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MATEMATICAL MODEL FOR DEFINING WRIGGLE FORCES

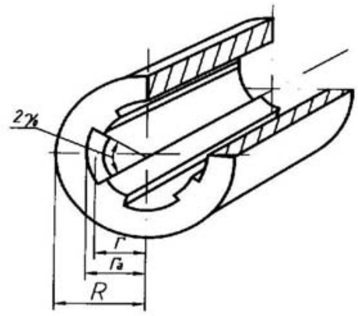
Mr Muhamed Lemeš
Vanjsko trgovinska komora Bosne i Hercegovine
Branislava Đurđeva 10, Sarajevo
Bosna i Hercegovina

ABSTRACT
Calculating of mathematical model, for defining wriggle forceson, on basic of experimental measurements for three variable influence parameters (type of material, logarithm degree of deformity cross-section and relation between outside and inside radius) with minim and maxim value on basic u plan factor ***2³*** *and on basic regression analyses...*
Key words: Wriggle forces, experimental measurement, influence parameters and plan factor 2^3.

1. EXPERIMANTAL MATHEMATICAL MODELS ON BASIC PLAN FACTOR 2^3

From experimental factor 2^3 it perform three influance factors on two levels and for cases of making of plan matrix. In this work models are made for wriggley forces and plan matrix for factors: influance od material type , logaritam degree of deformity and geometrical of example. Experimental research, are made on three time repeated trial for some combination factor level, what enable assessments of error detection experimant and getting value are value of wriggle forces , (table 1 and 2), used are intermediate values.

2. MATHEMATICAL MODELS OF WRIGGLE FORCES $F_p = F_p(k_{sr}; \varphi; R/r)$



Picture 1. Example after impress flute by wriggle the tool

Model for wriggle force is made in form: $F_p=F_p(k_{sr}; \varphi; R/r)$; $F_p=F_p(X_1;X_2;X_3)$

k_{sr} – specific deformation resistance of material

φ - logarithm degree of deformity

R/r –relation between outside and inside radius of example

X_1 – influence factor type of material over k_{sr},

$X_{1min} = k_{sr}$ *za AlCu5;* $X_{1max} = k_{sr}$ *for C10E.*

X_2 – influence factor logarithm degree of deformity chross-section of example

$X_{2min} = \varphi_{min} = ln((R^2-r^2)/(R^2-r^2-(r_{amax}^2-r^2)(m/6)));$

$X_{2max} = \varphi_{max} = ln((R^2-r^2)/(R^2-r^2-(r_{amin}^2-r^2)(m/6)));$

X_3 – factor influence geometrical radius of example
$X_{3min} = (R/r)_{min}$; $X_{3max} = (R/r)_{max}$;

As full tree factor ortogonalical plan ensure that, over basics factor effects, determine and effects intercourse function first and second form, elect for mathematical description of process model shape:

$$\hat{y} = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_{12}x_1x_2 + b_{13}x_1x_3 + b_{23}x_2x_3 + b_{123}x_1x_2x_3 \quad (1)$$

Search records are given in tablei 1. Coding of factors is made over transform equation:

$$x_1 = \frac{X_1 - 118{,}15}{28{,}45} \qquad x_2 = \frac{X_2 - 0{,}0210}{0{,}007} \qquad x_3 = \frac{X_3 - 2{,}51485}{0{,}46835} \quad (2)$$

Table 1. Plan matrix with results of experiment

Points	PLAN -MATRIX								EXPERIMENTAL RESULTS			
plan	X_0	X_1	X_2	X_3	X_1X_2	X_1X_3	X_2X_3	$X_1X_2X_3$	Y_1	Y_2	Y_3	$\bar{y}$
1	1	-1	-1	-1	1	1	1	-1	4,1	5,3	4,2	4,53333333
2	1	1	-1	-1	-1	-1	1	1	8,5	8,7	7,4	8,2
3	1	-1	1	-1	-1	1	-1	1	4,8	5	6,6	5,46666667
4	1	1	1	-1	1	-1	-1	-1	11	9,5	8	9,5
5	1	-1	-1	1	1	-1	-1	1	4,5	6,6	4,1	5,06666667
6	1	1	-1	1	-1	1	-1	-1	9	8,5	9	8,83333333
7	1	-1	1	1	-1	-1	1	-1	10	10	12	10,6666667
8	1	1	1	1	1	1	1	1	16	18	17	17
Basic level		118,2	0,021	2,515								8,65833333
Inter. varijation		28,5	0,007	0,468								
Uper level		146,6	0,028	2,983								
Lower level		89,70	0,014	2,047								
bi=Sxiy	bo	b1	b2	b3	b12	b13	b23	b123				

2.1. Calculation of model parameter

Method of more factors orthogonal plans make easier equations model [1,2,3,4,5], diffuse systems. On basic of characteristic orthogonal plans regarding to who correlation matrix exceding in diagonal get easy shape parameta equation function responding (coefficient complex regression).

Value of model parameters is:

$$b_i = \frac{1}{N}\sum_{u=1}^{N} x_{iu}\overline{y_u}, \qquad i = 0,1,2,\ldots,N; \quad N = 2^k \quad (3)$$

Calculating with a middle value term:
$b_0 = 8{,}65875$; $b_1 = 2{,}22375$; $b_2 = 2{,}00875$; $b_3 = 1{,}74125$;
$b_{12} = 0{,}43750$; $b_{13} = 0{,}29125$; $b_{23} = 1{,}44125$; $b_{123} = 0{,}27625$
So the model in coordinat coordinates:

$$\hat{y} = 8{,}6587 + 2{,}22375\,x_1 + 2{,}00875\,x_2 + 1{,}74125\,x_3 + 0{,}43750\,x_1x_2 + 0{,}29125\,x_1x_3 + 1{,}44125\,x_2x_3 + 0{,}27625\,x_1x_2x_3 \quad (4)$$

Crossing on natural coordinates $\hat{y} = f(X_1, X_2, X_3)$, with transform equation.

Table 2. Experimental data

Points plan	EXPERIMENTAL RESULTS								
	y1	y2	y3	$\bar{y}$	si2	ŷi	(yi-ŷ)2	$\Sigma y_i^2/(n-1)$	$(\Sigma y_i)^2/(n*(n-1))$
1	4,1	5,3	4,2	4,533333	0,443333	4,5787	0,002	31,27	30,827
2	8,5	8,7	7,4	8,2	0,49	8,1212	0,006	101,35	100,860
3	4,8	5	6,6	5,466667	0,973333	5,3912	0,006	45,8	44,827
4	11	9,5	8	9,5	2,25	9,5787	0,006	137,625	135,375
5	4,5	6,6	4,1	5,066667	1,803333	5,1487	0,007	40,31	38,507
6	9	8,5	9	8,833333	0,083333	8,7512	0,007	117,125	117,042
7	10	10	12	10,66667	1,333333	10,6212	0,002	172	170,667
8	16	18	17	17	1	17,0787	0,006	434,5	433,500
				8,658333	8,376667	8,6587	0,041893	1079,98	1071,603

On basic analysis from table (1), for given value of exit, for calculation of experimental mistake we have that equation of square sum on the same repeat:

$s^2(y) = s^2_E = 8{,}3767/16 = 0{,}52354375; \quad f_E = N(m-1) = 8(3-1) = 16,$ (5)

System of repeat experimental in example is orthogonal plan and dispersion of model parameters is regarding to equation [1,2,3,4,5]:

$$s^2(b_i) = \frac{s^2(y)}{Nn} = \frac{s_E^2}{Nn} = \frac{0{,}52354375}{8 \cdot 3} = 0{,}021814323 \quad \text{or} \quad s(b_i) = 0{,}147696726 \qquad (6)$$

2.2. Verification of significant model

Evaluation of signification any of model parameter first rows is executes independent from others. The aim of rating is control zero hypotheses $b_i=0$. That mean that does groupe insignificant parametrs can exclude from model without corrct thereby values others significants parametrs that remains in model.

$$t_{ri} = \frac{|b_i|}{s(b_i)},$$

Acquired calculated values t_r for t-criteria:

$t_{r0}=58{,}625$; $t_{r1} = 15{,}14926$; $t_{r2} = 13{,}60048$; $t_{r3} = 11{,}789318$;
$t_{r12}=2{,}9621$; $t_{r13} = 1{,}97194273$; $t_{r23} = 9{,}672733$; $t_{r123}=1{,}87038$;

For number of grade freedom $f_E = N(n-1) = 8(3-1) = 16$ (therby apply t-criteria for rating of signification model parametrs taker this number of grade freedom (f_E) with who is determine dispersion of experimental ($s^2(y)= s^2_E$), and adopted level of significance $\alpha=5\%$, being table value $t_t = 1{,}75$, So according to conditions of significants $t_{ri} > t_t$ continues that parametars are: b_0, b_1, b_2, b_3, b_{12}, b_{13}, b_{23} *i* b_{123} significants, So that model model final is:

$$\hat{y} = 8{,}6587 + 2{,}22375\, x_1 + 2{,}00875\, x_2 + 1{,}74125\, x_3 + 0{,}43750\, x_1x_2 + 0{,}29125\, x_1x_3 + 1{,}44125\, x_2x_3 + 0{,}27625\, x_1x_2x_3 \qquad (7)$$

or, in natural coordinates (use equation of transformation):

$$\hat{y} = -2{,}1339+0{,}1576776X_1 - 371{,}25X_2 + 1{,}83442X_3 - 3{,}788785X_1X_2 +$$

$89,693X_2X_3$ $-0,0621957X_1X_3$ $+2,96177X_1 X_2 X_3$ (8)

Model for defining wriggle force in function of includes influence factors have shape:

$$F_p = -2,1339+0,1576776k_{sr} -371,25\varphi +1,83442R/r -3,788785k_{sr}\varphi + 89,693\varphi R/r -0,0621957k_{sr}R/r +2,96177 k_{sr}\varphi R/r \quad (9)$$

2.3. Verification of adequate model

Verification of adequate model consist, in general responsibility, in classification of dispersion experimental results in respect on regression line (s^2_R) and dispersion experimental results in points multi factor space (s^2_E) over Fisher criteria, where is from s^2_R used s^2_E witch represent dispersion middle values experimental results in respect on regression line. Next step dispersion analyses in mathematical modeless of process includes check model adequacy. Because this is system experimental repeat n – times in every points of hypercube will be dispersion related for model adequacy:

$$S^2{}_{LF} = \frac{m(\bar{y}_1 - \hat{y}_1)}{Nm - (m+1) - N(m-1)}; \qquad f_{LF} = Nm - (m+1) - (m-1) \quad (10)$$

$$s^2_{LF} = \frac{3 \cdot 0,0419}{24-(3+1)-8(3-1)} = 0,031425; \qquad f_{LF} = 24-(3+1)-8(3-1) = 4 \quad (11)$$

Then is from equation: $F_{rLF} = \frac{s^2_{LF}}{s^2_E}$ (12)

Get calcualated value: $F_r = 0,0324975/0,52354375 = 0,060023636$ (13)

Since is geting calculated value lower then table value $F_t = 3$ *for* $f_{LF} = 4$, $f_E = 16$*and* α*=5%, i.e.* $F_r = 0,030011818 < F_t = 3$, therefore geting model (equation of multiple regression) on adecuate way discribed explicit three-factor proces.

2.4. Determination limit value of model reliability

Boundaries of reliable model parameters define equation:

$$b_i \pm \Delta b_i = b_i \pm ts(b_i) = b_i \pm t\sqrt{c_{ii}}s(y) \quad (14)$$

So that is:

$\Delta b_i = \pm ts(b_i) = \pm 1,75 \cdot 0,147696726 = \pm 0,25846927,$ (15)

Where is: $t = t_t = 1,75$ *za* $f_E = 16$ *i* α*=5%*

3. RECAPITULATION

Applying plan factor 2^3 and experimental research, derived with triplex repeated trials, for the same combination factor level, geted result that value of wriggley forces, in shape mathematical model witch includes three influence factors. In experimental par influence factors are varying from minimal to maximal values. Aimed, lower volumen, experimental research, with selection influence factors and with applying mentioned plan factor can be arrive to trustworth mathematical model for calculating needed value.

4. REFERENCE

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