# ANALYSIS OF SPIN OF THE VUK-T SAILPLANE

dr Dragan Cvetković Faculty of Business Information Science, University "SINGIDUNUM" Danijelova St. 32, 11000 Belgrade, Serbia

dr Časlav Mitrović, mr Dragoljub Bekrić Faculty of Mechanical Engineering, University of Belgrade Kraljice Marije St. 16, 11000 Belgrade, Serbia

## **ABSTRACT**

One of the most interesting aeronautical sports is sailplane flying - flying aircraft with no engine, either for pleasure or at competitions. Soaring is a the most challenging sport for all aviators. If for any reason (severe turbulence, pilot's error, etc.) the speed of sailplane drops below the stalling speed, at such high angle of bank the sailplane will most probably fall into a spin. Spin is a very dangerous and unpleasant maneuver, and in the history of aviation, very large number of both aircraft and sailplane accidents was caused by spin. So, good spin recovery characteristics are imperative for any modern sailplane. Vuk-T sailplane is modern single seat, all composite sailplane for advanced pilot training and competitions, designed at the Belgrade Faculty of Mechanical Engineering. Major assumptions that were a starting point in this subject are the adopted aerodynamic concepts of the sailplane and the full system of equations for the aircraft motion in the case of spin.

Keywords: spin, spin recovery, flight simulation, non-linear model

#### 1. INTRODUCTION

A spin is an interesting manouvre, if only for the reason that at one time there stood to its discredit a large proportion of all airplane accidents that had ever occurred. It differs from other manouvres in the fact that wings are "stalled". Wings are beyond the critical angle-off-attack, and this accounts for the lack of control which the pilot experiences over the movements of the airplane while spinning. It is form of "auto-rotation", which means that there is a natural tendency for the airplane to rotate of its own accord.

## 2. NON-LINEAR MATHEMATICAL MODELS OF FLIGHT DYNAMICS

Mathematical modeling of spin motion is a very complex task and, therefore, it is necessary to introduce series of assumptions and approximations. Mathematical model is based on the first and second non-linear equations systems, shown bellow. The first system (V=const and H=const) is:

$$\begin{split} \dot{\alpha} &= a_{11} \frac{1}{\cos \alpha} + a_{12} \frac{\beta p}{\cos \alpha} + a_{13} q + a_{14} \frac{\cos \theta \cos \gamma}{\cos \alpha} + a_{15} \\ \dot{\beta} &= a_{21} \beta + a_{22} r \cos \alpha + a_{23} p \sin \alpha + a_{24} \cos \theta \sin \gamma + a_{25} \\ \dot{p} &= a_{31} r q + a_{32} \beta + a_{33} p + a_{34} r + a_{35} + a_{36} \\ \dot{r} &= a_{41} p q + a_{42} \beta + a_{43} p + a_{44} r + a_{45} q + a_{46} + a_{47} \\ \dot{q} &= a_{51} p r + a_{52} \left| \beta \right| + a_{53} r + a_{54} q + a_{55} + a_{56} \dot{\alpha} + a_{57} \end{split}$$

$$\dot{\theta} = a_{61}r\sin\gamma + a_{62}q\cos\gamma$$

$$\dot{\gamma} = a_{71}p + a_{72}r\cos\gamma\tan\theta + a_{73}q\sin\gamma\tan\theta$$

Now, koefficients is given by:

$$\begin{split} a_{11} &= -\frac{S\rho V}{2m} C_z(\alpha) \quad a_{12} = -1 \quad a_{13} = 1 \quad a_{14} = \frac{g}{V} \quad a_{15} = -\frac{S\rho V}{2m} C_{z\delta_{hk}}(\alpha) \Delta \delta_{hk}(t) \quad a_{62} = 1 \quad a_{72} = -1 \\ a_{21} &= \frac{S\rho V}{2m} C_{y\beta}(\alpha) \quad a_{22} = 1 \quad a_{23} = 1 \quad a_{24} = \frac{g}{V} \quad a_{25} = \frac{S\rho V}{2m} C_{y\delta_{vk}}(\alpha) \Delta \delta_{vk}(t) \quad a_{31} = \frac{I_y - I_z}{I_x} \\ a_{32} &= \frac{Sl\rho V}{2I_x} C_{l\beta}(\alpha) \quad a_{33} = \frac{Sl\rho V^2}{2I_x} C_{l\rho}(\alpha) \quad a_{34} = \frac{Sl\rho V^2}{2I_x} C_{lr}(\alpha) \quad a_{35} = \frac{Sl\rho V^2}{2I_x} C_{l\delta_{vk}}(\alpha) \Delta \delta_{vk}(t) \\ a_{36} &= \frac{Sl\rho V^2}{2I_x} C_{l\delta_k}(\alpha) \Delta \delta_k(t) \quad a_{41} = \frac{I_z - I_x}{I_y} \quad a_{42} = \frac{Sl\rho V^2}{2I_y} C_{n\beta}(\alpha) \quad a_{43} = \frac{Sl\rho V^2}{2I_y} C_{n\rho}(\alpha) \\ a_{44} &= \frac{Sl\rho V^2}{2I_y} C_{nr}(\alpha) \quad a_{45} = -\frac{I_p \omega_p}{I_y} \quad a_{46} = \frac{Sl\rho V^2}{2I_y} C_{n\delta_{vk}}(\alpha) \Delta \delta_{vk}(t) \quad a_{51} = \frac{I_x - I_y}{I_z} \quad a_{71} = 1 \\ a_{47} &= \frac{Sl\rho V^2}{2I_y} C_{n\delta_k}(\alpha) \Delta \delta_k(t) \quad a_{52} = \frac{Sb\rho V^2}{2I_z} C_{m\beta}(\alpha) \quad a_{53} = \frac{I_p \omega_p}{I_z} \quad a_{54} = \frac{Sb\rho V^2}{2I_z} C_{mq}(\alpha) \\ a_{55} &= \frac{Sb\rho V^2}{2I_z} C_m(\alpha) \quad a_{56} = \frac{Sb\rho V^2}{2I_z} C_{m\dot{\alpha}}(\alpha) \quad a_{57} = \frac{Sb\rho V^2}{2I_z} C_{m\delta_{hk}}(\alpha) \Delta \delta_{hk}(t) \quad a_{61} = 1 \quad a_{73} = 1 \\ a_{73} &= 1 \\ a_{74} &= \frac{Sb\rho V^2}{2I_z} C_m(\alpha) \quad a_{75} &= \frac{Sb\rho V^2}{2I_z} C_{m\dot{\alpha}}(\alpha) \Delta \delta_{hk}(t) \quad a_{61} = 1 \quad a_{73} = 1 \\ a_{75} &= \frac{Sb\rho V^2}{2I_z} C_{m\dot{\alpha}}(\alpha) \quad a_{77} &= \frac{Sb\rho V^2}{2I_z} C_{m\dot{\alpha}}(\alpha) \Delta \delta_{h\dot{\alpha}}(t) \quad a_{61} = 1 \quad a_{73} = 1 \\ a_{75} &= \frac{Sb\rho V^2}{2I_z} C_{m\dot{\alpha}}(\alpha) \Delta \delta_{h\dot{\alpha}}(t) \quad a_{61} = 1 \quad a_{73} = 1 \\ a_{75} &= \frac{Sb\rho V^2}{2I_z} C_{m\dot{\alpha}}(\alpha) \Delta \delta_{h\dot{\alpha}}(t) \quad a_{61} = 1 \quad a_{73} = 1 \\ a_{75} &= \frac{Sb\rho V^2}{2I_z} C_{m\dot{\alpha}}(\alpha) \Delta \delta_{h\dot{\alpha}}(t) \quad a_{61} = 1 \quad a_{73} = 1 \\ a_{75} &= \frac{Sb\rho V^2}{2I_z} C_{m\dot{\alpha}}(\alpha) \Delta \delta_{h\dot{\alpha}}(t) \quad a_{61} = 1 \quad a_{73} = 1 \\ a_{75} &= \frac{Sb\rho V^2}{2I_z} C_{m\dot{\alpha}}(\alpha) \Delta \delta_{h\dot{\alpha}}(t) \quad a_{75} = \frac{Sb\rho V^2}{2I_z} C_{m\dot{\alpha}}(\alpha) \Delta \delta_{h\dot{\alpha}}(t) \quad a_{75}$$

The second system (V and H are not constant) is:

$$\dot{\alpha} = b_{11} \frac{\rho V}{\cos \alpha} + b_{12} \frac{\beta p}{\cos \alpha} + b_{13} q + b_{14} \frac{\cos \theta \cos \gamma}{V \cos \alpha} + b_{15} \frac{V \tan \alpha}{V} + b_{16} \frac{\rho V}{\cos \alpha}$$

$$\dot{\beta} = b_{21} \rho V \beta + b_{22} r \cos \alpha + b_{23} p \sin \alpha + b_{24} \frac{\cos \theta \cos \gamma}{V} + b_{25} \frac{\dot{V} \beta}{V} + b_{26} \rho V$$

$$\dot{p} = b_{31} r q + b_{32} \rho V^2 \beta - b_{33} \rho V^2 p + b_{34} \rho V^2 r + b_{35} \rho V^2 + b_{36} \rho V^2$$

$$\dot{r} = b_{41} p q + b_{42} \rho V^2 \beta + b_{43} \rho V^2 p + b_{44} \rho V^2 r + b_{45} q + b_{46} \rho V^2 + b_{47} \rho V^2$$

$$\dot{q} = b_{51} p r + b_{52} \rho V^2 |\beta| + b_{53} r + b_{54} \rho V^2 q + b_{55} \rho V^2 + b_{56} \rho V^2 \dot{\alpha} + b_{57} \rho V^2$$

$$\dot{\theta} = b_{61} r \sin \gamma + b_{62} q \cos \gamma$$

$$\dot{\gamma} = b_{71} p + b_{72} r \cos \gamma \tan \theta + b_{73} q \sin \gamma \tan \theta$$

$$\dot{V} = b_{81} \frac{\rho V^2}{\cos \alpha} + b_{82} V \dot{\alpha} \tan \alpha + b_{83} V q \tan \alpha + b_{84} \frac{V r \beta}{\cos \alpha} + b_{85} \frac{\sin \theta}{\cos \alpha} + b86 \frac{\rho V^2}{\cos \alpha}$$

$$\dot{H} = b_{91} V \cos \alpha \sin \theta + b_{92} V \sin \alpha \cos \theta \cos \gamma + b_{93} V \beta \cos \theta \sin \gamma$$

Applied koefficients are given by:

$$\begin{split} b_{11} &= -\frac{S}{2m} C_z(\alpha) \quad b_{12} = -1 \quad b_{13} = 1 \quad b_{14} = g \quad b_{15} = -1 \quad b_{16} = -\frac{S}{2m} C_{z\delta_{hk}}(\alpha) \Delta \delta_{hk}(t) \\ b_{21} &= \frac{S}{2m} C_{y\beta}(\alpha) \quad b_{22} = 1 \quad b_{23} = 1 \quad b_{24} = g \quad b_{25} = -1 \quad b_{26} = \frac{S}{2m} C_{y\delta_{vk}}(\alpha) \Delta \delta_{vk}(t) \\ b_{31} &= \frac{I_y - I_z}{I_x} \quad b_{32} = \frac{Sl}{2I_x} C_{l\beta}(\alpha) \quad b_{33} = \frac{Sl}{2I_x} C_{lp}(\alpha) \quad b_{34} = \frac{Sl}{2I_x} C_{lr}(\alpha) \quad b_{35} = \frac{Sl}{2I_x} C_{l\delta_{vk}}(\alpha) \Delta \delta_{vk}(t) \end{split}$$

$$b_{36} = \frac{Sl}{2I_{x}}C_{l\delta_{k}}(\alpha)\Delta\delta_{k}(t) \quad b_{41} = \frac{I_{z} - I_{x}}{I_{y}} \quad b_{42} = \frac{Sl}{2I_{y}}C_{n\beta}(\alpha) \quad b_{43} = \frac{Sl}{2I_{y}}C_{np}(\alpha)$$

$$b_{44} = \frac{Sl}{2I_{y}}C_{nr}(\alpha) \quad b_{45} = -\frac{I_{p}\omega_{p}}{I_{y}} \quad b_{46} = \frac{Sl}{2I_{y}}C_{n\delta_{vk}}(\alpha)\Delta\delta_{vk}(t) \quad b_{47} = \frac{Sl}{2I_{y}}C_{n\delta_{k}}(\alpha)\Delta\delta_{k}(t)$$

$$b_{51} = \frac{I_{x} - I_{y}}{I_{z}} \quad b_{52} = \frac{Sb}{2I_{z}}C_{m\beta}(\alpha) \quad b_{53} = \frac{I_{p}\omega_{p}}{I_{z}} \quad b_{54} = \frac{Sb}{2I_{z}}C_{mq}(\alpha) \quad b_{55} = \frac{Sb}{2I_{z}}C_{m}(\alpha)$$

$$b_{56} = \frac{Sb}{2I_{z}}C_{m\alpha}(\alpha) \quad b_{57} = \frac{Sb}{2I_{z}}C_{m\delta_{hk}}(\alpha)\Delta\delta_{hk}(t) \quad b_{61} = 1 \quad b_{62} = 1 \quad b_{71} = 1 \quad b_{72} = -1 \quad b_{73} = 1$$

$$b_{81} = -\frac{S}{2m}C_{x}(\alpha) \quad b_{82} = 1 \quad b_{83} = -1 \quad b_{84} = -1 \quad b_{85} = -g \quad b_{86} = -\frac{S}{2m}C_{x\delta_{hk}}(\alpha)\Delta\delta_{hk}(t)$$

$$b_{91} = 1 \quad b_{92} = -1 \quad b_{93} = -1$$

## 3. THE GENERAL MODEL

Modeling was carried out using the software package MATLAB, capable of solving mathematical problems, and analysing data and visualization. has the ability to integrate numerical analyzis, matrices calculus, processing, and graphic displaying. Simulation was performed by using the SIMULINK module, having the special feature to simulate a dynamic system within a graphic mode, where the linear, non-linear, time-continuous or discrete multivariable systems having concentrated parameters be analyzed. Simulation is achieved by creating the SIMULINK model and implementing appropriate functions for the numeric solving differential equations. In practice, SIMULINK models are represented as block diagrams, that are a special type of mathematical system model, and show dynamic system characteristics and major system variables, and system element links.

The algorithm developed for simulation is based on seven block diagrams. In general case, the entire block diagram shown in Figure 1 contains the sub block diagrams.

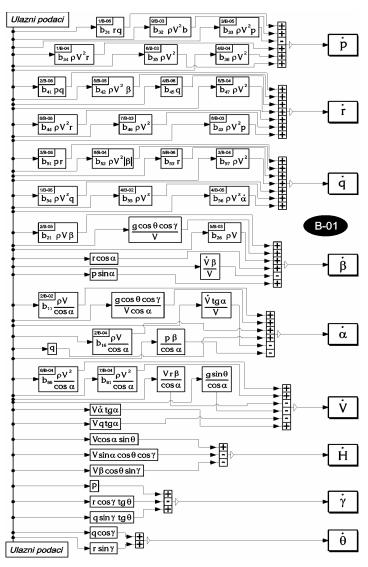


Figure 1. Main block diagram

#### 4. PROGRAM RESULTS

The programme has been tested on the VUK-T sailplane project, (Fig. 2) developed at the Institute for aeronautics, Faculty of Mechanical Engineering in Belgrade. The obtained results fully agree, and in

some cases are complementing with the in-flight testing results of this aircraft. Program results are diagrams, shown on Figure 3-4.

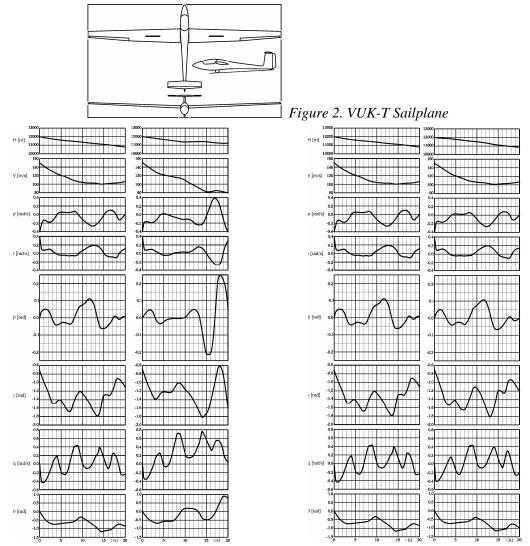


Figure 3. Basic diagram (left), S<sub>hk</sub> greater 40%

Figure 4. Basic diagram (left),  $S_{vk}$  smaller 40%

Practical application of this software has shown that computational design in flight mechanics gives exceptional results, allowing the designer to follow the development of calculation with a large number parameters in detail, and continuously be able to fulfill the criteria of tactical technical demands, performing all necessary optimizations.

## 5. REFERENCES

- [1] M. G. Kotik: Dinamika stopora samoleta, Masinostroenie, Moskva, 1976.
- [2] P. Wittwer and O. Masefield: *Spin Behaviour of the Pilatus PC-7 Turbo Trainer*, ICAS Proceedings, Volume 2, pp. 1032-1039, Seattle, Washington, USA, 1982.
- [3] William Bihrle, Jr. and Billy Barnhart: *Spin Prediction Techniques, Journal of Aircraft*, Vol. 20, No. 2, February 1983., pp. 97-101.
- [4] Rutan, E. L. and others: *Stall / Near Stall Investigation of the F-4E Aircraft*, (FTC-TR-70-20), Air Force Flight Test Center, Edwards AFB, California, August 1970.
- [5] G. S. Bjusgens and R. V. Studnev: *Dinamika samoleta Prostranstvenoe dvizenie*, Masinostroenie, Moskva, 1983.
- [6] G. S. Bjusgens and R. V. Studnev: *Aerodinamika samoleta Dinamika prodoljnogo i bokovogo dvizenia*, Masinostroenie, Moskva, 1979.
- [7] Papers from Internet