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ADAPTIVE CONTROL OF INTERCONNECTED LIQUID TANKS

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ABSTRACT
Control of an interconnected liquid tanks system as a two inputs – two outputs system is presented. A control algorithm based on polynomial theory and pole – placement is proposed. The algorithm in adaptive version is then used for control of the model. The results of the real-time experiments are also given.
Keywords: multivariable control, control algorithms, adaptive control, polynomial methods, pole assignment

1. INTRODUCTION
Many technological processes require that several variables relating to one system are controlled simultaneously. Each input may influence all system outputs. The interconnected liquid tank system is a typical multivariable nonlinear system with interactions between control loops. The design of a controller able to cope with such a system must be quite sophisticated. There are many different methods of controlling multivariable systems. In this paper polynomial theory approach [1] is used to control a multivariable system. A controller based on the configuration given in [2] is presented.

2. DESCRIPTION OF THE THREE – TANK - SYSTEM
The experiments were carried out with an experimental laboratory model three – tank – system. Such a system can be viewed as a prototype of many industrial applications in process industry, such as chemical and petrochemical plants, oil and gas systems. The typical control issue involved in the system is how to keep the desired liquid level in each tank. The principle scheme of the model is shown in Fig 1. The basic apparatus consists of three plexiglass tanks numbered from left to right as T1, T3 and T2. These are connected serially with each other by cylindrical pipes. Liquid, which is collected in a reservoir, is pumped into the first and the third tanks to maintain their levels. The level in the tank T3 is a response which is uncontrollable. It affects the level in the two end tanks. Each tank is equipped with a static pressure sensor, which gives a voltage output proportional to the level of liquid in the tank. Hmax denotes the highest possible liquid level. In case the liquid level of T1 and T2 exceeds this value the corresponding pump will be switched off automatically. Q_1 and Q_2 are the flow rates of the pumps 1 and 2. Two variable speed pumps driven by DC motor are used in this apparatus. These pumps are designed to give an accurate well defined flow per rotation. Thus, the flow rate provided by each pump is proportional to the voltage applied to its DC motor. There are six manual valves v1, v2...v6 that can be used to vary the configuration of the process or to introduce disturbances or faults. The pump flow rates Q_1 and Q_2 denote the input signals, the liquid levels of T1 and T2 are the output signals.

3. MATHEMATICAL MODEL OF THE SYSTEM
The examined system is a typical example of a two inputs – two outputs system with internal interactions between the control loops. The transfer matrix of the system is expressed as

$$\boldsymbol{Y}(z) = \boldsymbol{G}(z)\boldsymbol{U}(z) = \begin{bmatrix} G_{11}(z) & G_{12}(z) \\ G_{21}(z) & G_{22}(z) \end{bmatrix} \boldsymbol{U}(z) \qquad (1)$$

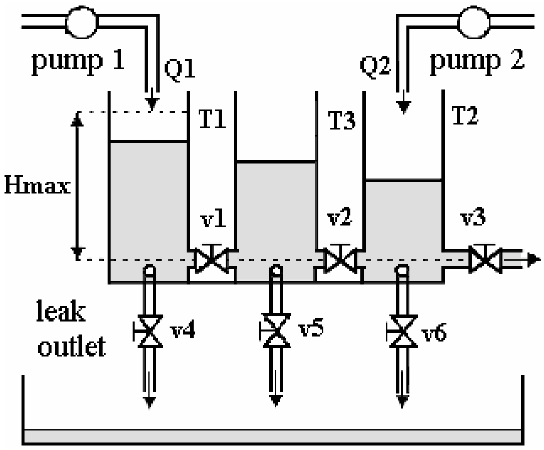

Figure 1. Principal scheme of three – tank – system

Where $\boldsymbol{U}(z)$ and $\boldsymbol{Y}(z)$ are vectors of the manipulated variables (inputs to the servo motors) and the controlled variables (tension and speed at the work station).

$$\boldsymbol{U}(z) = [u_1(z), u_2(z)]^T \qquad \boldsymbol{Y}(z) = [y_1(z), y_2(z)]^T \tag{2}$$

It is possible to assume that the dynamic behaviour of the system can be described in the neighbourhood of a steady state by a discrete linear model in the following form of the matrix fraction

$$\boldsymbol{G}(z) = \boldsymbol{A}^{-1}(z^{-1})\boldsymbol{B}(z^{-1}) = \boldsymbol{B}_1(z^{-1})\boldsymbol{A}_1^{-1}(z^{-1}) \tag{3}$$

Where polynomial matrices $\boldsymbol{A} \in R_{22}\lfloor z^{-1} \rfloor, \boldsymbol{B} \in R_{22}\lfloor z^{-1} \rfloor$ are the left indivisible decomposition of matrix $\boldsymbol{G}(z)$ and matrices $\boldsymbol{A}_1 \in R_{22}\lfloor z^{-1} \rfloor, \boldsymbol{B}_1 \in R_{22}\lfloor z^{-1} \rfloor$ are the right indivisible decomposition of $\boldsymbol{G}(z)$.

At first, the algorithm described bellow was designed for a model with polynomials of the first order. This model proved to be unsuitable for the coupled drives process and the control algorithm failed. Consequently, the polynomial orders were increased and the algorithm was designed for a model with second order polynomials. This model proved to be effective. The model has sixteen parameters:

$$\boldsymbol{A}(z^{-1}) = \begin{bmatrix} 1 + a_1 z^{-1} + a_2 z^{-2} & a_3 z^{-1} + a_4 z^{-2} \\ a_5 z^{-1} + a_6 z^{-2} & 1 + a_7 z^{-1} + a_8 z^{-2} \end{bmatrix} \quad \boldsymbol{B}(z^{-1}) = \begin{bmatrix} b_1 z^{-1} + b_2 z^{-2} & b_3 z^{-1} + b_4 z^{-2} \\ b_5 z^{-1} + b_6 z^{-2} & b_7 z^{-1} + b_8 z^{-2} \end{bmatrix} \tag{4}$$

4. DESIGN OF 2DOF CONTROLLER

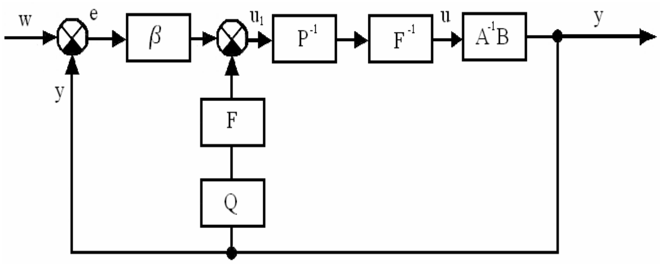

Figure 2. Block diagram of the closed loop system

The configuration of the closed loop, which is shown in Fig. 2, was presented in [2]. Generally, the vector of input reference signals $\boldsymbol{W}$ is given by

$$\boldsymbol{W}(z^{-1}) = \boldsymbol{F}_w^{-1}(z^{-1})\boldsymbol{h}(z^{-1}) \tag{5}$$

Here, the reference signals are considered from a class of step functions. In this case $\boldsymbol{h}\left(z^{-1}\right)$ is a vector of constants and $\boldsymbol{F}_w\left(z^{-1}\right)$ takes the form

$$\boldsymbol{F}_w\left(z^{-1}\right)=\begin{bmatrix}1-z^{-1} & 0\\ 0 & 1-z^{-1}\end{bmatrix} \tag{6}$$

The compensator $\boldsymbol{F}\left(z^{-1}\right)$ is a component formally separated from the controller. It has to be inherent in the controller to fulfil the requirement on the asymptotic tracking. If the reference signals are from the class of step functions, $\boldsymbol{F}\left(z^{-1}\right)$ is an integrator.
It is possible to derive the following equation for the system output (operator z^{-1} will be omitted from some operations for the sake of simplification)

$$\boldsymbol{Y} = \boldsymbol{A}^{-1}\boldsymbol{B}\boldsymbol{U} = \boldsymbol{A}^{-1}\boldsymbol{B}\boldsymbol{F}^{-1}\boldsymbol{P}^{-1}\boldsymbol{U}_1 \tag{7}$$

Where

$$\boldsymbol{U}_1 = \boldsymbol{\beta}\left(\boldsymbol{W}-\boldsymbol{Y}\right)-\boldsymbol{QFY} \tag{8}$$

The equation for the controller output, as shown in the block diagram in Fig 2, takes the form

$$\boldsymbol{U} = \boldsymbol{F}^{-1}\boldsymbol{P}^{-1}\boldsymbol{U}_1 \tag{9}$$

Substitution of $\boldsymbol{U}_1$ and $\boldsymbol{Y}$ results in

$$\boldsymbol{U} = \boldsymbol{F}^{-1}\boldsymbol{P}^{-1}\left[\boldsymbol{\beta}\left(\boldsymbol{W}-\boldsymbol{A}^{-1}\boldsymbol{BU}\right)-\boldsymbol{QFA}^{-1}\boldsymbol{BU}\right] \tag{10}$$

The equation (10) can be modified using the right matrix fraction of the controlled system to the form

$$\boldsymbol{U} = \boldsymbol{A}_1\left[\boldsymbol{PFA}_1+\left(\boldsymbol{\beta}+\boldsymbol{FQ}\right)\boldsymbol{B}_1\right]\boldsymbol{\beta W} \tag{11}$$

The closed loop system is stable when the following diophantine equation is fulfilled

$$\boldsymbol{PFA}_1+\left(\boldsymbol{\beta}+\boldsymbol{FQ}\right)\boldsymbol{B}_1 = \boldsymbol{M} \tag{12}$$

Where $\boldsymbol{M} \in R_{22}\left[z^{-1}\right]$ is a stable diagonal polynomial matrix.

$$\boldsymbol{M}\left(z^{-1}\right)=\begin{bmatrix}1+m_1z^{-1}+m_2z^{-2}++m_3z^{-3}+m_4z^{-4} & 0\\ 0 & 1+m_1z^{-1}+m_2z^{-2}++m_3z^{-3}+m_4z^{-4}\end{bmatrix} \tag{13}$$

The roots of this polynomial matrix are the ruling factor in the behaviour of the closed loop system. They must be inside the unit circle if the system is to be stable.
The degree of the controller matrices polynomials depends on the internal properness of the closed loop. The structure of the matrices $\boldsymbol{P}$, $\boldsymbol{Q}$ and $\boldsymbol{\beta}$ was chosen so that the number of unknown controller parameters equals the number of algebraic equations resulting from the solution of the diophantine equation using the uncertain coefficients method.

$$\boldsymbol{P}\left(z^{-1}\right)=\begin{bmatrix}1+p_1z^{-1} & p_2z^{-1}\\ p_3z^{-1} & 1+p_4z^{-1}\end{bmatrix}\qquad \boldsymbol{Q}\left(z^{-1}\right)=\begin{bmatrix}q_1+q_2z^{-1} & q_3+q_4z^{-1}\\ q_5+q_6z^{-1} & q_7+q_8z^{-1}\end{bmatrix}\qquad \boldsymbol{\beta}\left(z^{-1}\right)=\begin{bmatrix}\beta_1 & \beta_2\\ \beta_3 & \beta_4\end{bmatrix} \tag{14}$$

The solution to the diophantine equation results in a set of sixteen algebraic equations with unknown controller parameters. The control law apparent in the block diagram has the form

$$\boldsymbol{FPU} = \boldsymbol{\beta E}-\boldsymbol{FQY} \tag{15}$$

5. EXPERIMENTAL RESULTS

The model was connected with PC by the card Advantech PCL – 812. For its control the Matlab and the Real Time Toolbox [4] were used.

The three – tank – system is a nonlinear system with variable parameters which is, therefore, impossible to control deterministically. The nonlinear dynamics was described by the linear model in the neighbourhood of steady state. Adaptive control using recursive identification was performed. The recursive least squares method [3] proved effective for self-tuning controllers and was used as the basis for our identification algorithm.

The right side control matrix which resulted from a number of experiments has the form

$$\boldsymbol{M}\left(z^{-1}\right)=\begin{bmatrix} 1-0{,}9z^{-1}+0{,}19z^{-2}-0{,}009z^{-3}-0{,}002z^{-4} & 0 \\ 0 & 1-0{,}9z^{-1}+0{,}19z^{-2}-0{,}009z^{-3}-0{,}002z^{-4} \end{bmatrix} \quad (16)$$

The sampling period was chosen T_0=0,25 *s*.

The time responses of the control are shown in Fig. 3.

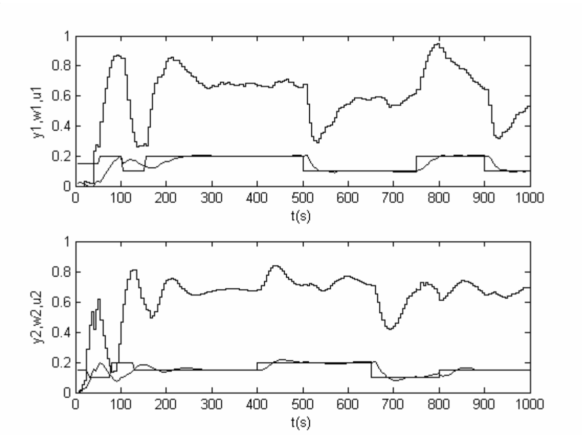

Figure 3. Adaptive control of the laboratory model

6. CONCLUSIONS

The multivariable adaptive control of the real three – tank – system was realized by means of polynomial theory. The control tests on the laboratory model gave satisfactory results despite the fact that the non-linear dynamics was described by the linear model. The objective laboratory model simulates a range of technological processes, which frequently occur in industry. The laboratory tests proved that the examined method could be implemented and used successfully to control such processes.

7. ACKNOWLEDGEMENTS

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