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ON THE USE OF THE VAUCANSON PLANETARY TRANSMISSION IN THE RENEWABLE ENERGY SYSTEMS. PART I: VELOCITIES AND TORQUES

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ABSTRACT

The paper main objective is to model and analyze the properties of the Vaucanson planetary transmission in order to establish the possibility of using it in renewable energy systems. The structural features, the kinematical and static modeling of the initial Vaucanson speed reducer are presented in the first part of the paper.
Keywords: Vaucanson planetary transmission, transmission ratio, speed reducer, speed amplifier.

1. STRUCTURAL CHARACTERIZATION

The speed reducers, of *„ideal“* type, have to ensure maximum transmission ratios and efficiencies, in the conditions of more reduced overall size and complexity. The famous French inventor *J. Vaucanson* (1709-1782), member of the French Academy, proposed a planetary reducer with bevel gears (Fig.1) [2], which aspirates to this title these days, too.
The main properties of this reducer are further modeled and interpreted; on the basis of these properties, then there are formulated useful conclusions for the design of the new speed reducers and speed multipliers destined to fit RES equipments.

2. STRUCTURAL CHARACTERIZATION

In Fig.1,a it is illustrated the scheme of the *Vaucanson's planetary reducer* (reproduced from *Doyon* and *Liaigre*), while in Fig.1,b and c there are represented the structural scheme (Fig.1,b) and the block scheme (Fig.1,c), according to the exigencies of mechanisms theory. On the structural scheme (Fig.1,b) there are specified the axes positive orientations, while on the block scheme (Fig.1,c) there are specified: the *degrees of freedom* and the *interior kinematical ratios* of the component units, the links between these units and the reducer external links (input a and output H); nearby, on the block scheme from Fig.1,d there are represented the torques signs that load the gears and the connecting shafts.
According to Fig.1,b and c, the reducer contains a *differential* (with the degree of freedom $M_1 = 2$), with bevel gears, of *symmetrical* type (with the interior kinematical ratio $i_{01} = i_{1,3}^{H} = -1$): 1-2-3-H. Between the sun gears 1 and 3, of this unit, it is introduced a *closing kinematical chain,* consisting of two bevel gear pairs with fixed axes: 4-5 (with the degree of freedom $M_2 = 1$) and 6-7 (with $M_3 = 1$). Because the power input is made through the shaft a (Fig.1,b and c), solidary to the gears 5 and 6, and the output through the carrier H shaft, the reducer has the external parameters: ω_a, T_a and ω_H, T_H.

Among the three component units (Fig.1,b and c), interfere Lc = 3 connections: 5 ≡ 6 ≡ a, 3 ≡ 4 and 7 ≡ 1; therefore, the reducer has the degree of freedom [1]:

$$M = M_1 + M_2 + M_3 - L_c = 2 + 1 + 1 - 3 = 1; \tag{1}$$

This means that the reducer has [1]: an *independent external motion* (for instance, speed ω_a) and, implicitly, a *dependent external torque* (for instance, $T_H = T_H(T_a)$); thus, the remained external motion is *dependent* ($\omega_H = \omega_a / i_{a,H}$), and the remained external torque (T_a) is *independent*.
For simplicity, it will be considered that the *independent external parameters* are equal to one: $\omega_a = 1$ and $T_a = 1$; the angular speeds and the torques, established in these conditions, are further called *reduced angular speeds* and *reduced torques*, respectively.

3. MODELING OF THE REDUCED SPEEDS

Firstly it is explained the basic idea used by *Vaucanson* and then there are modeled, analytically and numerically, the reducer *reduced angular speeds*.
The central idea, on which the *Vaucanson's* reducer is based (Fig.1,a and b), can be intuitively explained as follows:

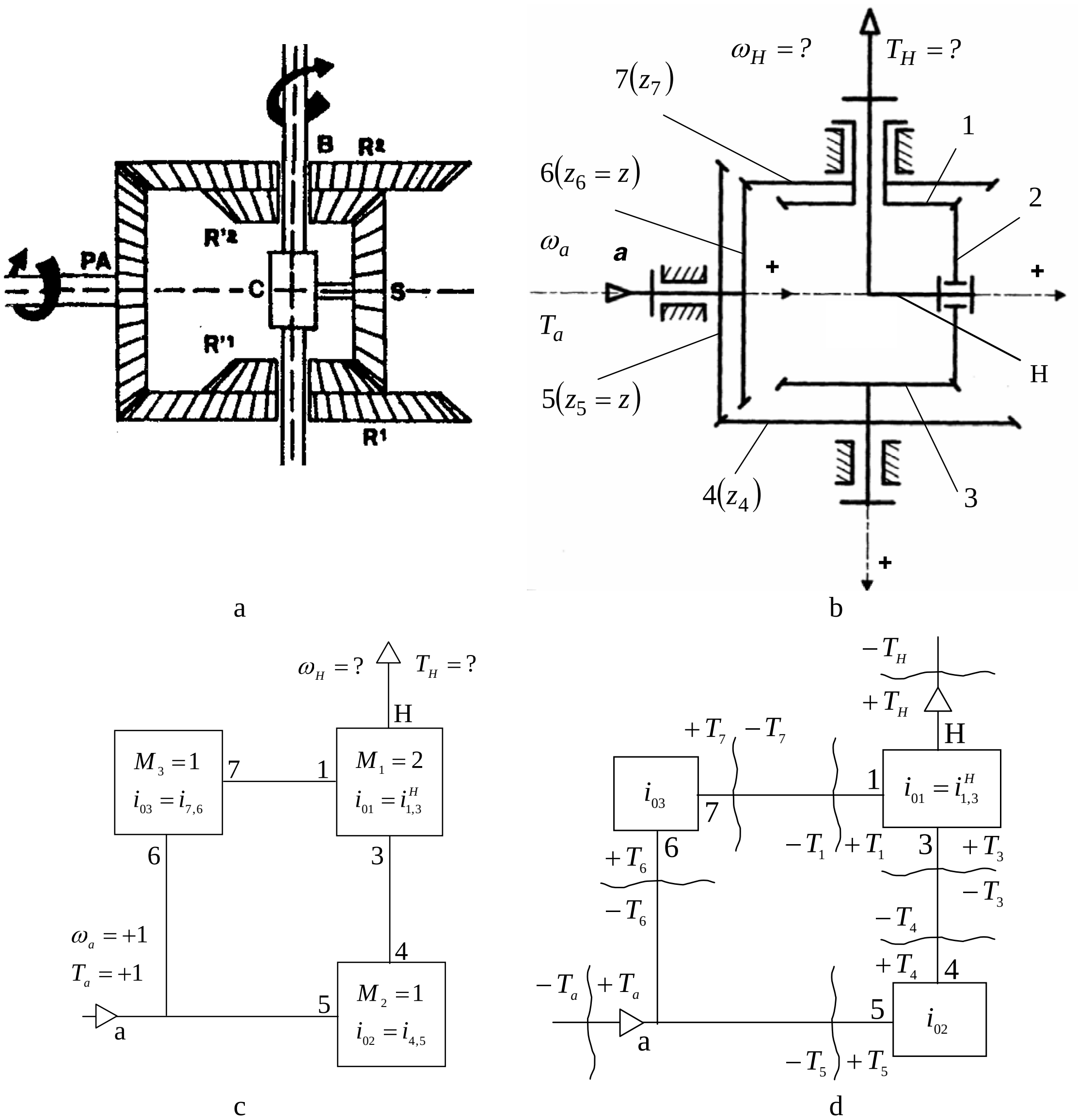

Figure 1. Vaucanson planetary reducer: a) the reducer scheme, after Doyon and Liaigre [2], b) structural scheme, c) block diagram and d) block diagram with the signs of moments that load the gears and shafts.

1°. If in the differential gear-set 1-2-3-H (Fig.1,b), considered isolated, the carrier H is blocked, then the gears speeds 1 and 3 are equal but backwards; *reciprocally*, if, in the differential, gears 1 and 3 are actuated with equal speeds backwards, then the carrier H stays still even if it is not fixed. Physically, this desideratum can be obtained by introducing a *closing chain* 3 ≡ 4-5 ≡ 6-7 ≡ 1 (Fig.1,b), in which the gears have the numbers of teeth: $z_4 = z_7$ and $z_5 = z_6$; in the obtained mechanism the transmission ratio is infinite, because for a rotation of the input shaft a ≡ 5 ≡ 6 (Fig.1,b), the output shaft H remains in repose and thus: $i_{a,H} = \omega_a/\omega_H = 1/0 = \infty$!

2°. If the teeth numbers z_4 and z_7 (Fig.1,b) are different, but very close, then gears 1 and 3 are rotating with *almost equal* speeds but backwards, and the carrier H is rotating *very slow*; for the considered numerical example of *Vaucanson* ($z_4 = 23$, $z_7 = 22$ and $z_5 = z_6 = 20$, Fig. 1,a and b), for 50,6 rotations of the input shaft, the output shaft makes a single rotation (in the negative direction of the axis). Therefore, it was obtained a relative simple planetary reducer that accomplishes a high speeds transmission ratio: $|i_{a,H}| = |\omega_a/\omega_H| = 50,6$!

According to Fig.1,b and c, the angular speeds modeling for the *Vaucanson's* reducer, is reduced to solving the following system of equations [2]:

$$\omega_1 = i_{01}\omega_3 + (1 - i_{01})\omega_H\ ;\ i_{01} = i_{1,3}^H = \omega_{1,H}/\omega_{3,H} = -z_3/z_1 = -1\ ;\ \omega_4 = \omega_5 i_{02}\ ; i_{02} = i_{4,5} = +z_5/z_4\ ;\ \omega_7 = \omega_6 i_{03}\ ;$$
$$i_{03} = i_{7,6} = -z_6/z_7\ ; \omega_1 = \omega_7\ ;\ \omega_3 = \omega_4\ ;\ \omega_5 = \omega_6 = \omega_a = 1 \qquad (2)$$

In the case of the considered numerical example, $z_4 = 23$, $z_7 = 22$ and $z_5 = z_6 = 20$ (Fig.1,b), the following values for the *reduced speeds* and for the speeds transmission ratio ($i_{a,H}$) result from system (2):

$$\omega_5 = \omega_6 = \omega_a = 1\ ;\ \omega_1 = \omega_7 = \omega_a i_{03} = 1 i_{03} = -20/22\ ;\ \omega_3 = \omega_4 = \omega_a i_{02} = 1 i_{02} = +20/23\ ;$$
$$\omega_H = \omega_1/(1 - i_{01}) - \omega_3 i_{01}/(1 - i_{01}) = (\omega_1 + \omega_3)/2 = -10/506\ ;$$
$$i = i_{a,H} = \omega_a/\omega_H = 1/(-10/506) = -50,6\ . \qquad (3)$$

this means that for 50,6 complete rotations of the shaft a ≡ 5 ≡ 6, in the positive sense of its axis (Fig.1,b), the carrier H makes 10 complete rotations, in the negative sense of its axis!

4. MODELING OF THE REDUCED MOMENTS

According to Fig.1,b,c and d, the torques modeling for the *Vaucanson's* reducer is reduced at solving the following system of equations [3]:

$$T_1 + T_3 + T_H = 0\ ;\ \omega_{1,H} T_1 \eta_{01}^{x1} + T_3\omega_{3,H} = 0 \Leftrightarrow T_1 i_{01}\eta_{01}^{x1} + T_3 = 0\ ,\ x1 = sgn(\omega_{1,H}T_1) = sgn[(\omega_1 - \omega_H)T_1] = \pm 1\ ;$$
$$\omega_4 T_4 \eta_{02}^{x2} + T_5\omega_5 = 0 \Leftrightarrow T_4 i_{02}\eta_{02}^{x2} + T_5 = 0\ ,\ x2 = sgn(\omega_4 T_4) = \pm 1\ ;$$
$$\omega_7 T_7 \eta_{03}^{x3} + T_6\omega_6 = 0 \Leftrightarrow T_7 i_{03}\eta_{03}^{x3} + T_6 = 0\ , x3 = sgn(\omega_7 T_7) = \pm 1\ ;$$
$$-T_3 - T_4 = 0\ ;\ -T_1 - T_7 = 0\ ;\ -T_5 - T_6 + T_a = 0\ ;\ T_a = +1\ . \qquad (4)$$

in which through η_{01}, η_{02} and η_{03} were denoted the *efficiencies of the associated fixed axes mechanisms* of the three component units (Fig. 1,b and d); for the next numerical calculus, initially, there were considered the following values: $\eta_{01} = \eta_{1,3}^H \cong \eta_{3,1}^H = 0{,}94^2 \cong 0{,}883$, $\eta_{02} = \eta_{4,5} \cong \eta_{5,4} = 0{,}94$ and $\eta_{03} = \eta_{7,6} \cong \eta_{6,7} = 0{,}94$.

The following expressions for the *reduced torques* are obtained by solving the system (4):

$$T_a = +1\ ;\ T_1 = T_a/\left[i_{03}\eta_{03}^{x3} - i_{01}i_{02}\eta_{01}^{x1}\eta_{02}^{x2}\right] = 1/\left[i_{03}\eta_{03}^{x3} + i_{02}\eta_{01}^{x1}\eta_{02}^{x2}\right];\ T_3 = -T_1 i_{01}\eta_{01}^{x1} = T_1\eta_{01}^{x1}\ ;$$
$$T_4 = -T_3 = T_1 i_{01}\eta_{01}^{x1} = -T_1\eta_{01}^{x1}\ ;\ T_7 = -T_1\ ;\ T_6 = T_1 i_{03}\eta_{03}^{x3}\ ;\ T_5 = -T_1 i_{01} i_{02}\eta_{01}^{x1}\eta_{02}^{x2} = T_1 i_{02}\eta_{01}^{x1}\eta_{02}^{x2}\ ;$$
$$T_H = -T_1\left(1 - i_{01}\eta_{01}^{x1}\right) = -T_1\left(1 + \eta_{01}^{x1}\right);$$
$$-T_H/T_a = \left(1 - i_{01}\eta_{01}^{x1}\right)/\left[i_{03}\eta_{03}^{x3} - i_{01}i_{02}\eta_{01}^{x1}\eta_{02}^{x2}\right] = \left(1 + \eta_{01}^{x1}\right)/\left[i_{03}\eta_{03}^{x3} + i_{02}\eta_{01}^{x1}\eta_{02}^{x2}\right]. \qquad (5)$$

In this phase, the expressions (5) can not be calculated because the exponents x_1, x_2 and x_3 are not known. Therefore, first there are determined the values of the expressions (5) in the *premise of neglecting friction*,

in which the exponents x_1, x_2 and x_3 have no effect: $\eta_{01}^{x1} = (1)^{x1} = 1$, $\eta_{02}^{x2} = (1)^{x2} = 1$ and $\eta_{03}^{x3} = (1)^{x3} = 1$. The torques obtained in this premise, called *theoretical torques* (without friction), and the *real homonym torques* (with friction) have different modules (the modules of the theoretical torques are bigger than of the real torques), but *have the same sign*! This means that the exponents x_1, x_2 and x_3 can be established by means of the *theoretical torques*, and afterwards can be calculated the real torques (with friction).
In the conditions of the considered numerical values, the following values for the *theoretical reduced torques* (without friction) are obtained (in order to avoid confusion, the reduced torques without friction were denoted *italic*):

$$T_a = +1;\ T_1 = 1/(i_{01} + i_{02}) = 1/(+20/23 - 20/22) = -506/20 = -25{,}3;\ T_3 = T_1 = -25{,}3;$$
$$T_4 = -T_3 = -T_1 = +25{,}3;\ T_7 = -T_1 = +25{,}3;\ T_6 = T_1 i_{03} = (-506/20)(-20/22) = +23;$$
$$T_5 = T_1 i_{02} = (-506/20)(+20/23) = -22;\ T_H = -T_1(1+1) = +50{,}6;\ -T_H/T_a = -50{,}6. \quad (6)$$

Therefore, in theoretical conditions (without friction), the analyzed reducer reduces the input speed 50,6 times and amplifies the same number of times the input torque. With the previous values for the theoretical reduced torques, the following values for the exponents x_1, x_2 and x_3 are obtained:

$$x1 = sgn(\omega_{1,H} T_1) = sgn[(\omega_1 - \omega_H) T_1] = +1;\ x2 = sgn(\omega_4 T_4) = +1;\ x3 = sgn(\omega_7 T_7) = -1. \quad (7)$$

Taking into account the values of the exponents x_1, x_2 and x_3, the following values for the *real reduced torques* (with friction) and for the *real torques' transmission ratio*, are obtained:

$$T_a = +1;\ T_1 = 1/\left[i_{03}\eta_{03}^{x3} + i_{02}\eta_{01}^{x1}\eta_{02}^{x2}\right] = -4{,}0837;\ T_3 = T_1\eta_{01}^{x1} = -3{,}6083;\ T_4 = -T_3 = +3{,}6083;$$
$$T_7 = -T_1 = +4{,}0837;\ T_6 = T_1 i_{03}\eta_{03}^{x3} = +3{,}9494;\ T_5 = T_1 i_{02}\eta_{01}^{x1}\eta_{02}^{x2} = -2{,}9494;\ T_H = -T_1\left(1 + \eta_{01}^{x1}\right) = -7{,}6920;$$
$$-T_H/T_a = -7{,}6920 \neq i_{a,H} = -\omega_a/\omega_H = -50{,}6 \quad (8)$$

Comparing the values (6) and (8), it is detected that, by introducing friction, the signs of the reduced torques remain unmodified but their modules are diminishing drastically. Thus, in *real conditions* (with friction), the analyzed reducer reduces the input speed 50,6 times and amplifies the input torque 7,692 times! In other words, due to friction, the module of the torques' transmission ratio ($-T_H/T_a$) becomes much smaller than the module of the speed transmission ratio: (ω_a/ω_H).

5. CONCLUSIONS

The structural characterization, the kinematical and static features of the planetary reducer of Vaucanson type are presented in this part of the paper.
1°.The *Vaucanson's* reducer (see Fig.1) is characterized through the following properties:
  a) it is a 1 DOF planetary mechanism, which has a relative reduced degree of complexity;
  b) it transmits the speed from input to output in the conditions of reducing it 50,6 times;
  c) in *real conditions* (with friction), the analyzed reducer amplifies the input torque 7,692 times.
2°. The properties derived from the analysis made in this paper are useful in the transmission optimization, from the dynamic point of view (see Part II of the paper); thus, useful conclusions for the use of the Vaucanson transmission in renewable energy systems are formulated in the second part of the paper.

6. REFERENCES

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