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THE ISENTROPIC COEFFICIENT AND THE ADIABATIC COMPRESSIBILITY OF ARGON DERIVED FROM THE SPEED OF SOUND

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ABSTRACT
A new method for deriving the isentropic coefficient and the adiabatic compressibility from speed of sound is recommended. It is based on numerical integration of differential equations connecting the speed of sound with other thermodynamic properties. The method requires initial values of density and heat capacity at a single temperature in the pressure range of interest. It is tested by deriving the isentropic coefficient and the adiabatic compressibility of gaseous argon in the temperature range 250-450 K, and in the pressure range 0.2-12 MPa. Estimated absolute average deviation between calculated and reference values of the isentropic coefficient and the adiabatic compressibility is 0.007% and 0.008%, respectively.
Key words: argon, isentropic coefficient, adiabatic compressibility, speed of sound

1. THEORY

1.1. The thermodynamic speed of sound
The thermodynamic speed of sound (i.e., the speed of sound at zero frequency) in a fluid, u, m/s, is defined by the Laplace's equation $u^2 = (\partial p/\partial \rho)_s$, where: ρ, kg/m^3, is the density of the substance, p, N/m^2, is the pressure, s, J/kg·K, is the specific entropy. Since $\rho = 1/v$ the Laplace's equation has the following form, Ref. [2, p.127]:

$$u^2 = -v^2\left(\frac{\partial p}{\partial v}\right)_s \tag{1}$$

Combining the above equation with important relationship, rarely mentioned in the literature, that determines the following derivative, Ref. [2, p.124]:

$$\left(\frac{\partial p}{\partial v}\right)_s = \left(\frac{\partial p}{\partial v}\right)_T - \frac{T}{c_v}\left(\frac{\partial p}{\partial T}\right)_v^2 \tag{2}$$

we obtain

$$u^2 = v^2\left[\frac{T}{c_v}\left(\frac{\partial p}{\partial T}\right)_v^2 - \left(\frac{\partial p}{\partial v}\right)_T\right] \tag{3}$$

An equivalent form of Eq. (3) can be found by replacing the derivative $(\partial p/\partial T)_v$ in terms of the cyclic equation, according to Ref. [1, p.636]:

$$\left(\frac{\partial p}{\partial T}\right)_v = -\left(\frac{\partial v}{\partial T}\right)_p\left(\frac{\partial p}{\partial v}\right)_T \tag{4}$$

so that, Eq. (3) results

$$u^2 = v^2 \left[\frac{T}{c_v} \left(\frac{\partial p}{\partial v} \right)_T^2 \left(\frac{\partial v}{\partial T} \right)_p^2 - \left(\frac{\partial p}{\partial v} \right)_T \right] \tag{5}$$

1.2. The isentropic coefficient and the adiabatic compressibility

The next relationship widely used in the thermodynamics to calculate the isentropic coefficient for a substance in various states is, Ref. [2]:

$$\kappa \equiv -\frac{v}{p} \left(\frac{\partial p}{\partial v} \right)_s , - \tag{6}$$

Since the partial derivative in the above equation is defined as, Ref. [2, p.117]

$$\left(\frac{\partial p}{\partial v} \right)_s = \frac{c_p}{c_v} \left(\frac{\partial p}{\partial v} \right)_T \tag{7}$$

Eq. (6) can be rewritten in the following form

$$\kappa = -\frac{v}{p} \frac{c_p}{c_v} \left(\frac{\partial p}{\partial v} \right)_T \tag{8}$$

However, combining Eqs. (6) and (2) leads to another form of the isentropic coefficient

$$\kappa = -\frac{v}{p} \left[\left(\frac{\partial p}{\partial v} \right)_T - \frac{T}{c_v} \left(\frac{\partial p}{\partial T} \right)_v^2 \right] \tag{9}$$

Another relationship also important in the thermodynamics, so-called the coefficient of adiabatic compressibility, is defined as, Ref. [2]:

$$\alpha_s \equiv -\frac{1}{v} \left(\frac{\partial v}{\partial p} \right)_s , \ \mathrm{Pa}^{-1} \tag{10}$$

Since the partial derivative in the above equation is defined as, Ref. [2, p.124]:

$$\left(\frac{\partial v}{\partial p} \right)_s = \left(\frac{\partial v}{\partial p} \right)_T + \frac{T}{c_p} \left(\frac{\partial v}{\partial T} \right)_p^2 \tag{11}$$

Eq. (10) has the following form

$$\alpha_s = -\frac{1}{v} \left[\left(\frac{\partial v}{\partial p} \right)_T + \frac{T}{c_p} \left(\frac{\partial v}{\partial T} \right)_p^2 \right] \tag{12}$$

1.3. Average absolute and relative deviation

Average absolute deviation (*AAD*) is calculated according to expression

$$AAD[\%] = \frac{100}{M(N-1)} \sum_{i=1}^{M} \sum_{j=2}^{N} \frac{\left| X_{i,j}^{\mathrm{cal}} - X_{i,j}^{\mathrm{eos}} \right|}{X_{i,j}^{\mathrm{eos}}} \tag{13}$$

and relative deviation (*RD*) according to expression

$$RD[\%] = 100 \frac{X_{i,j}^{cal} - X_{i,j}^{eos}}{X_{i,j}^{\mathrm{eos}}}, \quad i = 1, M \ \text{ and } \ j = 1, N \tag{14}$$

where: M is the number of isobars, N is the number of isotherms, X is the value of a coefficient, superscript *cal* denotes the calculated value of a coefficient, and superscript *eos* denotes the reference value of a coefficient obtained from the fundamental equation of state.

2. RESULTS AND CONCLUSION

Numerical procedure is used for deriving the isentropic coefficient and the adiabatic compressibility of gaseous argon from its speed of sound, Ref. [3], in the temperature range 250-450 K, and in the pressure range 0.2-12 MPa. The temperature range is divided into 5 isotherms (e.g. 250 K, 300 K, 350 K, 400 K, and 450 K), and the pressure range is divided into 7 isobars (e.g. 0.2 MPa, 2 MPa, 4 MPa, 6 MPa, 8 MPa, 10 MPa, and 12 MPa). The set of equations (1) to (5) is solved numerically by combined Adams-Moulton, Ref. [4, p.404], and Runge-Kutta, Ref. [5, p.93], method. All pressure derivatives are estimated by Lagrange interpolating polynomial, Ref. [6, p.55], of sixth-degree. Figs. 1-4 give an impression of the results obtained. Initial values of ρ and c_p, Ref. [3], are specified along isotherm at 250 K, and therefore this isotherm is omitted. Estimated absolute average deviation of calculated values of the isentropic coefficient and the adiabatic compressibility, with reference to corresponding reference values, Ref. [3], is 0.007% and 0.008%, respectively.

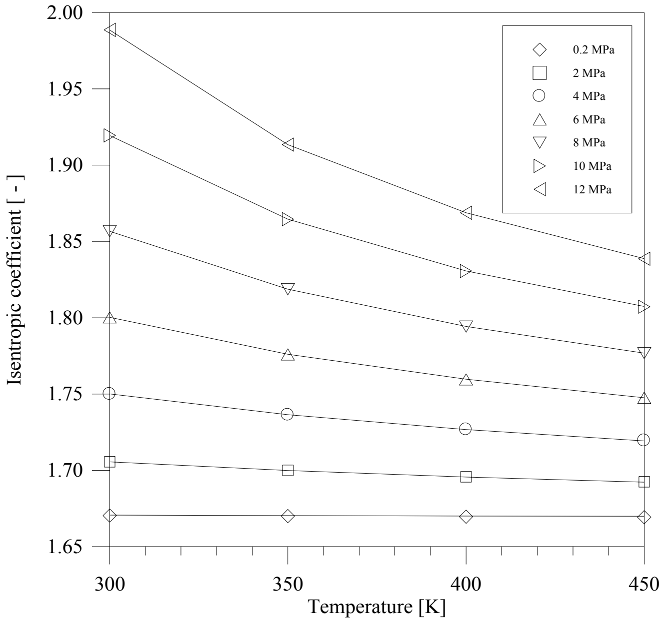

Figure 1. Isentropic coefficient of argon vs. temperature;full line this work; symbols [3].

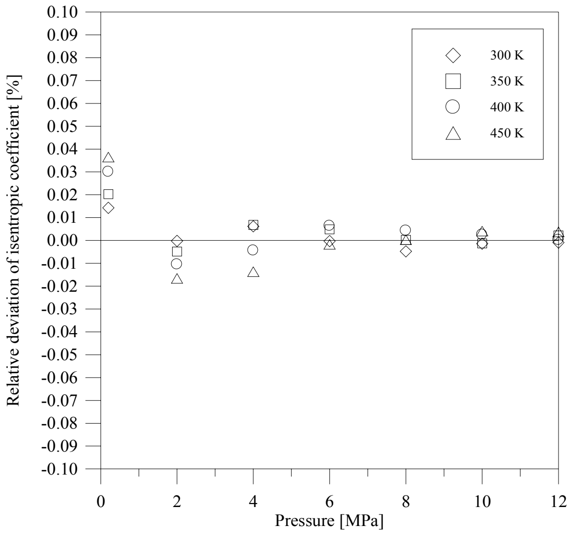

Figure 2. Relative deviation of argon isentropic coefficient vs. pressure; symbols this work; full line [3].

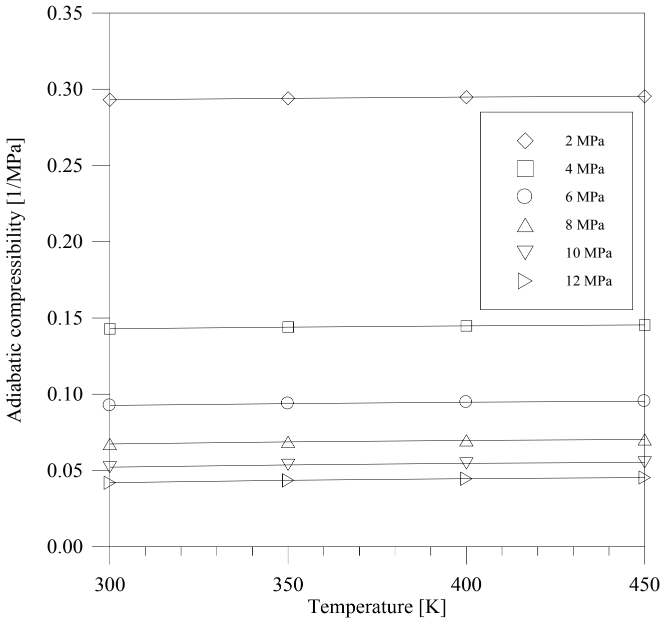

Figure 3. Adiabatic compressibility of argon vs. temperature;full line this work; symbols [3].

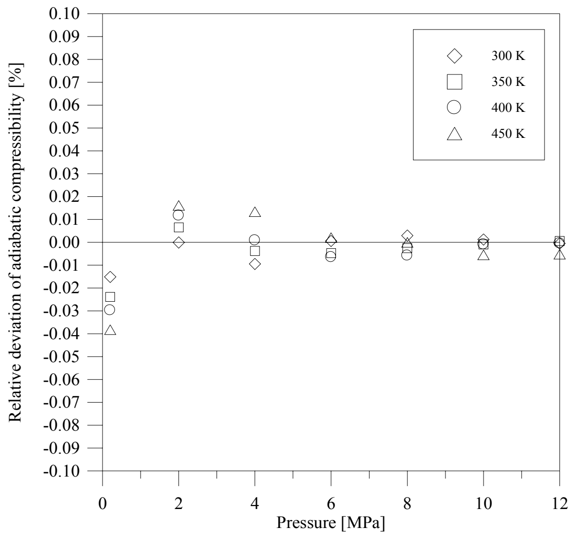

Figure 4. Relative deviation of argon adiabatic compressibility vs. pressure; symbols this work; full line [3].

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