12th International Research/Expert Conference
”Trends in the Development of Machinery and Associated Technology”
TMT 2008, Istanbul, Turkey, 26-30 August, 2008

UTILIZATION OF COMPUTER PROGRAMS IN MODELLING OF GEARS WITH ASYMMETRIC INVOLUTE TEETH

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ABSTRACT
This paper studies the equations of rack cutters for generating helical gears with asymmetric involute teeth. By applying profile equations of the rack cutter, the principle of coordinate transformation, the theory of differential geometry, and the theory of gearing, the mathematical models of involute helical gear is given. The paper presents aspects regarding the utilisation of computer programmes for computer aided design graphical modelling of cylindrical gears with asymmetric involute teeth. Software applications in the domain of computing the points that define the profile and the line of tooth flanks, as well as their 2 and 3 D representation are performed.
Keywords: Involute profile, modeling, computer aided design

1. INTRODUCTION
Gears are important machine elements, which are widely used in automobile, aircraft, and machine tools. Involute-shaped gears are widely used in industries because of their simple manufacturing process by straight-sided cutters and their constant ratio of transmission due to their center distance tolerance [1]. Litvin’s vector approach [2] enables accurate representation of gear tooth profiles for computer-aided design and finite element modeling. This approach has the flexibility for application of profile modifications to spur and helical gearing. Based on Litvin’s vector approach, Yang proposed a mathematical model of the helical gear with asymmetric involute teeth [3].

Asymmetric teeth are well suited for cases where the torque is transmitted mainly in one direction. The performance of the primary contacting profile can be improved by degrading the performance of the opposite profile. Designers are able to create gear drives capable of handling greater torque in the same amount of space, or they are able to reduce the amount of space required to handle the same amount of torque.[4,5]

In this study, the design of rack-type generating tools (rack cutter or hub) with asymmetric teeth is considered. A mathematical model of involute gears with asymmetric teeth is given based on the theory of gearing and the geometry of the straight-sided rack cutter. Computer aided design of involute gears are based on the utilisation of the following programs: MATHLAB for automatic generation of the tooth flanks (involute and trochoid), AutoCAD for 2D and 3D drawing. The output file of the tooth profile generation programme which is written in MATHLAB software is a script file involving the coordinates of gear geometry. This file when run in AutoCAD programme automatically draws the gear geometry. The methodology given combines some of the facilities of the above-mentioned programs, resulting in the reduction of design and modelling time for spur and helical gears with asymmetric involute teeth.

2. GEAR TOOTH SURFACES

Figure 1 presents the design of the normal section of a rack cutter Σ_n, where regions $\overline{ac}$ and $\overline{bd}$ are the left- and right-side top lands, regions $\overline{ce}$ and $\overline{df}$ are the left- and right-side fillets and, regions $\overline{eg}$ and $\overline{fh}$ are the left- and right-side working regions. In simulating the rack cutter surface for the helical gear generation, the normal section of the rack cutter Σ_n, attached to the coordinate system S_n with its origion O_n, is translated along the line $\overline{O_nO_c}$ as shown in Fig.1. Therefore, $\rho = \left|\overline{O_nO_c}\right|$ is also one of the design parameters of the rack cutter surface, and β is the helix angle of the generated helical gear. The equation of the transformation is as follows :

$$\mathbf{R}_c^i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\beta & -\sin\beta & -\rho\sin\beta \\ 0 & \sin\beta & \cos\beta & \rho\cos\beta \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{R}_n^i \qquad (1)$$

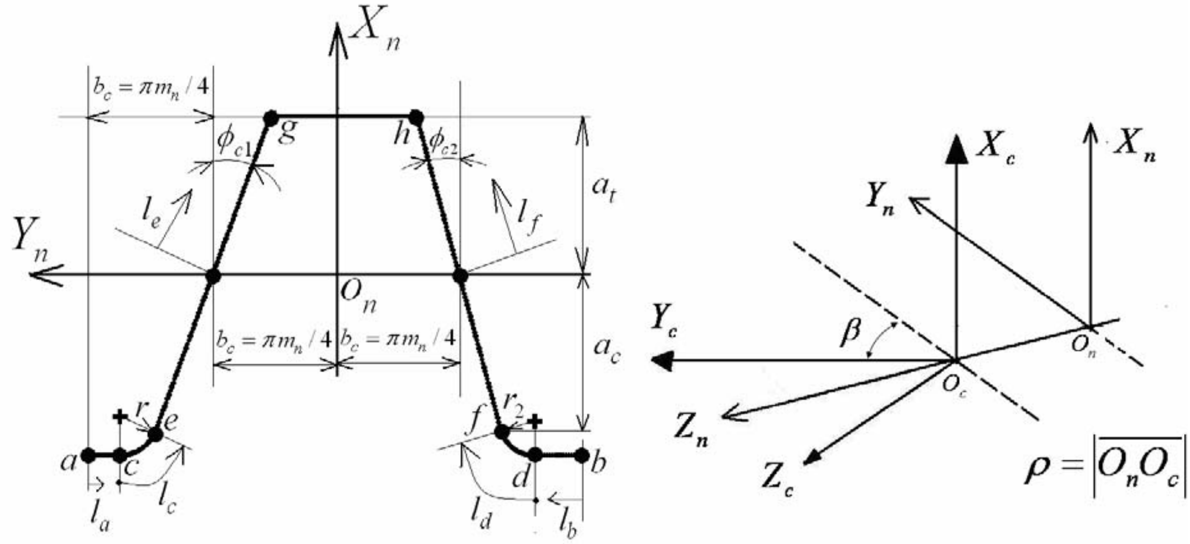

Figure 1. Formation of rack cutter surface for helical gear generation

As shown in Fig. 1., regions $\overline{eg}$ and $\overline{fh}$ of the rack cutter are used to generate different sides of the working tooth surface of the asymmetric helical gears. The symbol m_n represents the normal module. a_c is the design parameter used to determine addendum of the cutter, $b_c = \pi m_n / 4$ half tooth width of the cutter, ϕ_{c1} and ϕ_{c2} are the normal pressure angles.

To derive the mathematical model for the complete tooth profile of involute helical gears with asymmetric teeth, coordinate systems $S_c(X_c, Y_c, Z_c)$, $S_1(X_1, Y_1, Z_1)$ and $S_h(X_h, Y_h, Z_h)$ should be set up as depicted in Fig 2. During the generation process, the rack cutter translates a distance $S = r_{p1}\ \phi_1$ while the gear blank rotates by an angle ϕ_1.

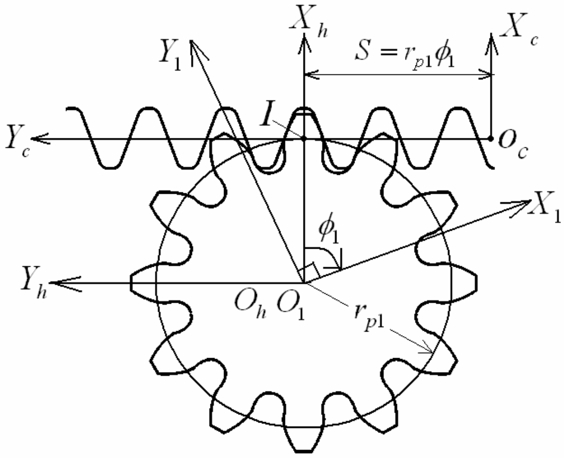

Figure 2. Kinematic relationship between the rack cutter and the generated gear

The mathematical model of the generated gear tooth surface is a combination of the meshing equation and the locus of the rack cutter surfaces according to gearing theory [2]. Applying the following homogeneous coordinate transformation matrix equation makes it possible to obtain the locus of the cutter represented in coordinate system S_1 as follows :

$$\mathbf{R}_1^i = [M_{1c}]\mathbf{R}_c^i, \tag{2}$$

Transformation matrix $[M_{1c}]$ describes tool rack rolling without sliding around the pitch circle of the gear, as in the case of hobbing.

The equation for the tooth profile of the generated driving gear can be obtained by solving the locus equation and the meshing equation simultaneously. After substitutions, working tooth profile which is generated by regions $\overline{eg}$ and $\overline{fh}$ of the cutter can be represented by,

$$\begin{cases} x_1^{eg} = l_e \cos\phi_{c1}\cos\phi_1 - (b_c - l_e \sin\phi_{c1})\cos\beta\sin\phi_1 + \rho\sin\beta\sin\phi_1 + r_{p1}(\cos\phi_1 + \phi_1\sin\phi_1) \\ y_1^{eg} = l_e \cos\phi_{c1}\sin\phi_1 + (b_c - l_e \sin\phi_{c1})\cos\beta\cos\phi_1 - \rho\sin\beta\cos\phi_1 + r_{p1}(\sin\phi_1 - \phi_1\cos\phi_1) \\ z_1^{eg} = (b_c - l_e\sin\phi_{c1})\sin\beta + \rho\cos\beta \\ \phi_1 = ((b_c\sin\phi_{c1} - l_e)\cos\beta - \rho\sin\beta\sin\phi_{c1})/(r_{p1}\sin\phi_{c1}) \end{cases} \tag{3}$$

$$\begin{cases} x_1^{fh} = l_f \cos\phi_{c2}\cos\phi_1 + (b_c - l_f \sin\phi_{c2})\cos\beta\sin\phi_1 + \rho\sin\beta\sin\phi_1 + r_{p1}(\cos\phi_1 + \phi_1\sin\phi_1) \\ y_1^{fh} = l_f \cos\phi_{c2}\sin\phi_1 - (b_c - l_f \sin\phi_{c2})\cos\beta\cos\phi_1 - \rho\sin\beta\cos\phi_1 + r_{p1}(\sin\phi_1 - \phi_1\cos\phi_1) \\ z_1^{fh} = (-b_c + l_f\sin\phi_{c2})\sin\beta + \rho\cos\beta \\ \phi_1 = ((-b_c\sin\phi_{c1} + l_f)\cos\beta - \rho\sin\beta\sin\phi_{c2})/(r_{p1}\sin\phi_{c2}) \end{cases} \tag{4}$$

3. COMPUTER AIDED DESIGN OF GEARS

In order to establish the coordinates of points defining the tooth profile, a program was prepared, called *simprog2D* (fig.3), which is based on the parametric equations of the gear tooth profile. The program generates a *scr* file (fig.3) for profile drawing in AutoCAD software.

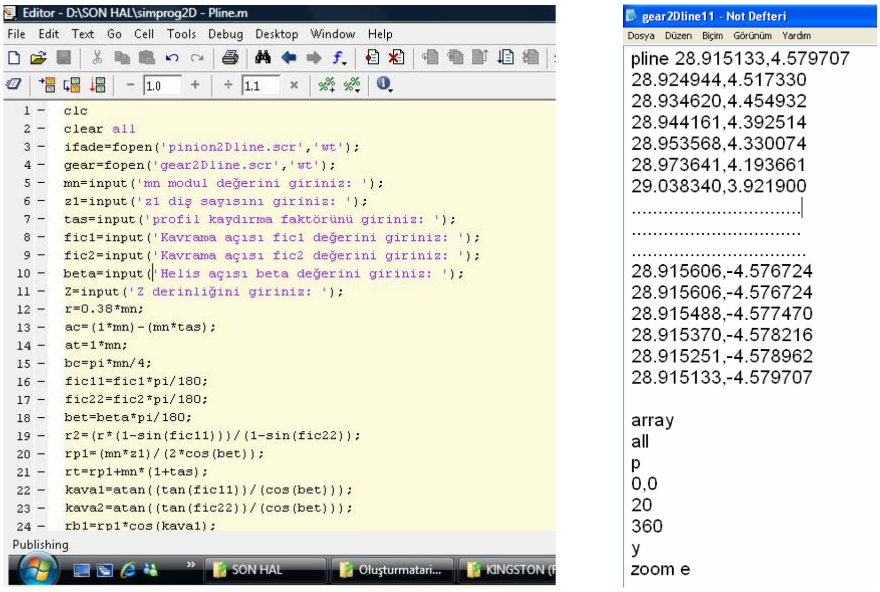

Figure 3. The code in Matlab for calculating coordinates and a Script file for profile drawing

By running this script file, first a single tooth is obtained and then by using polar array command a gear wheel is constructed as depicted in Fig 4.

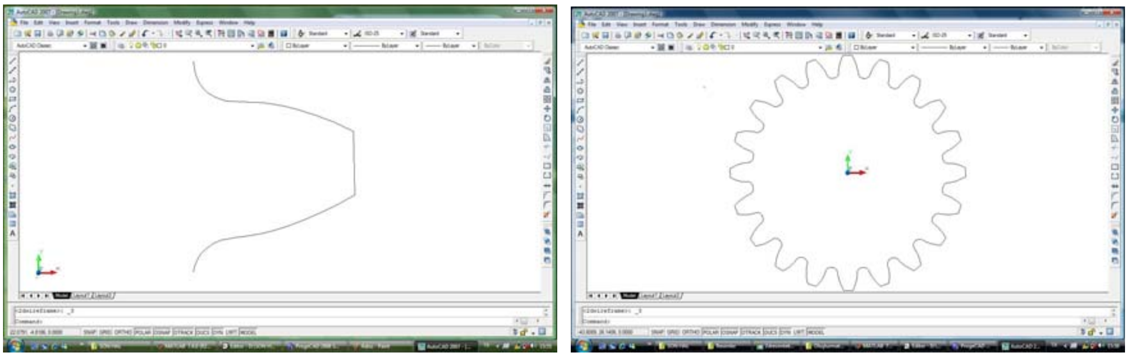

Figure 4. Two dimensional gear wheel design by using polar array

4. CONCLUSION

In this study the equations of rack cutters for generating helical gears with asymmetric involute teeth is used for the the utilisation of computer programmes for CAD modeling of gears. The methodology described combines some of the facilities of the above-mentioned programs, resulting in the reduction of design and modelling time for wheels and gears. The gear tooth models that were obtained may be used for further numerical stress analysis by using specialised programs (ANSYS, NASTRAN, etc.).

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