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MATHEMATICAL MODELING AND STATIC STRESS ANALYSIS OF STEEL WIRE ROPE STRAND USING FINITE ELEMENT METHOD

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ABSTRACT
Wire rope theory relies on the modeling of the wire ropes using the nonlinear equilibrium equations. The construction of the nonlinear equilibrium equations are given on the well known classical treatise on elasticity by Love in 1944. General nonlinear equilibrium equations acting on a thin wire is presented. The modeling difficulties are taken into account and a realistic model of a wire rope strand is constructed. A finite element model is presented and the numerical solution using this model is solved and the results are presented. Stresses and deformations are presented along a simple straight wire rope strand and the results shows the typical behavior of a strand under the axial loading.
Keywords: Wire rope, wire strand, nonlinear equilibrium equation, modeling of wire rope strand

1. INTRODUCTION
Wire rope theory is based on a well known classical treatise on elasticity by Love in 1944. General nonlinear equilibrium equations are derived and presented in [1]. The mechanical behaviors of the wire ropes are investigated in [2] based on torsion and shed light to the bending analysis of the open-coiled helical springs in axial plane by bending moment and lateral load. Green and Laws, in general theory of rods [3], mentioned to a restricted and linearized form to determine stresses in helical constituent wires in cables. Governing equilibrium equations are taken as a starting point in most of the analytical analysis. Costello et al. presented the general behavior of the wire ropes in different aspects in his papers and, gathered his works in [4].
Due to its complex shapes it is still more difficult task to model and analyze wire ropes using numerical methods. Most of the analysis are based on the modeling arc length of a simple strand to see the mechanical behavior of the strands and compare the numerical solution with the experiment or analytical solution if exists. The aim of this study is to model not an arc length of a strand but to make a realistic model of a strand using by using a modeling application Solidworks® and a computer-aided engineering tool Abaqus/CAE®.

2. GENERAL THEORY OF WIRE ROPE STRANDS
The general behavior of a strand is analyzed using the nonlinear equilibrium equations given by Love in [1]. The six governing differential equations are presented. Using the direction cosines and summing the forces $N+dN$, $N'+dN'$ and $T+dT$ along the element length ds respectively for x, y and z axis gives [1],

$$\frac{dN}{ds} - N'\tau + T\kappa' + X = 0, \frac{dN'}{ds} - T\kappa + N\tau + Y = 0, \frac{dT}{ds} - N\kappa' + N'\kappa + Z = 0, \qquad (1)$$

Similarly the couples $G+dG$, $G'+dG'$ and $H+dH$ for the moments in x, y and z axes will give,

$$\frac{dG}{ds}-G'\tau+H\kappa'-N'+K=0,\frac{dG'}{ds}-H\kappa+G\tau+N+K'=0,\frac{dH}{ds}-G\kappa'+G'\kappa+\Theta=0\,, \tag{2}$$

where N, N' and T shows the forces, G, G' and H shows couples. The equations (1)-(2) are the six differential equations which constitutes the equations of equilibrium for the thin wire loaded and depicted in Figure (1-a). When the cross-section of the wire rope is considered circular area within radius R, the changes in curvature and twist per unit length to the internal loads are given by;

$$G=\frac{\pi R^4}{4}E(\kappa-\kappa_0);\quad G'=\frac{\pi R^4}{4}E(\kappa'-\kappa_0');\quad H=\frac{\pi R^4 E}{4(1+\nu)}(\tau-\tau_0) \tag{3}$$

where ν and E are respectively represents Poisson's ratio and Young's Modulus while curvatures κ, κ' and the twist per unit length τ are given by; $\kappa=0$, $\kappa'=\cos^2\alpha/r$ and $\tau=\sin\alpha\cos\alpha/r$. For a circular cross section the tension T in the wire is given by $T=\pi R^2 E\xi$, where ξ shows the axial wire strain.

2.1 Analytical Solution Theory

A simple straight strand with the cross section given as in Figure (1-b) will be analyzed. The helix angle of an outside strand α_2 can be obtained by the relation, $\tan\alpha_2=p_2/2\pi r_2$, where p_2 is the pitch of an outside wire and $r_2=R_1+R_2$. The initial curvature and the twist per unit length are given,

$$\kappa_2=0;\quad \kappa_2'=\frac{\cos^2\alpha_2}{r_2}\quad and\quad \tau_2=\frac{\sin\alpha_2\cos\alpha_2}{r_2}. \tag{4}$$

The wires of the strand are deformed by the total axial force *F*, and the total axial twisting moment M_t. It will be assumed that an outside wire is not subjected to external bending moments per unit length in each direction, $K_2=K'_2=\Theta_2=0$. Also components of the external line load per unit length of the centerline in *y* and *z* directions are assumed to be zero, $Y_2=Z_2=0$, and the axial wire tension T_2 is assumed to be constant along the length of the wire. Using the equations (3) and (4), the equilibrium equations given before in equations (1)-(2) becomes to,

$$-\frac{\sin\alpha_2\cos\alpha_2}{r_2}N_2'(s)+\frac{\cos^2\alpha_2}{r_2}T_2(s)+X_2(s)=0\,,\ \frac{dN_2'}{ds}(s)=0\,,\ \frac{dT_2}{ds}(s)=0 \tag{5}$$

$$\frac{dG_2(s)}{ds}-\frac{\sin\alpha_2\cos\alpha_2}{r_2}G_2'(s)+\frac{\cos^2\alpha_2}{r_2}H_2(s)-N_2'(s)=0\,, \tag{6}$$

$$\frac{dG_2'(s)}{ds}+\frac{\sin\alpha_2\cos\alpha_2}{r_2}G_2(s)=0\,,\ \frac{dH_2(s)}{ds}-\frac{\cos^2\alpha_2}{r_2}G_2(s)=0\,, \tag{7}$$

where subscript *2* refers to the outside wires. The system of equations (5)-(7) are solved using Maple® in [5] and it has been shown that the resulting equations are harmonious with the equations found by Costello [4], the following results are obtained,

$$H_2=\frac{\sin\alpha_2}{\cos\alpha_2}G_2'+\frac{r}{\cos^2\alpha_2}N_2'\,, \tag{8}$$

$$N_2'=\frac{\cos^2\alpha_2}{r}H_2-\frac{\sin\alpha_2\cos\alpha_2}{r}G_2',\ X_2=\frac{\sin\alpha_2\cos\alpha_2}{r}N_2'-\frac{\cos^2\alpha_2}{r}T_2\,. \tag{9}$$

Using equation (3) with the geometrical properties defined on the cross section of the strand [5], the following equations can be written for G_2', H_2 and T_2,

$$G_2'=\frac{\pi}{4}ER_2{}^4\Delta\kappa_2',\ H_2=\frac{\pi}{4(1+\nu)}ER_2{}^4\Delta\tau_2\,,\ T_2=\pi\xi_2 ER_2{}^2\,, \tag{10}$$

A projection of the forces, acting on the outside wires, in the axial direction of the strand yields F_2 and, the total axial twisting moment is denoted by M_2. Both F_2 and M_2 are given below,

$$F_2 = m_2\left[T_2\sin\alpha_2 + N_2'\cos\alpha_2\right], M_2 = m_2\left[H_2\sin\alpha_2 + G_2'\cos\alpha_2 + T_2Er_2\cos\alpha_2 - N_2'Er_2\sin\alpha_2\right]. \quad (11)$$

The axial force and the axial twisting moment on the center wire are F_1 and M_1 respectively,

$$F_1 = \pi\xi_1 ER_1^{\,2},\ M_1 = \frac{\pi}{4(1+\nu)}ER_1^{\,4}\tau_s. \quad (12)$$

The total axial force is $F = F_1 + F_2$ and the twisting moment is $M_t = M_1 + M_2$ acting on the strand.

3. MODELING OF A SIMPLE STRAIGHT STRAND

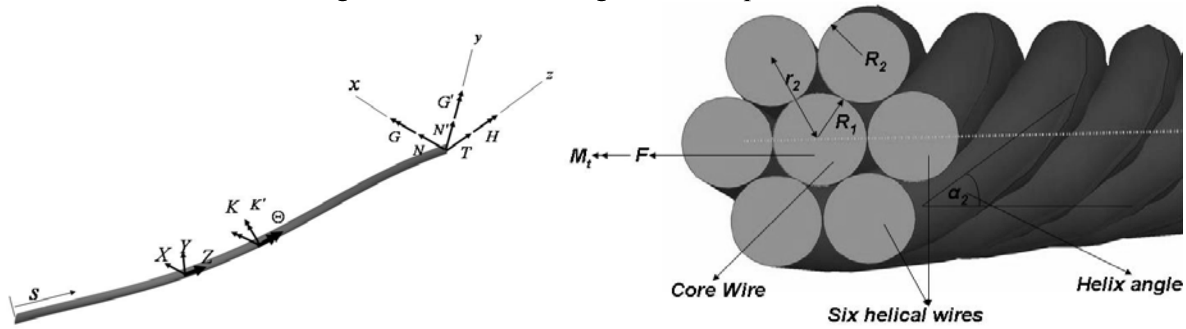
A simple straight strand model is given in Figure (1-b). The model is constructed with a center wire of radius R_1, surrounded by six helices around with the helix angle α_2. To construct (1+6) wire rope strand model Solidworks and Abaqus/CAE programs are used simultaneously. Core wire is created in the Abaqus/CAE to get rid of the meshing problems. But it is not possible to simply create a helical wire using Abaqus/CAE at the moment. First of all a helical path have to be created in Solidworks. To do this helix and spiral curve should be used. Selecting the front plane, a circle with the radius $R_1 + R_2$ is created which will be the radius of the helical path. The pitch of the helix, starting angle of the helix and number of revolutions parameters are given as $247.65mm$, 0° and 1 respectively. When this is accomplished a circle will be created at the beginning of the helical path with radius R_2. Then from the features, Swept Boss/Base will be selected to surround the path with this circle. As a profile the circle with radius R_2 and as a path the helical path should be selected. Doing this a helical wire with a thickness of $2.5654mm$ has been created. At the end of this work this part should be passed to the Abaqus/CAE with Parasolid file extension. In Abaqus/CAE the helical part can be imported as a part. The helical wire part and circular wire part can be assembled in Abaqus/CAE. At the end a radial pattern procedure with 6 numbers of instances for 360° total angle should be applied. The simple straight strand geometry with $(1+6)$ wires is ready at the moment for analysis. Only the necessary boundary conditions, contacts and the loads have to be applied to the assembly. After that it is necessary to create a mesh for solving the problem. The element type and the mesh size are very important to solve the problem. If the mesh is coarse then problem could not converge and there will be no solution for this reason. C3D8R: A-8 node linear brick, reduced integration hourglass control type element is used for the analysis. There are 14859 numbers of nodes, 11340 numbers of elements.

3.1 Numerical Example

***3.1.1 Analytical Solution*:** Consider a simple straight strand cross-section given as in Figure (1-b) with the parameters [4]; $R_1 = 2.6162\ mm$, $R_2 = 2.5654\ mm$, $p_2 = 247.65\ mm$, $E = 196497.52\ N/mm^2$, $\nu = 0.25$ and $m_2 = 6$. Outside wires are assumed not touching each other and $r_2 = R_1 + R_2 = 5.1816\ mm$, $\alpha_2 = 82.510641^\circ$. The angle of twist per unit length of the strand is $\tau_s = 0$, which means that the strand is not allowed to rotate and $\xi_1 = \varepsilon = 0.003$. $R_2\Delta\kappa_2'$ and $R_2\Delta\tau_2$ can be computed as $R_2\Delta\kappa_2' = -0.00005564$, $R_2\Delta\tau_2 = -0.0001838$. Equations (8)-(12) yields;

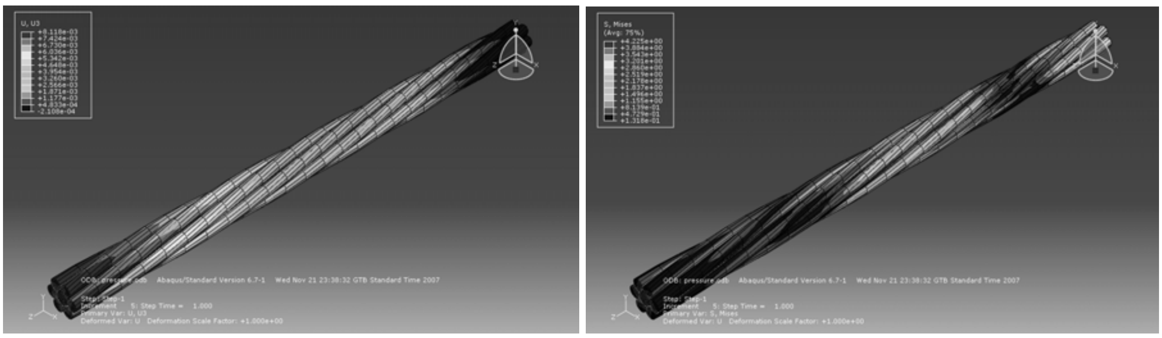
$$F = F_1 + F_2 = 12675.65 + 70970.48 = 83646.12\ N,\ M_t = M_1 + M_2 = 45877.83\ Nmm.$$

***3.1.2 Numerical Solution*:** Encastre boundary condition is given to one side of the wire rope strand, while force is applied to the other side. Each of the outer wires are loaded with force $11828.6N$ and the center wire is loaded with $12677.4N$. Wire material is selected as steel with the Young's modulus of $E = 196497.52N/mm^2$ and the Poissons ratio is taken as $\nu = 0.25$. Also the wires are constrained at the loaded side with another boundary condition which allows the strain and displacements can occur only in the $u_3(z)$ directions. The other directions are prohibited to strain/displacement affects in $u_1(z)$ and $u_2(z)$ directions. The proposed model is solved and the deformations can be seen in Figure (2-a), von-Misses stress can be seen in Figure (2-b). It can be concluded that strain distribution along the strand is harmonious with the general behavior of a wire rope strand and also in a good agreement with the analytical solution of 0.003 strain given in the example above. Strains are going to be stable near to the encastre side of the strand. Also von-Misses stress distribution shows the good distribution along the wire rope strand.



(a) Loads acting on a thin wire *(b) (1+6) wire straight strand model*

Figure 1. Loads and cross-section of a wire rope strand



a. Strains and displacements *b. von-Misses stress*

Figure 2: Deformation and stress distribution on a wire rope strand

4. CONCLUSION

After discussing the general theory of a wire rope strand, analytical solution of the nonlinear equilibrium equation for a simple straight strand is given. Modeling of a Simple Straight Strand using the Solidworks and the numerical definition of the experiment problem using Abaqus/CAE is defined. After the wire rope strand problem is solved using Abaqus/CAE, results are presented. The results shows that both deformation and von-Misses stress distribution over the simple straight strand are harmonious with the general behavior of a wire rope strand. The result shows that the strain distribution is in good agreement with the analytical solution given in example.

5. REFERENCES

[1] Love A.E.H., A treatise on the mathematical theory of elasticity, 4th ed., New York: Dover, 1944 Publications, 1944, First American printing 1944, Chapter XVIII-XIX, pp. 381-426.

[2] Timoshenko S., 1878-1972, *Strength of materials*, New York: Van Nostrand, [1955-56], Vol.2, 292-299.

[3] Green A.E. and Laws N., "A general theory of rods", Proceedings of the Royal Society of London, Series A, *Mathematical and Physical Sciences*, Vol. 293, No. 1433, (Jul. 26, 1966), pp. 145-155.

[4] Costello GA. *Theory of wire rope*. Berlin: Springer, 1990.

[5] Erdönmez C., İmrak C.E., General Nonlinear Equilibrium Equation Solution of the Straight Wire Strand, ICNAAM, Corfu, Greece, September 16-20, 2007