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LOCATION PROBLEMS SOLUTION AND THEIR APPLICATION

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ABSTRACT
The special class of optimization task is made up of the location problems that go up the significant interests in the modern operating researches. In the common case, the location problem is position /location/ determination of some objects in existing space in which other relevant objects are placed. The new objects are usually some center kinds that offer services called provider. Existing objects are service users or clients called users. We can distinguish two kinds of location problems: continuous location problems and discrete location problems.
Keywords: location problems, optimization, algorithms, continuous location problems, discrete location problems

1. INTRODUCTION
One of key indicators of society development lays in degree of progress of its networks (school, post office, telecommunications, roads (transportation) and others). Location problems are intensively used for designing of above named networks. The problem of optimal location determination of one or more objects in one network is reduced to choice of construction location, therefore minimizing overall path, travel time or total expenses during serving given network nodes. In terms of service that is endangering life environment, it is necessary for certain objects not to be located close to nodes they are serving. Position of object is therefore depending upon the type of serving and it directly influences the quality of serving and overall system costs. It is necessary to predetermine specific parameters and criteria that would dictate the way of location choice. The conclusion is that location theory should be able to answer the following questions: what is the necessary number of objects in the network, where to locate the object, and in what way to determine set of clients that are served by a certain object?

2. CLASSIFICATION OF LOCATION PROBLEMS
Proceeding from the characteristics of thus far developed models, it is possible to make the following classification of location problems [1]:
- According to the number of objects in the network (only one object needs to be identified in the network and larger number of objects need to be identified in the network).
- According to the allowed positions for locating objects (it is possible to locate objects in any given point of observed area (continuous location problems) and it is possible to locate objects only in specific, predefined points (discrete location problems).
- According to the type of objects in the network (**medians** – it is necessary to locate one or more objects in the network in order to minimize average distance between objects and service users; **centers** – it is necessary to locate one or more objects in the network in order to minimize distance between the most remote user and requirement problems – objects with predefined system performances).
- According to the type of algorithm for solving location problems (exact algorithms and heuristic algorithms).

- According to the number of criteria functions, based on which object location is determined (only one criteria function exist, and series of criteria functions exist (multi-criteria optimization problems)).
The variety of location problem classification is designed to show different aspects of certain location problem. Large number of algorithms has thus far been developed for different types of location problem solutions (determination of medians and centers, and requirement problem solutions). The best criteria for determination of locations in one network are number of population per serving node and time zones that present limitations of maximum allowed distances between clients and nodes they are being served in. According to the aforesaid criteria, there are two different approaches to this problem:
- determine optimal locations of predefined number of objects in the network with minimum total distance (median determination problem)
- determine optimal number and optimal locations of objects when number of objects in the network is not predefined (group covering problems – specific group ''requirement problems'').
Prior to stating algorithms for noted problem solutions, we need to explain the concept of medians. Medians are type of node network that are important in a way that by choosing these nodes for object locations we minimize total distance between clients and nodes they are being served in. Additionally, their choice can produce smaller number of required objects that satisfy users' needs. Problem p - median can be presented in non-oriented network $G = (N, A)$ that has n nodes. Necessary parameters are:

a_i - number of serving demands in node i,

d_{ij} - distance between node i and node j,

p - number of objects that need to be located in the network.

We can introduce binary variables x_{ij} that are defined in the following way:

$$x_{ij} = \begin{cases} 1, \text{if } i \text{ receives service in node } j \\ 0, \text{otherwise} \end{cases} \qquad \text{... (1)}$$

Having in mind thus far said, problem p – median can be formulated in the following way:

$$\text{Minimize } F = \sum_{i=1}^{n} \sum_{j=1}^{n} a_i d_{ij} x_{ij} \qquad \text{.... (2)}$$

with limitations

$$\left.\begin{array}{l} \sum_{j=1}^{n} x_{ij} = 1, \ i = 1,2,...,n \\ \sum_{j=1}^{n} x_{jj} = p \\ \mathrm{x_{jj} \geq x_{ij}, \ i, j = 1,2,...,n; \ i \neq j} \\ x_{ij} \in \{0,1\}, \ i, j = 1,2,...,n \end{array}\right\} \qquad \text{.... (3)}$$

Where first limitation in 3) shows that each client is being served in only one node; secondly that network contains total p objects; and third that each client located in certain node is served in that object.

3. ALGORITHMS FOR LOCATION PROBLEM SOLUTIONS

In order to solve the problem p - median increasing number of algorithms has lately been developed, that belong in one of the following categories [2]:
- Algorithm for generating allowable solutions set;
- Algorithm based on chart theory;
- Heuristic algorithms;

- Algorithms based on mathematical programming.

Algorithm for generating allowable solutions set implies examination of all possible solutions for locations p - median. Their number is $\binom{n}{p}$. It is clear that this approach can be applied on networks with smaller number of nodes. We must emphasize the fact that certain algorithms for determination p - median based on chart theory, have proven to be efficient when the network was presenting wood [3].

3.1. Algorithm for determination of one median network

This algorithm was defined by Hakimi. It belongs to a group of algorithms that are used for determination of allowable solution set, and it is used for determination of one median in non-oriented network. It is composed of following steps:

STEP 1: Calculate shortest paths lengths d_{ij} between all node pairs (i, j) of network G and show them in shortest paths matrix D (nodes i present possible median locations, while nodes j present client locations that demand service).

STEP 2: Multiply column j of shortest paths matrix with number of service demands a_j from node j. Element $a_j \cdot d_{ij}$ of matrix $[a_j \cdot d_{ij}]$ present "distance" taken by users from node j, when they are being served in node i. Matrix $[a_j \cdot d_{ij}]$ needs to be marked with D'.

STEP 3: Perform addition along each row i of matrix D'. Expression $\sum_{j=1}^{n} a_j \cdot d_{ij}$ presents total ''distance'' performed by users in case when object is located in node i.

STEP 4: Node, whose row equals to minimum total ''distance'' performed by users, presents location for median.

3.2. Determination of problem *p* - median using algorithm for generating allowable solution set

Algorithm for generating allowable solution set is examining all possible solutions for location p - median, calculation of criteria function value, and determination of optimal solution. It is noted that this algorithm can be used in cases of smaller values $\binom{n}{p}$. We can mark the following: n - total number of nodes in the network, d_{ij} - length of shortest paths from node i to node j, $d'_{ij} = a_j \cdot d_{ij}$ - ''distance'' taken by users from node j, when they are being served in node i, D' - matrix whose elements are d'_{ij}, $X_p = \{v_{j1}, v_{j2}, ..., v_{jp}\}$ - one of possible subsets of p nodes. For each of the $\binom{n}{p}$ subsets of p nodes, we need to calculate summary $\sum_{j=1}^{n} \min\{d'_{j1j}, d'_{j2j}, ..., d'_{jnj}\}$. Subset of p nodes that is equal to minimum sum presents node set where p -median need to be located.

3.3. Algorithm for solving set covering problems

Set covering problems present special group of ''demanding problems''. Besides determining object location, method also shows number of objects needed in the network. It was defined by Larson and Odoni.

Firstly, node sets $B_n = \{b_1, b_2, ..., b_n\}$ are determined, where demands for particular service occur, and $A_m = \{a_1, a_2, ..., a_m\}$ that presents candidates for service location objects. Maximum allowed distance d^* between points $a_j \in A_m$ and $b_i \in B_n$ is proposed. If distance is $d(a_j, b_i) \le d^*$, it can be said that point a_j "covers" point b_i.

$$p(i,j) == \begin{cases} 1, & \text{for } d(i,j) \le d^* \\ 0, & \text{otherwise} \end{cases} \qquad \ldots (4)$$

Shortest distance matrix $[d(i,j)]$ is formed between nodes $b_i \in B_n$ and $a_j \in A_m$, and then transformed to covering matrix $[p(i,j)]$, where $p(i,j)$ is matrix element $[p(i,j)]$. Set covering problem is focused on obtaining minimal number of points x^* from set A_m points, required for covering all set B_n points. Algorithm that Larson and Odoni proposed is the following:

STEP 1: If there is at least one matrix covering column whose elements are equal to zero, then algorithms is finished, since there is no allowable solution. (It is necessary to either increase the number of points where service objects are placed, or change maximum allowed distance between service objects and points it serves).

STEP 2: If any column has at least one unit, in row i^*, then point that equals row i^* must contain object. That point is included in the list of points that need to comprise objects, while row i^* and all columns in whose cross-section with row i^* elements equal 1, are erased from the matrix.

STEP 3: If any row i'' has elements that are smaller or equal to corresponding elements of other row i' (only if $p(i'',j) \le p(i',j)$ za $\forall j$), then row i'' needs to be eliminated.

STEP 4: If any column j'' has elements that are bigger or equal to corresponding elements of other column j' (only if $p(i,j'') \ge p(i,j')$ za $\forall j$), then column j'' needs to be eliminated.

STEP 5: Repeat steps 2 - 4 until:

a) Covering matrix is completely empty, or

b) Passages through steps 2 - 4 are no longer allowed to eliminate any rows or columns.

In case a) minimum required number of objects and their location have been determined. In case b) detailed examination needs to be executed or some other algorithm must be applied.

4. CONCLUSION

This paper stipulates that for solving location problems different algorithms can be used, depending upon classification of location problems. In order to successfully solve location problems we need to examine complex operating conditions and possibilities in current and future situations. In this way we can come closer to highly developed countries in terms of service provisions to users. The task of location optimization is equal to the following demands: profit needs to be as high as possible, costs reduced and service quality improved, with all other necessary specific and other demands fulfilled. Today there exist large number of commercial software; however, sometimes computer resources are not sufficient for finding problem solutions. Above mentioned algorithms increase the possibilities for analysts to model their problems in simple and correct way, and create conditions for greater use of optimization methods with location problems.

5. REFERENCES

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