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ON THE INCIDENCE ANGLE OPTIMIZATION OF THE DUAL-AXIS SOLAR TRACKERS

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ABSTRACT
This paper presents the modelling of the sun-ray unit vector and of a solar panel normal unit vector in the reference system of an observer from Earth. Using the resulted formulas set there is modelled and numerical simulated the sun ray-solar panel incidence angle variation. The comparative analysis of the results gives useful conclusions regarding the incidence angle minimization.
Keywords: incidence angle, sun-ray unit vector, solar panel, normal unit vector.

1. INTRODUCTION
A solar tracker main function is the sun ray-solar panel incidence angle minimization achieved by minimizing the angle between the sun-ray unit vector and the solar panel normal unit vector. The paper presents the incidence angle modelling for some representative cases: equatorial dual-axis tracker, azimuthal dual-axis tracker, horizontally fixed solar panel (its normal unit vector is coincident with the zenital axis) and slopped solar panel (between its normal unit vector and the zenital axis there is the angle φ-δ, see Fig. 2). Using the resulted formulas set, there are made representative numerical simulation; the comparative analysis of the results accentuates the solar tracker key role in the incidence angle optimization.

2. THE SUN-RAY UNIT VECTOR
Fig.1 illustrates the specific angles of the Sun-Earth geometry and the considered ± angle signs. Using Fig.1 there are obtained the following correlations between these angles (their names are mentioned in the figure title) [1], [2]:

$$\delta = 23.45^o \sin\frac{360^o(n-80)}{365}; \ \omega = 15^o(12-T); \qquad (1), (2)$$

$$\omega_{sr,ss} = \pm\cos^{-1}(-\tan\varphi\tan\delta); \ \alpha = \sin^{-1}(\sin\delta\sin\varphi + \cos\delta\cos\varphi\cos\omega); \qquad (3), (4)$$

$$\psi = (\mathrm{sgn}\,\omega)\cos^{-1}\frac{\cos\delta\cos\omega\sin\varphi - \cos\varphi\sin\delta}{\cos\alpha}; \ \bar{e}_{sun-ray} = \begin{bmatrix} \cos\alpha\cdot\sin\psi \\ -\cos\alpha\cdot\cos\psi \\ \sin\alpha \end{bmatrix}; \qquad (5), (6)$$

In the correlation (1), n is the day number of the year (n = 1 is 1st of January), in the correlation (2) T is the solar time and in the correlation (3) ω_{sr} and ω_{ss} represent the sun rise hour angle and the sun set hour angle. The matrix (6) models the sun ray unit vector in a $X_0Y_0Z_0$ trihedron (X_0=East, Y_0=North and Z_0= Z zenital axis), whose origin is the observer Q (see Fig.1).

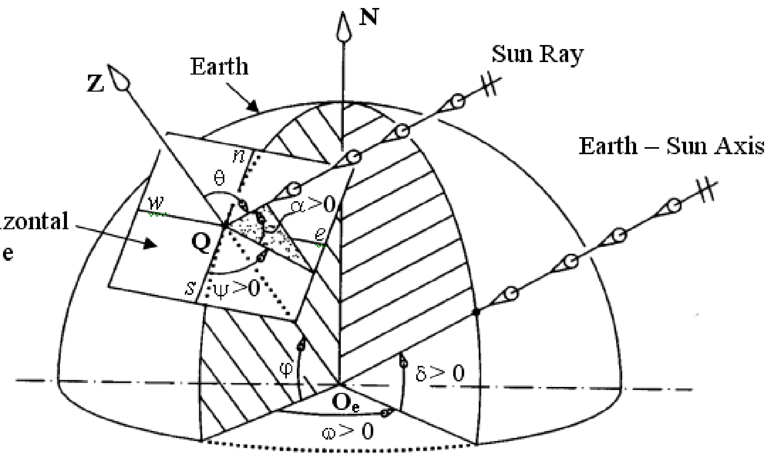
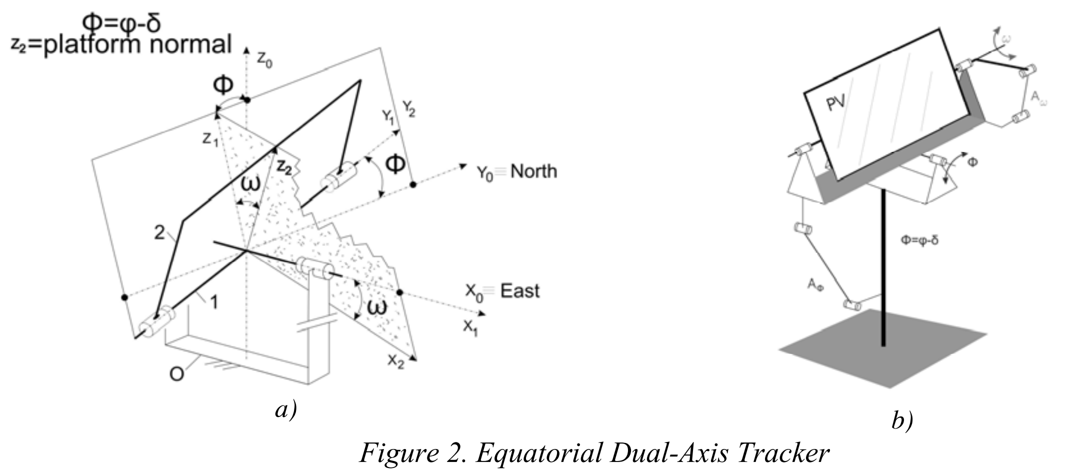


Figure 1. Earth- Sun angles: declination δ, hour angle ω, altitude α, azimuth ψ, zenith θ, latitude φ.

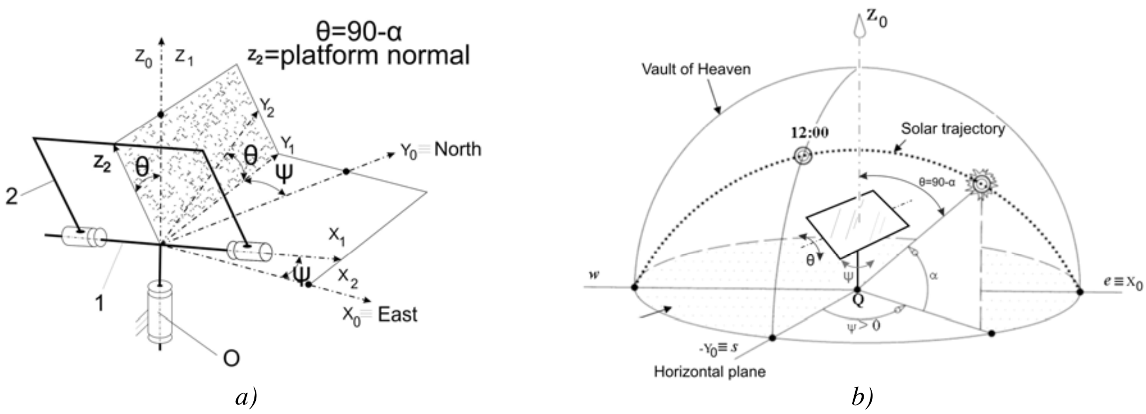
3. THE NORMAL UNIT VECTOR OF THE SOLAR PANEL

Fig. 2 and Fig. 3 illustrate the calculus diagram and the basic diagram of an equatorial solar tracker (where A_Φ and A_ω are the linear actuators for the angular displacements Φ and ω, see Fig. 2). According to Fig. 2 and Fig. 3, in the $X_0Y_0Z_0$ trihedron, result the following solar panel normal unit vector correlations: (7) for the equatorial dual-axis solar tracker unit vector (Fig.2), (8) for the azimuthal dual-axis solar tracker unit vector (Fig. 3), (9) for the horizontally fixed solar panel unit vector and (10) for solar panel slopped at Φ = φ-δ (the angle between the solar panel normal unit vector and the zenital axis).



a) *b)*

Figure 2. Equatorial Dual-Axis Tracker



a) *b)*

Figure 3. Azimuthal Dual-Axis Tracker

$$\bar{e}_{PV-eq} = [k_2]_{x_0y_0z_0} = \begin{bmatrix} \sin\omega \\ -\cos\omega \cdot \sin(\varphi-\delta) \\ \cos\omega \cdot \cos(\varphi-\delta) \end{bmatrix};\ \bar{e}_{PV-az} = [k_2]_{x_0y_0z_0} = \begin{bmatrix} \cos\alpha \cdot \sin\psi \\ -\cos\alpha \cdot \cos\psi \\ \sin\alpha \end{bmatrix} \qquad (7),(8)$$

$$\bar{e}_{PV-fix-horiz} = [k_2]_{x_0y_0z_0} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}_{x_0y_0z_0};\ \bar{e}_{PV-fix-\delta} = [k_2]_{x_0y_0z_0} = \begin{bmatrix} 0 \\ -\sin(\varphi-\delta) \\ \cos(\varphi-\delta) \end{bmatrix}_{x_0y_0z_0} \qquad (9),(10)$$

4. THE INCIDENCE ANGLE MODELLING AND ITS SIMULATIONS

The angle of incidence correlations result from the dot product of the sun-ray unit vector (6) and:
a) the solar panel normal unit vector for the equatorial solar tracker (7), b) the solar panel normal unit vector for the azimuthal solar tracker (8), c) the horizontally fixed solar panel normal unit vector (9), d) the slopped solar panel normal unit vector (10):

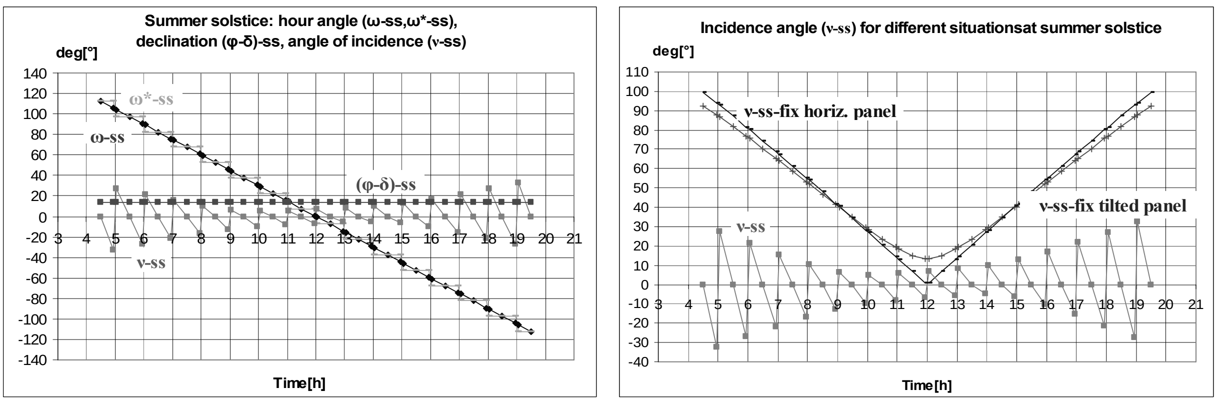
$$\begin{aligned}\cos\nu &= \bar{e}_{sun-ray} \cdot \bar{e}_{PV-eq} = \\ &= \cos\alpha \cdot \sin\psi \cdot \sin\omega^* + \cos\alpha \cdot \cos\psi \cdot \cos\omega^* \cdot \sin(\varphi-\delta) + \sin\alpha \cdot \cos\omega^* \cdot \cos(\varphi-\delta)\end{aligned} \qquad (11)$$

$$\begin{aligned}\cos\nu &= \bar{e}_{sun-ray} \cdot \bar{e}_{PV-az} = \\ &= \cos\alpha \cdot \sin\psi \cdot \cos\alpha^* \cdot \sin\psi^* + \cos\alpha \cdot \cos\psi \cdot \cos\alpha^* \cdot \cos\psi^* + \sin\alpha \cdot \sin\alpha^*\end{aligned} \qquad (12)$$

$$\cos\nu = \bar{e}_{sun-ray} \cdot \bar{e}_{PV-fix-horiz} = \sin\alpha = \cos\theta \qquad (13)$$

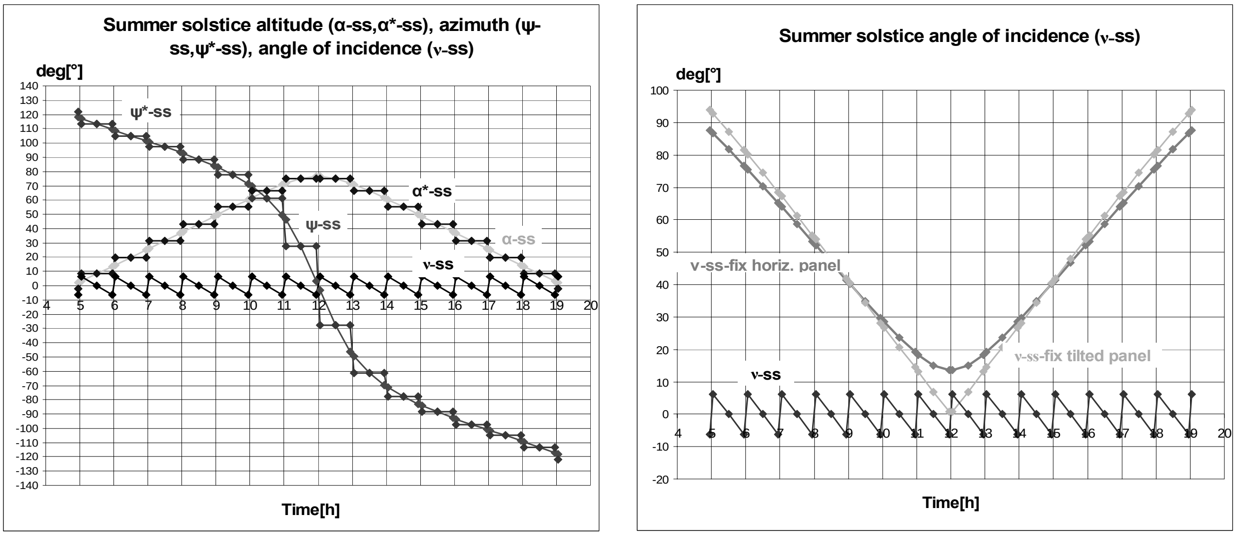
$$\cos\nu = \bar{e}_{sun-ray} \cdot \bar{e}_{PV-fix-\delta} = \cos\alpha \cdot \cos\psi \cdot \sin(\varphi-\delta) + \sin\alpha \cdot \cos(\varphi-\delta) \qquad (14)$$

By economic reasons, the mobile panels displacement is made discontinuously (in steps); therefore, in the correlations (11) and (12) the solar panel variables have discreet variation and are marked with asterisk: ω^*, ψ^* and α^* (in order to distinguish them from the sun-ray variables, which have continuous variation: ω, ψ and α).
Using the previous correlations, there were made numerical simulation, considering as input data: $\varphi = 37°$ N, day n =172 Summer Solstice (June 21st) ⇔ $\delta \approx +23{,}45°$, an orientation cycle of the solar panel consists in: aprox. 1 min. displacement + aprox. 59 min. standing. Some of the simulations' results, made with Excel Software, are illustrated in Fig. 4, 5 and 6.



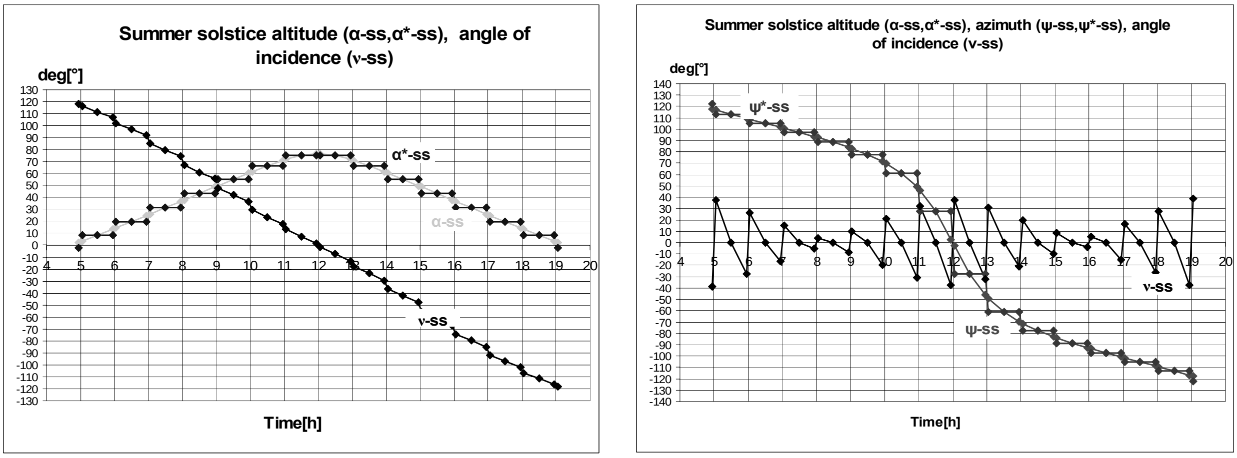
a) *b)*

Figure 4. Summer solstice variations of ω, ω, Φ=Φ*and ν for an equatorial dual-axis tracker (a) and variation of the incidence angle ν for an equatorial dual-axis tracker vs. a horizontally fixed solar panel and a slopped solar panel(b).*



a) b)

Figure 5. Variations of ψ, ψ, α, α*,ν for an azimuthal dual-axis tracker (a) and ν variation for an azimuthal dual-axis tracker vs. a horizontally fixed solar panel and a slopped solar panel(b).*



a) b)

*Figure 6. Summer solstice for α, α*and ν for an azimuthal single-axis tracker with ψ*=0 (a) and variation of ψ, ψ* and for an azimuthal single-axis solar tracker with α*=* constant ≈ 40° *(b)*

5. CONCLUSIONS

a) In the considered numerical situation, the fixed solar panels have incidence angles similar variations (see Fig.4,b and 5,b) and comparable with the single-axis azimuthal solar tracker variation with $\psi^*=0$ (see Fig.6,a).

b) The minimal incidence angles are achieved with the dual-axis azimuthal solar tracker (see Fig. 5); but this tracker type is less economic.

c) During a day, the dual-axis equatorial tracker works practically in single-axis operation mode and achieves more reduced incidence angles around noon (see Fig.4); a significant improvement can be achieved if the time step is smaller in the morning and in the evening.

d) Azimuthal solar tracker with $\alpha^*\approx 40°$ = constant (see Fig.6,b) works in single axis operation mode and achieves incidence angles similar to the equatorial tracker incidence angles (except from noon); as a conclusion, the azimuthal displacement has a much higher influence compared to the altitudinal displacement.

6. REFERENCES

[1] Messenger, R., Ventre, J.: Photovoltaic System Engineering, London, CRC Press, 2000.

[2] Diaconescu, D. a.o.: Analysis of the Sun-Earth angles used in the design of the solar collectors' trackers, Bulletin of the *Transilvania* University of Brasov, Vol.13(47), 2006, pg. 99-105.