

A NEW ARTIFICIAL POTENTIAL FOR NOMINAL CONTROL OF REPETITIVE ROBOT MOTION

Branko M. Novakovic
Dubravko N. Majetic
Josip J. Kasac
Danko S. Brezak
FSB - University of Zagreb
Ivana Lucica 5, Zagreb
Croatia

ABSTRACT

In this paper a new artificial potential, suitable for nominal control of the repetitive robot motion, is proposed. Starting with this potential a normalized energy equation has been established. This equation was the source for derivation of a nominal robot velocity as the function of the initial and final positions, as well as the limited maximal nominal velocity of the robot motion. For derivation of nominal robot acceleration, as the function of the same parameters, the artificial potential relation has been employed. Both nominal velocity and acceleration equations satisfy the condition of the repetitive motion: boundary velocities are equal to zero and boundary accelerations are positive on the first side and negative on the second one. The nominal transient position relation is twice continuously differentiable resulting in the mentioned nominal velocity and acceleration equations. The obtained nominal velocity and acceleration relations are very simple for calculation and can be applied to the real time nominal control of repetitive robot motion.

Keywords: artificial potential, nominal velocity and acceleration, nominal repetitive robot control.

1. INTRODUCTION

There exist a lot of nominal robot control algorithms mostly given as nominal velocity profiles of the robot motion. The main problem of the existing nominal robot control algorithms is in their real-time realization. Because of complex nominal velocity profiles and complex interactions in the robot structure, the related control of the fast robots can not be calculated in the real-time. Especially, in the case of the repetitive nominal control both nominal velocity and acceleration equations must satisfy the main condition of the repetitive motion: boundary velocities should be equal to zero and boundary accelerations should be positive on the first side and negative on the second one. An important subject in control of mechanical systems is tracking repetitive nominal signals and attenuating periodic disturbances. Many tracking systems, such as computer disk drivers, rotation machine tools and assembly robots, have to deal with repetitive nominal signals and disturbances.

Generally, repetitive controllers can be classified as being either internal model based or external model based [1]. The control algorithms with internal models are linear and have repetitive signal generators [2, 3]. On the other side, the control algorithms with the external models are based on the feedforward compensation of the inverse dynamics and their disturbance model is placed outside the basic feedback loop [4, 5]. The main advantage of the control algorithms with internal models is that they are linear, making the analysis and implementation easier. The main disadvantage of that control algorithms is that the stability is almost entirely governed by the feedback loop of the repetitive compensators. In the case of the external model approach there is no significant influence on the stability margin of the control system. Meanwhile, the analysis and implementation of the control

algorithms are more complex by the external model approach. In the reference [6] a new class of passive internal model based repetitive controllers for robot manipulators is proposed. In this approach the passive interconnection of the controller with nonlinear mechanical system has no influence to the stability margin like in exact feed-forward compensation of robot dynamics [7, 8]. An interesting approach to synthesis of the nominal control of the repetitive robot motion has been presented in [9]. This approach is based on the analogy to the basic equations of the universe motion [10]. The main disadvantage in that approach is the condition that the generalized (joint) coordinates should be different from zero. Of course, this limitation can always be avoided by using the related coordinate transformation, but it slows down the calculation of the nominal control variables.

In order to find out a simple and fast nominal control algorithm in this paper a new artificial potential, suitable for nominal control of the repetitive robot motion, is proposed. Starting with this potential a normalized energy equation has been established. This equation was the source for derivation of a nominal robot velocity as the function of the initial and final positions, as well as the limited maximal nominal velocity of the robot motion. For derivation of nominal robot acceleration, as the function of the same parameters, the artificial potential relation has been employed. Both nominal velocity and acceleration equations satisfy the condition of the repetitive motion: boundary velocities are equal to zero and boundary accelerations are positive on the first side and negative on the second one. The obtained nominal velocity and acceleration relations are very simple for calculation and can be applied to the real time nominal control of repetitive robot motion.

This paper is organized as follows. The synthesis procedure of the nominal control of the repetitive robot motion, based on an artificial potential, is presented in the second section. Some comments and conclusions are emphasized by the third section, while reference list is given by the fourth (last) section.

2. SYNTHESIS OF NOMINAL CONTROL OF REPETITIVE ROBOT MOTION

Following the idea to employ an artificial potential for design a nominal control of repetitive robot motion one can establish the related normalized energy equation:

$$\frac{\dot{q}_i^2}{2} + \frac{v_{m_i}^2}{2}(1 - K_i q_i)^2 = \frac{v_{m_i}^2 k_i^2}{2}, \quad i = 1, 2, \dots, n, \quad (1)$$

where q_i and \dot{q}_i are generalized (joint) coordinate and velocity of the i -th robot link, v_{m_i} is the limited maximal nominal velocity, K_i and k_i are related constants and n is the number of the robot links. The first part on the left side of the equation (1) represents the normalized kinetic energy, the second one is the normalized potential energy, while the right side of the equation (1) represents the normalized total (kinetic + potential) mechanical energy. An artificial scalar potential V_{p_i} of an artificial potential field in the equation (1) is described by the following relation:

$$V_{p_i} = \frac{v_{m_i}^2}{2}(1 - K_i q_i)^2, \quad i = 1, 2, \dots, n. \quad (2)$$

The normalized energy equation (1) can be transformed into the new relation:

$$\frac{\dot{q}_i^2}{2} - K_i q_i v_{m_i}^2 \left(1 - \frac{K_i q_i}{2}\right) = \frac{v_{m_i}^2}{2}(k_i^2 - 1). \quad (3)$$

Consequently, the related velocity equation can be derived from (3) in the following form:

$$\dot{q}_i = \pm \left[2K_i q_i v_{m_i}^2 \left(1 - \frac{K_i q_i}{2}\right) + (k_i^2 - 1)v_{m_i}^2 \right]^{1/2}. \quad (4)$$

The notation (+) is valid for motion in a positive direction, while the notation (-) is related to the negative one. The acceleration equation can be derived directly from the artificial scalar potential (2):

$$\ddot{q}_i = -\frac{\partial V_{p_i}}{\partial q_i}, \quad \rightarrow \quad \ddot{q}_i = K_i v_{m_i}^2 (1 - K_i q_i). \quad (5)$$

The velocity equation (4) has two zeros, where the first one is at the initial position q_{oi} and the second one is at the final position q_{ei} :

$$q_{oi} = q_{i_{\min}} = \frac{1 - k_i}{K_i}, \quad q_{ei} = q_{i_{\max}} = \frac{1 + k_i}{K_i}. \quad (6)$$

From the relations (6) one can calculate the parameters k_i and K_i as the functions of the initial and final positions:

$$k_i = \frac{q_{ei} - q_{oi}}{q_{oi} + q_{ei}}, \quad K_i = \frac{2}{q_{oi} + q_{ei}}. \quad (7)$$

Substituting parameters k_i and K_i from (7) into the relation (4) one obtains the velocity equation as the function of the initial and final positions:

$$\dot{q}_i = \pm \frac{2v_{mi}}{q_{oi} + q_{ei}} \left[(q_{oi} + q_{ei})q_i - q_i^2 - q_{oi}q_{ei} \right]^{1/2}. \quad (8)$$

On the other side, the substitution of the parameters k_i and K_i from (7) into the relation (5) gives the acceleration equation also as the function of the initial and final positions:

$$\ddot{q}_i = \frac{2v_{mi}^2}{q_{oi} + q_{ei}} \left(1 - \frac{2q_i}{q_{oi} + q_{ei}} \right). \quad (9)$$

From the relation (9) we can see that the zero acceleration is occurred at the position q_{ci} :

$$\ddot{q}_i = 0, \quad \rightarrow \quad q_i = q_{ci} = \frac{q_{oi} + q_{ei}}{2}. \quad (10)$$

For $q_i < q_{ci}$ the acceleration is positive and for $q_i > q_{ci}$ the acceleration is negative. Including $q_i = q_{ci}$ into the velocity equation (8) one obtains the maximal velocity at that position:

$$q_i = q_{ci} \quad \rightarrow \quad \dot{q}_i = \dot{q}_{i_{\max}} = k_i v_{mi} = \frac{q_{ei} - q_{oi}}{q_{oi} + q_{ei}} v_{mi}, \quad (11)$$

$$(q_{ei} - q_{oi}) \leq (q_{oi} + q_{ei}) \quad \rightarrow \quad \dot{q}_{i_{\max}} \leq v_{mi}.$$

Thus, the maximal velocity is always less or equal to the limited maximal nominal velocity v_{mi} . At the initial and final positions we have the following values of the velocities and accelerations:

$$\begin{aligned} q_i = q_{oi} &\rightarrow \dot{q}_i = \dot{q}_{oi} = 0, \quad q_i = q_{ei} \rightarrow \dot{q}_i = \dot{q}_{ei} = 0, \\ q_i = q_{oi} &\rightarrow \ddot{q}_i = \ddot{q}_{oi} = K_i k_i v_{mi}^2 = \frac{2(q_{ei} - q_{oi})v_{mi}^2}{(q_{oi} + q_{ei})^2}, \\ q_i = q_{ei} &\rightarrow \ddot{q}_i = \ddot{q}_{ei} = -K_i k_i v_{mi}^2 = \frac{-2(q_{ei} - q_{oi})v_{mi}^2}{(q_{oi} + q_{ei})^2}. \end{aligned} \quad (12)$$

The relations (12) show that both nominal velocities and accelerations satisfy the condition of the repetitive motion: boundary velocities are equal to zero and boundary accelerations are positive on the first side and negative on the second one. Now, one can established the following vector relations:

$$\mathbf{q} = [q_1 \ q_2 \ \dots \ q_n]^T, \quad \dot{\mathbf{q}} = [\dot{q}_1 \ \dot{q}_2 \ \dots \ \dot{q}_n]^T, \quad \ddot{\mathbf{q}} = [\ddot{q}_1 \ \ddot{q}_2 \ \dots \ \ddot{q}_n]^T, \quad \mathbf{U} = [u_1 \ u_2 \ \dots \ u_n]^T, \quad (13)$$

where \mathbf{q} is a real n -vector of generalized (joint) coordinates, $\dot{\mathbf{q}}$ and $\ddot{\mathbf{q}}$ are the related generalized velocity and acceleration vectors and $\mathbf{U}(t)$ is a real n -vector of the nominal robot control. Following

the relations (8) to (13) one can implement the nominal control algorithm for control of the nominal repetitive robot motion:

$$U(t) = E(q)\ddot{q} + F(q, \dot{q}). \quad (14)$$

In the relation (14), $E(q)$ is a real $(n \times n)$ -inertial matrix and $F(q, \dot{q})$ is a real n -vector of centrifugal, Coriolis and gravity forces. Both $E(q)$ and $F(q, \dot{q})$ include the actuator parameters and related constant connections between manipulator and actuators. Thus, the nominal control of the repetitive robot motion can be calculated in a real-time by employing very simple relations (8) to (13) and the equation (14).

3. CONCLUSION

The synthesis procedure of the new nominal control of the repetitive robot motion, based on an artificial potential, is presented and the related normalized energy equation has been established. This equation was the source for derivation of a nominal robot velocity relation as the function of the initial and final positions, as well as the limited maximal nominal velocity of the robot motion. The nominal robot acceleration equation, as the function of the same parameters, has been derived directly from the mentioned artificial potential. It is shown that both nominal velocity and acceleration equations satisfy the condition of the repetitive motion: boundary velocities are equal to zero and boundary accelerations are positive on the first side and negative on the second one. The obtained nominal velocity and acceleration relations are very simple for calculation and can be applied to the real time nominal control of repetitive robot motion.

4. REFERENCES

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