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COMPUTER AIDED DESIGN OF HELICAL GEARS WITH ASYMMETRIC INVOLUTE TEETH

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ABSTRACT
This paper studies the equations of rack cutters for generating involute gears with asymmetric teeth. The asymmetry means that different pressure angles are applied for driving and coast sides, respectively. By applying the equations of designed profile of rack cutter, the principle of coordinate transformation, the theory of differential geometry, and the theory of gearing, the mathematical models of involute helical gear is given. Trochoidal envelope traced by cutter is investigated. Computer graphs of asymmetric involute helical gears are presented based on the given model, and also generation simulation is illustrated.
Keywords: Gear design, Asymmetric teeth, Generation simulation

1. INTRODUCTION
Involute gears are the most popular power transmission devices for parallel axes owing to their simple geometry, easy manufacturing, and constant gear ratio even when the centre distance has been changed. An accurate geometrical representation of gear tooth surfaces is the fundamental starting point for developing a reliable computerized gear design which includes tooth contact analysis under light and heavy loads and stress analysis [1]. In practice, cylindrical gears are produced by generation cutting using rack and pinion type shaping cutters and hobs. Considering the kinematic motions in the generation process and the cutting tool profile, it is possible to obtain the gear profile as the envelope to the locus of the tool surfaces [2]. Different methods are reported in the literature [3-5] based on the above approach. Yang proposed a mathematical model of the helical gear with asymmetric involute teeth [6]. Different standard pressure angles are used in the front and back profiles of an asymmetric tooth. This gear type can reduce the size and the weight of gear and increase its load capacity.

In this study, the design of rack-type generating tools (rack cutter or hub) with asymmetric teeth is considered. By applying the equations of designed profile of rack cutter, the principle of coordinate transformation, the theory of differential geometry, and the theory of gearing, the mathematical models of involute helical gear is given. In addition to the mathematical model proposed by Yang [6], the trochoidal envelope of the cutter tip which generates the gear tooth root fillet during the cutting process is also investigated. Furthermore, a computer simulation program is developed to generate the tooth profile of asymmetric involute gears and to illustrate the effect of tool geometry on generated surfaces. The motion path of cutter during generating process is also illustrated.

2. RACK CUTTER SURFACES
Due to the asymmetry of the rack cutter, left and right sides of the cutter are considered separately. Figure 1 presents the design of the normal section of a rack cutter Σn, where regions $\overline{ac}$ and $\overline{bd}$ are the left- and right-side top lands, regions $\overline{ce}$ and $\overline{df}$ are the left- and right-side fillets and, regions

$\overline{eg}$ and $\overline{fh}$ are the left- and right-side working regions. In simulating the rack cutter surface for the helical gear generation, the normal section of the rack cutter Σn, attached to the coordinate system S_n with its origin O_n, is translated along the line $\rho = \overline{O_n O_c}$ as shown in Figure 1. Therefore, $\rho = \left|\overline{O_n O_c}\right|$ is one of the design parameters of the rack cutter surface, and β is the helix angle of the generated helical gear.

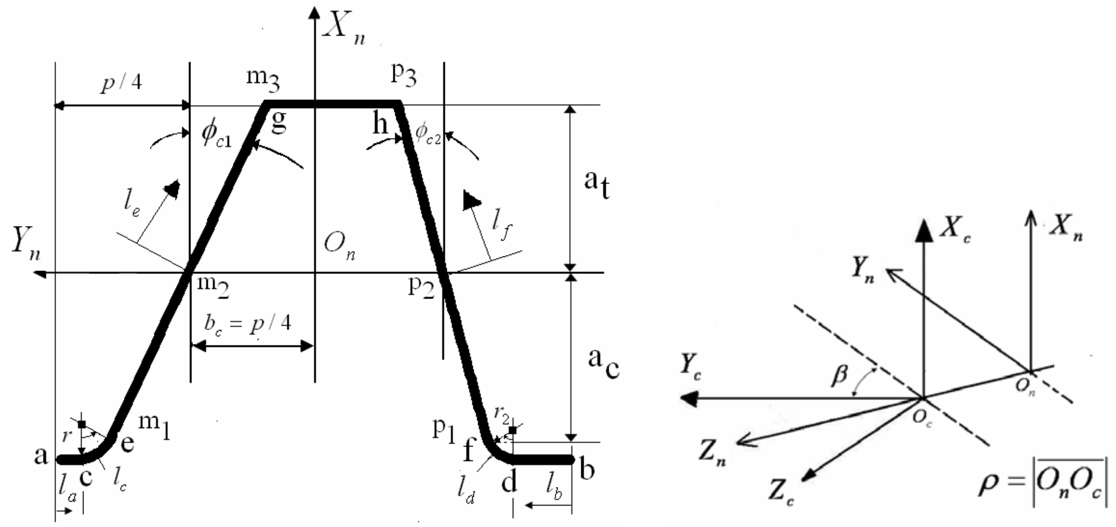

Figure 1. Diagram of rack cutter surface

As shown in Figure 1, two straight edges $\overline{eg}$ and $\overline{fh}$ of the rack cutter is used to generate the left- and right-side tooth surface of the asymmetric helical gear, respectively. The symbol m_n represents the normal module. The position vector of regions $\overline{eg}$ and $\overline{fh}$ are represented in the coordinate system S_c as follows:

$$\mathbf{R}_c^{eg} = \begin{bmatrix} x_c^{eg} \\ y_c^{eg} \\ z_c^{eg} \end{bmatrix} = \begin{bmatrix} l_e \cos\phi_{c1} \\ (\frac{\pi m_n}{4} - l_e \cos\phi_{c1} + c_y \pi m_n)\cos\beta - \rho\sin\beta \\ \frac{\pi m_n}{4} - l_e \cos\phi_{c1} + c_y \pi m_n)\sin\beta + \rho\cos\beta \end{bmatrix} \tag{1}$$

and

$$\mathbf{R}_c^{fh} = \begin{bmatrix} x_c^{fh} \\ y_c^{fh} \\ z_c^{fh} \end{bmatrix} = \begin{bmatrix} l_f \cos\phi_{c1} \\ (-\frac{\pi m_n}{4} + l_f \cos\phi_{c1} + c_y \pi m_n)\cos\beta - \rho\sin\beta \\ (-\frac{\pi m_n}{4} + l_f \cos\phi_{c1} + c_y \pi m_n)\sin\beta + \rho\cos\beta \end{bmatrix} \tag{2}$$

where l_e and l_f are the design parameters of the rack cutter surface which determine the location of points on the working surface. l_e and l_f are limited by $-a_c/\cos\phi_{c1} \le l_h \le a_t/\cos\phi_{c1}$ and $-a_c/\cos\phi_{c2} \le l_h \le a_t/\cos\phi_{c2}$ for the left- and right-side of the rack cutter respectively.

Based on the differential geometry, the unit normal vectors of regions $\overline{ac} \sim \overline{fh}$ of the rack cutter surface represented in coordinate system S_c can be obtained by

$$\mathbf{n}_c^i = \frac{\dfrac{\partial \mathbf{R}_c^i}{\partial l_j} \times \dfrac{\partial \mathbf{R}_c^i}{\partial \rho}}{\left| \dfrac{\partial \mathbf{R}_c^i}{\partial l_j} \times \dfrac{\partial \mathbf{R}_c^i}{\partial \rho} \right|} \quad \begin{matrix} (i = \overline{ac}, ..., \overline{fh}) \\ (j = a, ..., f) \end{matrix} \tag{3}$$

3. GENERATED TOOTH SURFACES

To derive the mathematical model for the complete tooth profile of involute helical gears with asymmetric teeth, coordinate systems $S_c(X_c, Y_c, Z_c)$, $S_1(X_1, Y_1, Z_1)$ and $S_h(X_h, Y_h, Z_h)$ should be set up and they are attached to the rack cutter, involute gear, and gear housing, respectively as shown in Figure 2.

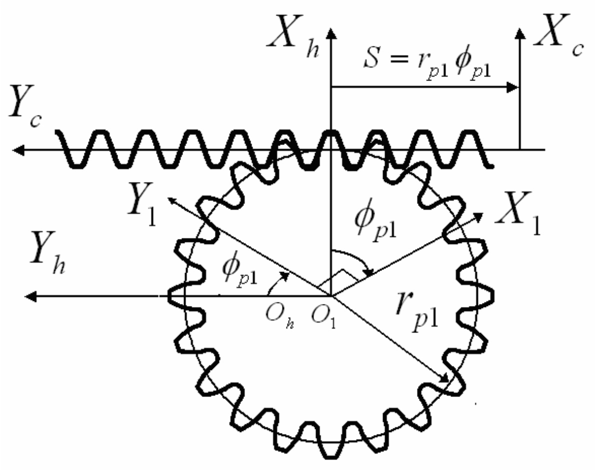

Figure 2. The Kinematic relationship between the rack cutter and generated gear

The mathematical model of the generated gear tooth surface is a combination of the meshing equation and the locus of the rack cutter surfaces according to gearing theory [2]. Thus, the mathematical model of the generated gear tooth surface is

$$\mathbf{R}_1^i = [M_{1c}]\mathbf{R}_c^i \tag{4}$$

and

$$\frac{X_c^i - x_c^i}{n_{cx}^i} = \frac{Y_c^i - y_c^i}{n_{cy}^i} = \frac{Z_c^i - z_c^i}{n_{cz}^i}, \tag{5}$$

where

$$[M_{1c}] = \begin{bmatrix} \cos\phi_{p1} & -\sin\phi_1 & r_{p1}(\cos\phi_{p1} + \phi_{p1}\sin\phi_{p1}) + e\cos\phi_{p1} \\ \sin\phi_{p1} & \cos\phi_{p1} & r_{p1}(\sin\phi_{p1} - \phi_{p1}\cos\phi_{p1}) + e\sin\phi_{p1} \\ 0 & 0 & 1 \end{bmatrix}.$$

4. COMPUTER IMPLEMENTATION

This study also develops a computer program based on the gear mathematical model, making it possible to display the computer graphs. Using one of the high-level programming languages and graphing software, the geometric models of the driving gear can be evaluated and plotted. Figure 3 displays the trochoidal envelope of the rack cutter with asymmetric teeth and the generated tooth profile. The center of the rounded corner at the tip traces out a trochoid. The rounded tip envelopes another curve called as secondary trochoid, defining the root fillet [7-8].

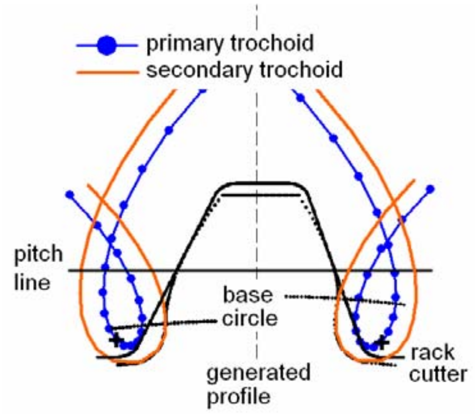

Figure 3. The trochoidal envelope of the rack cutter tip

Completed generation simulation of helical gear with asymmetric involute teeth showing the addendum circle of the blank is depicted in Figure 4. Animated files can also be obtained using appropriate software.

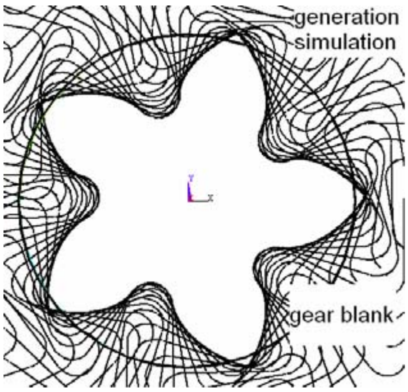

Figure 4. Generation simulation of gear with asymmetric involute teeth

5. CONCLUSION

In this study, the design of rack-type generating tools (rack cutter or hub) with asymmetric teeth is considered. The developed program based on the mathematical model of driving gear is used to obtain computer graphs of generating tool and generated surfaces. The illustrations show the efficiency of the given mathematical model and developed computer program for helical gear with asymmetric involute teeth. Variations on the tooth form and effects of changing tool parameters on the produced tooth form can be investigated before it is manufactured.

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