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ABOUT KINEMATIC AND DYNAMIC BEHAVIOR OF THE ROTATING MACHINERY AS NONHOLONOMIC SYSTEMS

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ABSTRACT
In the rotating machinery with variable speed drive, constraints are described by differential equations which can not be integrated, so geometric characteristics can not be easily obtained. In this paper the rotating machinery with frictional variable speed drive with automatic regulations will be analyzed. The non-holonomic connections exist at the points of the physical contact between discs. During dynamical analysis Appell`s equations will be used. As a result differential equations of motion are obtained. They describe behaviour the rotating mechanical non-holonomic system. In many cases it is not possible to solve these differential equations analytically, so they must be solved on computers using numerical methods and answer concerning the working stability as well as the dynamical and kinematics behaviour of observed nonholonomic system will be obtained.
Keywords: dynamic, kinematic, non-holonomic systemm, variable speed drive

1. INTRODUCTION

Different kinds of mechanical the variable-speed drives are used for changing speed in agricultural machines, cutting machines the cable, carpet and paper industries, mining machines, account machines, etc. Am of this paper is to give the dynamic analysis of a general example of this class of machine element. These mechanical systems are non-holominc, because the constraints are differential – they are function of speed and acceleration. In the rotating machinery with variable speed drive, constraints are described by differential equations which can not be integrated, so geometric characteristics can not be easily obtained. This is the basis of non-holonomic mechanics and gives the difference from holonomic mechanics where are all constraints are geometric and there are no limitations of speed and acceleration for the system.

2. APPEL'S EQUATIONS FOR DYNAMIC ANALYSIS OF NON-HOLONOMIC SYSTEMS

Appell`s equations, which are very suitable for dynamic analysis are given in the well known form:

$$\frac{\partial S^*}{\partial \ddot{q}} = Q_\nu^* \quad (\nu = 1,2, ..., p) \tag{1}$$

Where: S* - energy of acceleration; $\ddot{q}$ - generalized accelerations ; Q_ν^*-generalized forces

$$S = \sum_{j=1}^{N} \frac{m_j \vec{a}_j^2}{2} = \frac{1}{2}\sum_{j=1}^{N} m_j (\ddot{\vec{r}}_j)^2 \tag{2}$$

$$Q_\nu^* = \sum_{j=1}^{N} \vec{F}_j \vec{A}_{j\nu} \tag{3}$$

Where: $\vec{F}_j$ - external forces in material point j; $\vec{A}_{j\nu}$ - Appell`s vector

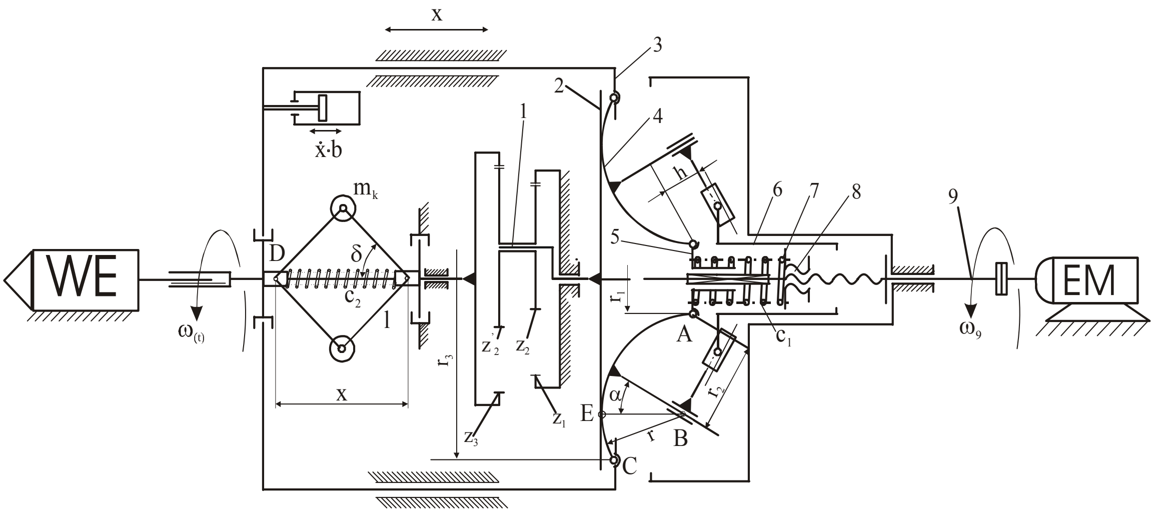
Non-holonomic connection is given by:

$$\sum_{j=1}^{n} A_{\rho i}\dot{q}_i + A_\rho = 0 \quad (\rho = 1,2,...,l) \tag{4}$$

Where $A_{\rho i}$ and A_{ρ} are coefficients in function of q_i and t.

3. THE PLANETARY VARIABLE-SPEED DRIVES AS NONHOLONOMIC SYSTEM

The frictional planetary variable-speed drives, is shown in Fig.1. Variation of the angular velocity happens due to the changing of contact point E position with respect to the position of the rotating axis B. In that way the distance between the contact point and rotating axis is changed and gives the variable transmission relation between elements 2 and 4. The closed automatic regulation is made in such a way that the slider is connected at point D; the elastic element keeps a contact sliding element of regulator with elements 3 and 4. The work of elastic element c_2 proportional to that of elastic



element 7 c_1. This type of the planetary variable-speed drives is very suitable for application in many different kind of complex mechanical systems.

Figure 1. Frictional planetary variable-speed drive, 1- carrier of planets z_2 and z_2', 2 - driven disk , 3 - slider with contacts in point C, 4 - mushrooms planet, 5 - driver, 6 - carrier of mushrooms planet 4, 7 - spring with stiffness c_1,8 - spiral shaft, 9 - input shaft locked with shaft 8.

Symbols in Fig.1. are:

$r_1, r', r_\alpha, r, r_2, r_3$ - radii of driving element 5, driven element 2, contact point of elements 4 and 2, mushroom 4, contact point A with respect to the B and position of point C

x and x_0 - position and start position of regulator slider

γ, δ - angle of conical input element 1 , position angle of the regulators bar

$\Delta\ell, c$ - start load of elastic element and elasticity of elastic element

$\dot{\phi}_1, \dot{\phi}_2, \dot{\phi}_4, \dot{\phi}_K$ - angular speeds of leading balls, leaded discs, balls, and element K

m_D , m_3, m_D – mass of slider in point D, sliding element 3 and regulators ball

J_5 , J_3 , J_2 , J_4 , J_{o4} , J_{z3} , J_{os} - the moments of inertia of elements in Fig.1

J_r , J_6 - reduced inertial moment of all rotating masses of working elements and reduced moment of inertia on the axle of carrier 6

M_5 and M_3 - torques on leading and leaded elements $M_5 = M_E$, $M_3 = M_w$

$\dot{x}$ - speed of regulation, change of position for slider D

The general form of Appell's equations is given by:

$$\frac{\partial S}{\partial \varphi} = Q_\varphi ; \qquad \frac{\partial S}{\partial \ddot{x}} = Q_X \tag{5}$$

Non-holonomic connection is:

$$\dot{\varphi}_5 - i_v \cdot \dot{\varphi}_2 = 0 \tag{6}$$

After transformations, non-holonomic connection given by eq.6 can be described in following form:

$$\dot{\varphi}_5 - \frac{r_1 + r_3}{r_1 \cdot [1 + \frac{2 \cdot r_3 \cdot r \cdot \sin\alpha}{r_2 \cdot (r_1 + r_2 - 2 \cdot h \cdot \sin\alpha)}]} \cdot \dot{\varphi}_2 = 0 \qquad (7)$$

Considering the mechanical system in Fig.1 one could see that there are five generalized coordinates from which only two are independent, the rotating angle $\varphi_{(t)}$ and sliding $x_{(t)}$ of a point D. Geometric relations in Fig. 1 are given by:

$$\delta = \delta_0 + \arccos(\frac{x_{(t)} - x_0}{2 \cdot l}) ; \alpha = \alpha_0 + \arccos(\frac{x_{(t)} - x_0}{r}) ; x_5 = r_2 \cdot \sin\alpha ; \ \omega_2 = i_r \cdot \dot{\varphi}_{(t)} \qquad (8)$$

Transmission ratio of planetary gear box is:

$$\omega_{z_2} = \left(1 - \frac{z_1}{z_2}\right) \cdot i_r \cdot \dot{\varphi}_{(t)} \qquad (9)$$

The energy of acceleration is determinated from:

$$S = \frac{1}{2} \cdot J_5 \cdot \ddot{\varphi}_5^2 + \frac{m_5}{2} \cdot \ddot{x}_5^2 + \frac{1}{2} \cdot (J_6 + J_{r6}) \cdot \ddot{\varphi}_6^2 + \frac{1}{2} \cdot J_4 \cdot \ddot{\varphi}_4^2 + \frac{1}{2} \cdot J_{04} \cdot \ddot{\alpha}^2 + \frac{1}{2} \cdot (J_2 + J_{kr} +$$
$$+ J_{0S}) \cdot \ddot{\varphi}_2^2 + \frac{1}{2} \cdot J_S \cdot \ddot{\varphi}_S^2 + \frac{1}{2} \cdot (J_{Z_3} + J_r) \cdot \ddot{\varphi}_{(t)}^2 + \frac{1}{2} \cdot (m_D + m_3) \cdot \ddot{x}_{(t)}^2 + m_k \cdot a_K^2 \qquad (10)$$

Where: $\ddot{\varphi}_2, \ddot{\varphi}_4, \ddot{\varphi}_5, \ddot{\varphi}_6, \ddot{\varphi}, \ddot{\alpha}, \ddot{\delta}$ - the angular accelerations of axles, $\ddot{x}$ - the acceleration of point D

The acceleration of the referent center points of regulator balls is:

$$a_k^2 = (l \cdot \dot{\delta}^2 \cdot \cos\delta + l \cdot \ddot{\delta} \cdot \sin\delta)^2 + \left(l \cdot \dot{\delta}^2 \cdot \sin\delta + l \cdot \ddot{\delta} \cdot \cos\delta - \dot{\varphi}_{(t)}^2 \cdot l \cdot \sin\delta\right)^2$$
$$+ \left(\ddot{\varphi}_{(t)} \cdot l \cdot \sin\delta + 2 \cdot \dot{\varphi}_{(t)} \cdot l \cdot \dot{\delta} \cdot \cos\delta\right)^2 \qquad (11)$$

$$\dot{\delta} = -\frac{\dot{x}_{(t)}}{\sqrt{4l^2 - (x_{(t)} - x_0)^2}} ; \quad \ddot{\delta} = -\frac{(x_{(t)} - x_0) \cdot \dot{x}_{(t)}^2}{\sqrt{(4l^2 - (x_{(t)} - x_0)^2)^3}} - \frac{\ddot{x}_{(t)}}{\sqrt{4l^2 - (x_{(t)} - x_0)^2}}$$

$$\dot{\alpha} = -\frac{\dot{x}_{(t)}}{\sqrt{4r^2 - (x_{(t)} - x_0)^2}} ; \quad \ddot{\alpha} = -\frac{(x_{(t)} - x_0) \cdot \dot{x}_{(t)}^2}{\sqrt{(4r^2 - (x_{(t)} - x_0)^2)^3}} - \frac{\ddot{x}_{(t)}}{\sqrt{4r^2 - (x_{(t)} - x_0)^2}}$$

The generalized forces for independents coordinates $x_{(t)}$, $\varphi_{(t)}$ and velocities $\dot{x}_{(t)}, \dot{\varphi}_{(t)}$ are given by:

$$Q_X = -b \cdot \dot{x}_{(t)} - \frac{\partial \Pi_1}{\partial x_{(t)}} - \frac{\partial \Pi_2}{\partial x_{(t)}} ; \quad Q_\varphi = M_W + M_5 \cdot i_{tot} - \frac{\partial \Pi_1}{\partial x_{(t)}} - \frac{\partial \Pi_2}{\partial x_{(t)}}$$
$$M_5 = M_E = A - B \cdot \dot{\varphi} \quad M_3 = M_W = D - C \cdot t \qquad (12)$$

Where the total transmission ratio is given by:

$$i_{tot} = i_r \cdot i_v \qquad (13)$$

The potential energy of Watt's regulator and spring in the input shaft 5 are:

$$\Pi_2 = \frac{1}{2} \cdot c_2 \cdot \left(\Delta l_2 + x_{(t)} - x_0\right)^2 \qquad \Pi_1 = \frac{1}{2} \cdot c_1 \cdot \left(\Delta l_1 + x_5\right)^2 \qquad (14)$$

Energy of acceleration of planetary train gears and small sliding displacement of input disc 5 are neglected, so the energy of acceleration can be written in form:

$$S_r = \frac{1}{2} J_{r2} \ddot{\varphi}_2^2 + \frac{1}{2} J_4 \ddot{\varphi}_4^2 + \frac{1}{2} J_{r6} \ddot{\varphi}_6^2 + \frac{1}{2} J_5 \ddot{\varphi}_5^2 + \frac{1}{2} m_T \ddot{x}^2 + \frac{1}{2} J_{02} \ddot{\alpha} + m_k \Big[\left(l\dot{\delta}^2 \cos\delta + l\ddot{\delta} \sin\delta\right)^2 +$$
$$+ \left(l\dot{\delta}^2 \sin\delta + l\ddot{\delta} \cos\delta - \dot{\varphi}^2 l \sin\delta\right)^2 + \left(\ddot{\varphi} l \sin\delta + 2\dot{\varphi} l \dot{\delta} \cos\delta\right)^2 \Big] \qquad (15)$$

Now the angular accelerations are given as:

$$\ddot{\varphi}_5 - \frac{di_v}{dt} \cdot \dot{\varphi}_2 - i_v \cdot \ddot{\varphi}_2 = 0 ; \ \ddot{\varphi}_6 = C_1 (\frac{di_v}{dt} \cdot \dot{\varphi}_2 + i_v \cdot \ddot{\varphi}_2) ; \ \ddot{\varphi}_4 = C_1 \cdot C_2 \cdot (\frac{di_v}{dt} \cdot \dot{\varphi}_2 + i_v \cdot \ddot{\varphi}_2) \qquad (16)$$

Where: $C_1 = r_1 / (r_1 + r_3)$ $\quad C_2 = 1 + r_3 / r_2$

After transformations, differential equations of motion are obtained (eq. 17.). By substituting $\dot{x} = v$, $\dot{\varphi} = \omega$ and solving the system (17) on digital computer using numerical method of integration it is possible to denote the parameters x, V and ω in function of time t

$$\left[J_{r2} + i_v^2\left(J_4 C_1^2 C_2^2 + J_{r6} C_1^2 + J_5\right) + m_k l^2 \sin^2\delta\right]\cdot\ddot{\varphi}\cdot i_r^2 +$$

$$+\left[i_r^2\cdot i_v\cdot\frac{di_v}{dt}\left(J_4 C_1^2 C_2^2 + J_{r6} C_1^2 + J_5\right) - \frac{4m_k l^2}{\sqrt{4l^2-(x_{(t)}-x_0)^2}}\dot{x}_{(t)}\cdot\sin\delta\cdot\cos\delta\right]\cdot\dot{\varphi}_{(t)} = M_W + M_5\cdot i_{tot}$$

$$\frac{2l^2\cdot\ddot{x}_{(t)}}{\sqrt{4l^2-(x_{(t)}-x_0)^2}} + \frac{2l^2\cdot\left(x_{(t)}-x_0\right)\cdot\dot{x}_{(t)}}{\sqrt{4l^2-(x_{(t)}-x_0)^2}} - 2l^2\cdot\dot{\varphi}^2\sin\delta\cdot\cos\delta = b\cdot\dot{x}_{(t)} + c_1(\Delta l_1 + x_{(t)} - x_0)\cdot$$

$$\frac{r_2\left[\sqrt{r^2-\left(x_0-x_{(t)}\right)^2}\cdot\sin\alpha_0 + (x_0-x_{(t)})\cdot\cos\alpha_0\right]}{r\cdot\sqrt{r^2-(x_0-x_{(t)})^2}} + c_2\left(\Delta l_2 + x_{(t)} - x_0\right) \qquad (17)$$

4. EXAMPLE

For the given data, in the diagram in Fig. 2 the parameters $\varphi, \dot{\varphi}$ are found.

Parameters:

$J_{r2} = 0.3\text{kgm}^2, J_B = 0.23\text{kgm}^2, J_4 = 0.21\text{kgm}^2,$

$J_5 = 0.25\text{kgm}^2, J_{r6} = 0.25\text{kgm}^2, m_r = 1.8\text{kg},$

$m_k = 0.2\text{kg}, A = 557.9\text{Nm}, B = 5.33\text{Nms}, D = 10\text{Nm},$

$C_M = 0.5241\text{Nms}^{-1}$, $M_W = D - Ct, M_5 = A - B\dot{\varphi}$

$\ell = 0.1\text{m}, \Delta\ell_1 = \Delta\ell_2 = 0.005\text{m}, c_1 = c_2 = 120\text{N/m},$

$b = 550\text{Nsm}^{-1}, t_0 = 0\text{s}, x_0 = 0.1\text{m}, \alpha_0 = 0°, \dot{\varphi}_{20} = 105\text{s}^{-1}$

$r_1 = 0.0125\text{m}, r_2 = 0.035\text{m}, r_3 = 0.07\text{m}, r = 0.0325\text{m},$

$i_r = 2.5$

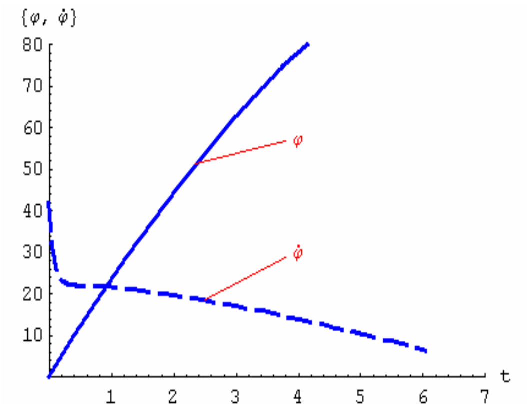


Figure 2. Time histories of φ and $\dot{\varphi}$

5. CONCLUSIONS

In this paper the rotating machinery with frictional variable speed drive with automatic regulations is analyzed. Solving the system of differential equations it is possible to determine the parameters x, $\dot{x} = v, \varphi, \dot{\varphi} = \omega$, in function of time t. In Fig.2 the functions $\varphi, \dot{\varphi} = \omega$ are shown. Then it is possible very simple to denote the stability of this system. The motion becomes stable often 5,0 second (see Fig.2). This example is very good for presentation of dynamically behaviour of non-holonomic mechanical variable-speed drives.

6. REFERENCES

[1] Artobolevskij I.I., Zinoviev V.A., Umnov V.N.: The equation of motion for a mechanical system with a variator, J. of Mechanisms, 1967.,

[2] Murinova G.M.: Osobennosti rasceta frikcionnogo variatora, Sbornik statej. peredatočnyje mechanizmy, Moskva, 2000.,

[3] Zlokolica M. Ž.., Popov R.V.: Dynamic Analyses of Variators with Half Ball and Two Disc as Non-Holonomic Systems, Design Eng. Conference, Beverly Hills, California, 1980.,

[4] Zlokolica M. Ž., Cvetićanin L. J.: About the Dynamics of the Planetary variator with Two Conic Disc as Non-holonomic System, The Fourth International Symposium on Ling. and Comp. Aided Design Met., III-2, Bucharest, 1985.,

[5] Zlokolica M. Ž., Čavić M. V., Kostić M. D.: About Application and Dynamic Behavior of Mechanical Nonholonomic System, Proceedings of the IC-HBRSD `97, Xi`an, P.R. China, 1997.