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HYPERBOLIC PROGRAMMING – AN METHOD FOR CHOOSING INVESTMENT PROJECTS

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ABSTRACT
The term investment implies investing capital in a business venture, production or a project, whose effects will manifest in the future. It is assumed that the invested funds will return and that the realization of the investment will result in profit. In a way, an investment represents renouncing assured pleasures in the present for the sake of future ones. An investment decision is considered as the readiness of the decision makers for temporary sacrifice and waiting.
This paper will put forward a model of hyperbolic programming to be used in investment determination, during the decision-making in choosing a project. The criteria function, which has to be maximized, is the relation between the net present value of a project and the return period.
Key words: hyperbolic programming, criteria function, investments, optimal solution

1. INTRODUCTION
Investments mean investing capital in a business venture, production or a project, whose effects will show in the future. It is assumed that the invested capital will be returned and that the investment realization will result in profit. In that sense, an investment means renouncing assured pleasures in the present for the sake of future ones. An investment decision is considered as the readiness of the decision makers for temporary sacrifice and waiting.

Quantitative models and methods are very efficient assets that can be used in economic analysis and planning. Although many mathematical methods and models are known in the investment decision-making process, the problem of choosing the most convenient investment project is not still fully solved. In light of that, this problem is still ongoing.

2. MODELS FOR CHOOSING INVESTMENT PROJECTS
The effects of investment decisions are not visible in the decision-making period. They appear much later. The time that passes from decision making to its realization is considerably long, which brings certain problems during the implementation of a mathematical model. For that reason, it cannot be said any method is ideal for solving the problem of investment decision-making.

Investment decision-making demands, in the first place, the definition of criterion that is used as a measure for comparing and evaluating various solutions. On the basis of the chosen criterion we choose the best solution. The question of decision-making choice, i.e. criteria function, is very important for the investment project choice. Speaking about investment decision-making, i.e. decision procedure investment by the means of quantitative models and methods, we have to make a choice whether to choose the investment projects according to one or more criteria and which criterion will be most suitable for the target wanted.

In practice we most often use models with one investment-choosing criterion. However, in that case we face another problem – how to choose the most adequate criterion for assessing investment projects? The most often used and the most important single-criteria models in investment choice are:

a) The Net Present Value of an Investment (NPV)
b) The Internal Profitability Rate
c) The Rate of Return on Investment

a) The most commonly applied model for choosing investments projects is **The Net Present Value Model**. The Net **Present Value** of an investment project is calculated as the current value of the residual between the profit and the cost of the observed project. The greatest advantage of this model compared to the other models is in that it displays cash flow in different periods of using the project.

According to NPV model it is necessary to find the maximum of the criteria function

$$(\max) z_0 = \sum_{j=1}^{n} v_j x_j \,, \tag{1}$$

Under the constraints

$$\sum_{j=1}^{n} a_{ij} x_j \le b_i, \quad i = 1,2,...,m; \quad x_j = 0 \text{ or } 1, \quad j = 1,2,...,n. \tag{2}$$

Where: v_j is The Net Present Value of an investment j; $x_j = 1$ - the project is adopted; $x_j = 0$ - the project is not adopted; a_{ij} -the present value of the costs of the project j in the year i ; b_j - the present value of the available investment funds in the year i ; n - the number of project; m - the number of years of investment.

In the system of constraints (2) available funds for investments during the project implementation period are included. The costs of the chosen investment project, at any time of the project duration, should not exceed the available funds for that year, that is

$b_i - \sum_{j=1}^{n} a_{ij} x_j > 0$, for any i . This condition means that the unused funds for one year have not been transposed in the following year. If the unused funds are transposed to the following year, it is necessary to introduce one more variable w_i , which represents unused funds in the year i . The unused funds, which are to be transposed to the following year, equal

$$w_i = b_i - \sum_{j=1}^{n} a_{ij} x_j, \quad i = 1,2,...,m \tag{3}$$

If the unused funds for one year are transposed to the following year the new system of constraints is obtained:

$$\sum_{j=1}^{p} a_{ij} x_j \le b_i + w_{i-1}, \quad i = 1,2,...,m; \tag{4}$$

that is

$$\sum_{j=1}^{p} a_{ij} x_j - w_{i-1} \le b_i, \quad i = 1,2,...,m; \tag{5}$$

b) The **Internal Profitability Rate** is a discount factor by which the net present value is discounted to zero, that is, the value of the Internal Profitability Rate is obtained when the net present value equals zero. According to this method the rate is determined in advance (capital cost), but an investment project is adopted only if the Internal Profitability Rate is greater than the capital cost.
c) **The Rate of Return on Investment** is the simplest model for choosing investment projects. This model shows how long the return period of initially invested funds, that is, the numbers of years needed for the return on assets.

3. HYPERBOLIC PROGRAMMING

When under linear constraint, we look for the maximal (minimal) value of rational-fraction function, with linear denominator and numerator, whose graph is a hyperbola (the function of one variable) is searched, the answer lies in the hyperbolic programming model. Hyperbolic programming is defined in the following way: We need to find the maximum (minimum) linear-fraction criteria function

$$z = \frac{\sum_{j=1}^{n} c_j x_j + \alpha}{\sum_{j=1}^{n} t_j x_j + \beta} = \frac{z_1(x)}{z_2(x)} \tag{6}$$

under the constraints

$$\sum_{j=1}^{n} a_{ij} x_j \le b_i, \quad i = 1,2,...,m; \; j = 1,2,...,n \tag{7}$$

$$x_j \ge 0, \quad j = 1,2,...,n\,. \tag{8}$$

The denominator in criteria function (6) can be interpreted as the investment value for the realization of a production program x, while a numerator represents the economic effects of these investments. Therefore the main problem is in determining such a production program x which will make the ratio between effects and investments, in certain conditions, the greatest possible. Thus we talk about the problem of investment efficiency maximization.

In considering economic problems, i.e. for the criteria function to have sense, it needs two more constraints:

(1) The set of plausible solutions for hyperbolic programming $F = \{x : Ax \le b, x \ge 0\}$ is not empty. It is constrained, which means that no economic activity that appears in a model can be unlimited. Normally, the set of possible solutions is, as in linear programming, convex, that is, it has a finite number of extremes.

(2) Criteria function numerator (6), that is, $z_2(x) \ne 0$, for each value of the variable x in the set of plausible solutions (F). Namely, since $z_2(x)$ is linear and continual function, it does not change its sign (for each x which *fulfills the condition of linear constraints system)*. If the criteria function numerator has a negative sign, it can be transferred to the denominator and for that we need the constraint $z_2(x) > 0$. This constraint enables the possibility of obtaining a super-ordinate program. The condition $z_2(x) \ne 0$ is of little mathematical significance, but considerably narrows the task effectiveness. However, it is of great economical importance.

Linear constraints in fraction linear programming tasks and the existence of an optimal in an extreme point in the convex set of possible solutions, enable the implementation of the simplex method for solving hyperbolic programming.

4. CHARNES – COOPER TRANSFORMATION

By combining the net present value of the project and the rate of return on investment the model of hyperbolic programming is obtained, wherein the criteria function is to be maximized

$$(\max) z = \frac{\sum_{j=1}^{p} v_j x_j}{\sum_{j=1}^{p} t_j x_j} \tag{9}$$

$$\begin{aligned} &\sum_{j=1}^{p} a_{ij} x_j - w_{i-1} \le b_i, \quad i = 1,2,\ldots,m; \\ &x_j = 0 \ ili \ 1; \ j = 1,2,\ldots,n; \quad w_i \ge 0; \ i = 1,2,\ldots,m. \end{aligned} \tag{10}$$

Where t_j is the return period on assets in the project j.

Model (9)-(10) represents a model of hyperbolic programming and can be solved via the Charnes and Cooper's Transformation. Namely, if we assign $\gamma_0 = \frac{1}{t_j x_j}$, and introduce a change $y = x\gamma_0$, the model of hyperbolic programming is transformed into a model of linear programming.

$$(\max) z = \sum_{j=1}^{n} v_j y_j \tag{11}$$

$$\begin{aligned} &\sum_{j=1}^{n} a_{ij} y_j - (b_i + w_{i-1})\gamma_0 \le 0, \quad i = 1,2,\ldots,m; \\ &\sum_{j=1}^{n} t_j y_j = 1 \\ &y_j \ge 0 \ ; \ j = 1,2,\ldots,n; \quad \gamma_0 \ge 0. \end{aligned} \tag{12}$$

Problem (11) - (12) can be solved via the Simplex Method because it is a mixed problem of linear programming, and the optimal solution for the hyperbolic programming problem is obtained from the relation $x^* = \frac{1}{\gamma_0} y^*$, where x^* is the optimal solution for the hyperbolic programming and y^* - the optimal solution for the linear programming.

5. CONCLUSION

All projects do not give the same effects when fulfilling system targets and all cannot be realized simultaneously. That is the reason why we need to choose between them on the basis of predetermined criteria.

6. REFERENCES

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