to the yield surface. The phenomenon whereby the yield stress increases with further plastic straining is known as strain hardening. In strain hardening theory, one must relate the hardening parameters to the experimental uniaxial stress/strain curve. A stress variable called effective stress is already defined in equation (2). The corresponding strain variable, called effective plastic strain, is defined using the concept of plastic work. Once the material is yielded, its behavior will be partly elastic and partly plastic. During any incremental change of loading, the change in strain is assumed to be decomposed additively into elastic and plastic components. Using the consistency condition, which requires the state of stress to remain on the yield surface during plastic flow, the incremental stress/strain relationship is obtained. Finally, the incremental stresses and strains of k-th orthotropic lamina, is obtained in terms of stress resultants as

$$\begin{Bmatrix} dN \\ dM \\ dP \\ dV \\ dZ \end{Bmatrix} = \begin{bmatrix} [A] & [B] & [E] & [\varphi] & [\varphi] \\ [B] & [D] & [F] & [\varphi] & [\varphi] \\ [E] & [F] & [G] & [\varphi] & [\varphi] \\ [\varphi] & [\varphi] & [\varphi] & [S_1] & [S_2] \\ [\varphi] & [\varphi] & [\varphi] & [S_2] & [S_3] \end{bmatrix} \begin{Bmatrix} d\varepsilon^p \\ d\varepsilon^f \\ \alpha\, d\varepsilon^w \\ d\varepsilon^s \\ \alpha\, d\varepsilon^{sw} \end{Bmatrix} \qquad (3)$$

The matrices [A], [B], [D], [E], [F] and [G], and [S_1], [S_2] and [S_3] are obtained using the planar and transverse shear stress constitutive equations, $[Q^{ep}]_{(k)}$ and $[R^{ep}]_{(k)}$, for each lamina, as given below.

$$\{[A] \quad [B] \quad [D] \quad [E] \quad [F] \quad [G]\} = \sum_k \int_{h_{k-1}}^{h_k} [Q^{ep}]_{(k)} \left(1 \quad z \quad z^2 \quad z^3 \quad z^4 \quad z^6\right) dz \qquad (4a)$$

$$\{[S_1]\ \ [S_2]\ \ [S_3]\} = \sum_k \int_{h_{k-1}}^{h_k} [R^{ep}]_{(k)} \left(1 \quad z^2 \quad z^4\right) dz$$ Superscript ‘ep’ refers to elastic-plastic properties (4b)

The variables *dN*, *dM*, *dP*, *dV* and *dZ*, and $d\varepsilon^p$, $d\varepsilon^f$, $d\varepsilon^w$, $d\varepsilon^s$, and $d\varepsilon^{sw}$ represent in-plane, flexural, warp, shear and shear-warp incremental stress resultants and incremental strains, respectively.

The PPLATE program has been developed as a finite element program for an elastic-plastic analysis of laminated composite plates. The program is capable of handling three finite elements; 8-node isoparametric, Heteriosis and 9-node Langrangian elements. An iterative and incremental solution of equation (3) is obtained using PPLATE program. Convergence criterion is used to terminate the iterative cycle when the solution is considered sufficiently accurate. In case of orthotropic laminated composite plates based on HSDT, a force convergence criterion based on the unbalanced nodal forces does not lead to a convergent solution. Convergence criteria both in terms of out of balance nodal forces and in terms of incremental nodal displacements are used for convergence. The results discussed in next section are obtained using 9-node Langrangian element with selective reduced integration.

3. RESULTS AND DISCUSSION

In this example, the elastic-plastic model presented in previous sections is used to examine the behavior of a $(0°/45°)_s$ laminated plate with a circular hole in the middle of the plate. In-plane tensile load in x-direction is applied. Material properties are: E_1=E_2=3.0E04 MPa, G_{12}=G_{13}=G_{23}=1.154E04 MPa, υ_{12}=0.3, E_p=300 MPa and G_p=E_p/3. Initial anisotropic parameters of plasticity are: σ^0_{11}=30 MPa, σ^0_{13}= σ^0_{23}=17.3 MPa, σ^0_{22}=40 MPa, and σ^0_{12}=20.2 MPa.

The plate is 1m x 1m and contains a circular hole of 0.25m diameter. The size of the elements are small around the holes, where stress concentrations are anticipated, and gets larger towards the edge of the plate as shown in Figure 1(a). The plate is assumed to be free from initial stresses. The response of the plate is observed during the course of loading in terms of the development of plastic zones. The loading is applied uniformly to the edge of the plate and is increased gradually.

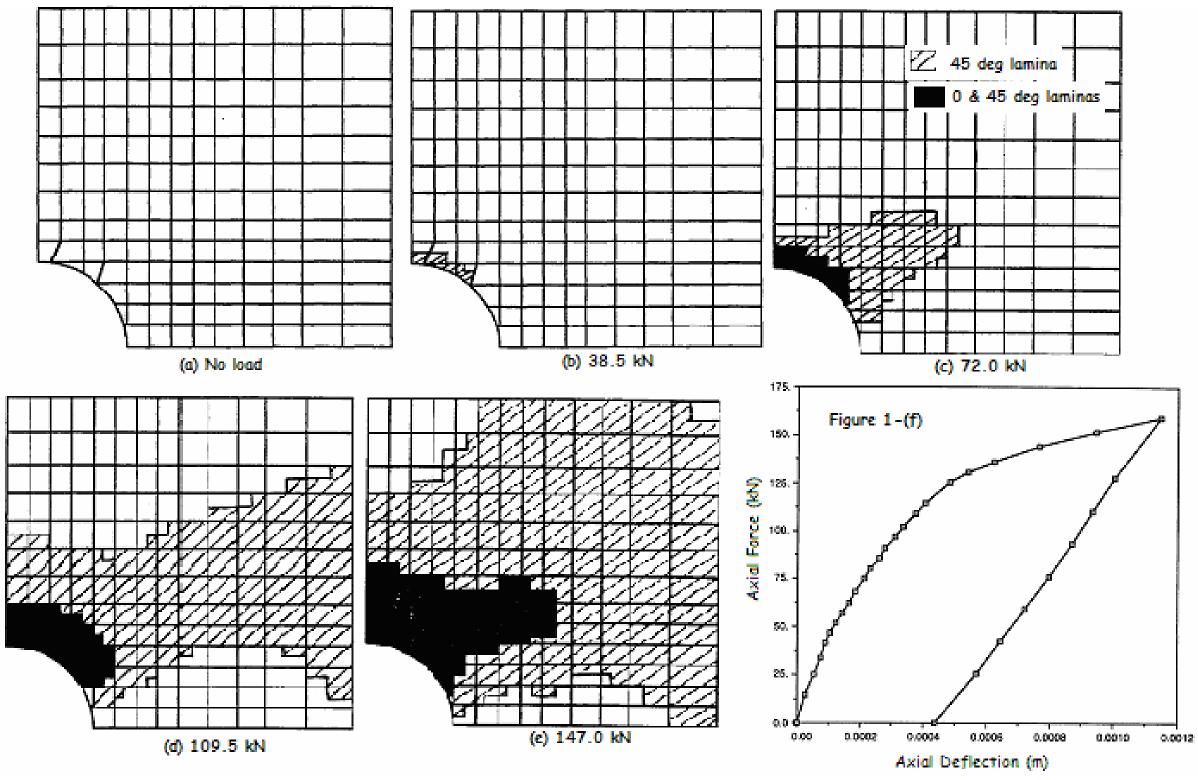

Figure 1: Spread of plastic zones in an angle ply laminated square plate at varying loads

First yielding is observed relatively early in the loading process at about 38.5 KN. This takes place in the 45° lamina at the edge of the circular hole as shown in Figure 1(b). Figures 1(c) to 1(e) show the growth of plastic zones as loading continues. The plastic zones in the 45° lamina spread rapidly. It can be observed that the 45° lamina has yielded almost completely at about 150 KN. Yielding in 0˚ lamina on the other hand, is somewhat delayed and the growth of plastic zones remain comparatively small and contained to the immediate vicinity of the hole. For example, at 137 KN only 34 percent of the 0˚ lamina has yielded in comparison to about 82 percent yielding in 45° lamina at the same level of loading. It has been observed that the yielding starts at lower load levels similar to the isotropic laminated plates. Therefore, the fiber reinforcement strengthens the matrix material substantially and is the principal source of composite stiffness, but they have a relatively small effect on the effective stress level which causes the onset of plastic flow.

The complete removal of load results in permanent deformation of plate as shown in figure 1(e). The laminated anisotropic plate is first unloaded by a small unloading step followed by 8 unloading increments. After the first unloading step, all laminas became elastic and remain elastic till the end of unloading. At the end of loading/unloading cycle, the plate is left with 30 percent of the total deformation as the permanent deformation. However, the magnitude of the permanent strain in 45° lamina is larger than the 0° lamina. This causes the induced residual stresses in each lamina.

4. CONCLUSIONS

This work is aimed for the finite element elastic-plastic analysis of an orthotropic laminated composite plates with a circular hole. A higher-order shear deformation theory is employed herein to obtain the elastic-plastic response of laminated fibrous composite plates. The HSDT contains two more independent variables (warp on xz-plane and yz-plane) as compared to previous analysis conducted by other researchers. It results in accurate prediction of deflections and stresses. It has been found that the different types of shear deformation theories have little effect on deflections, however, higher order shear deformation theory gives much accurate results for stresses than first order shear deformation theory. Spread of plastic zones has been predicted at varying load. Plastic zones have a tendency to spread along the weaker layer in laminated plates. It also demonstrates the significance of plasticity in fibrous composites for laminated plates. Elastic behavior is of limited significance in applications which hope to utilize the high strength of composite. Misleading results and predictions of stiffness and strength of laminated plates may occur if plastic deformation of the composite is disregarded.

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