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RESEARCH ON COLD RIFLING PROCESS LIMITS USING LONGITUDINAL CRACKS CRITERION

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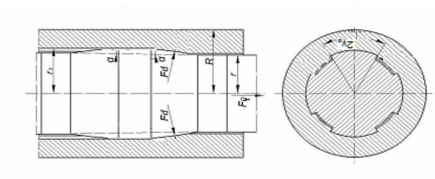
ABSTRACT
This paper deals with process limits in rifling performed by cold extrusion with rifled conical tool. The method of characteristics and sliding lines, and experimental method were used. Influence parameters were varied: sample geometry, tool geometry and limit deformation rate. This research lead to process limits when longitudinal cracks start to appear.
Keywords: Method of characteristics and sliding lines, experimental research, sample geometry, tool geometry, limit deformation rate, process limits, longitudinal cracks

1. STRESS ESTIMATION WITH CONTACT CONDITIONS



r - inner radius of sample
R - external radius of sample
r_a – tool radius
α - half of the central angle of rifled part of the tool
γ_0 - half of the central angle of tool rifle
F_d - deformation force
F_p – extrusion force

Figure 1. Scheme of rifling process by cold extrusion with rifled conical tool.

R. Hill's methodology [1] analyses this process by theory of characteristics and sliding lines through double centred fan field of sliding lines. Under ideal contact conditions, without friction forces (fig. 2a) and with friction forces (fig. 2b), occur at extrusion depth of $r_a - r$. At the conical tool surface (figs. 2a and 2b), with half of the central angle α , the field of sliding lines is limited by triangle ABC and determined with two families of orthogonal straight lines. Having in mind that tool and sample surfaces are assumed to be smooth, there are no tangential contact stresses ($\tau_\alpha = 0$), and sliding lines cross the tool contour under 45° angle (contour condition). Real rifling process occurs under condition of contact between the working piece and the tool (fig. 2b), which leads to occurrence of tangential stresses which influence contour conditions. Inside the triangle ABC, balanced pressure acts between the tool and the sample, p (p' - normal pressure under contact friction conditions). At the AC and BC sides of the triangle, two centred fields ADC and BCE appear, which consist out of two families of orthogonal sliding lines. The first sliding is formed by lines from points A and B, and another family consists of concentric circles with centres in points A and B.

The angles DAC = γ and CBE = δ are determined from conditions when limit lines ADO and BEO, who distinct plastic from rigid zone, cross the free surface in point O under angle of 45°. According to sliding line properties, the relation between these angles and the half of the central tool angle α is obtained. According to Hencky's plasticity integrals, we can assume that:

Direction $O \to D \to C$: $\sigma_C = \sigma_0 + 2k_s(\omega_0 + \omega_C - 2\omega_D) = \sigma_0 + 2k_s(2\gamma + \alpha)$ . ... (1)

where: k_s (MPa) - reduced value of specific deformation resistance.

Direction $O \to E \to C: \sigma_C = \sigma_0 + 2k_s(2\omega_E - \omega_C - \omega_0) = \sigma_0 + 2k_s(2\delta - \alpha)$. ... (2)

Reducing equations (1) and (2) we obtain the relation between angles:

$$\alpha = \delta - \gamma . \quad \text{... (3)}$$

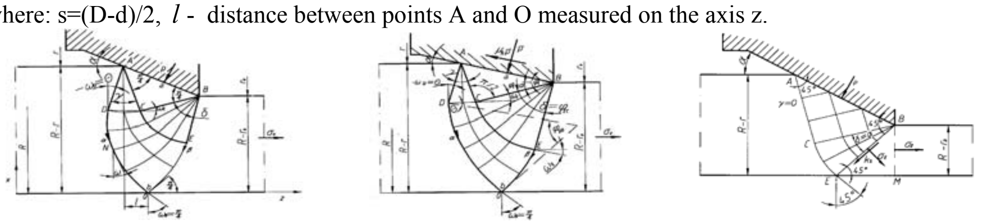
If around some point N (x,z) at the sliding line ADO arc element with length *ds* is spotted (fig. 2a), then its projections are:

$$dx = ds\cos\omega \quad \text{and} \quad dx = ds\sin\omega . \quad \text{... (a)}$$

The distances between endpoints of sliding lines AO measured at the abscissa x and ordinate z are:

$$\int_0^A ds\cos\omega = \int_0^S dx = s \quad i \int_0^A ds\sin\omega = \int_0^l dx = l . \quad \text{... (b)}$$

where: s=(D-d)/2, l - distance between points A and O measured on the axis z.



a) b) c)

Figure 2. Field of sliding lines a) under ideal contact conditions (by R. Hill), b) under contact friction and c) with limit reduction ratio.

The force from tool is transferred to the sample within the deformation zone, and there are no forces on the sliding line O-A, in the direction of tool motion. As seen in fig. 2a, this condition can mathematically expressed through the following equation [3]:

$$F_z = \int_0^A (\sigma\cos\omega - k_s\sin\omega)ds = 0 . \quad \text{... (4)}$$

$$\int_0^A \sigma\cos\omega ds = k_s\int_0^A ds\sin\omega = k_s l . \quad \text{... (4a)}$$

This leads to the conclusion: the sliding line OA, which divides the plastic and the rigid zone, is not chosen arbitrarily, but it has to obey the condition (4) or (4a). Using Hencky's equations for the point N (x,z) and the point O at the sliding line OA: $\sigma + 2k_s\omega = \sigma_0 + 2k_s\omega_0$, the stress in the point N is:

$$\sigma = \sigma_0 + 2k_s(\omega_0 - \omega) . \quad \text{... (c)}$$

From (c) and (4): $\int_0^A [\sigma_0 + 2k_s(\omega_0 - \omega)]\cos\omega ds = k_s l$, $\sigma_0 s + 2k_s\omega_0 s - 2k_s\int_0^A \omega\cos\omega ds = k_s l$

The stress in point O:

$$\sigma_C = 2k_s[\frac{1}{s}(\frac{l}{2} + \int_0^A \omega\cos\omega ds) + (2\gamma + \alpha - \frac{\pi}{4})] . \quad \text{... (5)}$$

If normal distribution of normal stress p is assumed, over the whole contact surface (figs. 2a and 2b), for the angle $\theta = 0$ the system of equations (1.71 - [2]) gives the major stresses of the process: $\sigma_x = \sigma_1,\ \sigma_z = \sigma_3\ and\ \tau_{xz} = 0$, and according to the equation (6.1) [2], $\omega = 45^0$. Thus we are transferred to the system of major axes, and equations (6.5) [2] lead to:

$$\sigma_1 = \sigma_c + k_s ; \quad \sigma_3 = \sigma_C - k_s ; \quad \tau_{xz} = 0 . \quad \text{... (d)}$$

From (d), the normal pressure between the tool and the sample is:

$$p = -\sigma_1 = -(\sigma_c + k_s) . \quad \text{... (7)}$$

$$p = -2k_s[\frac{1}{s}(\frac{l}{2} + \int_0^A \omega\cos\omega ds) + (2\gamma + \alpha - \frac{\pi}{4} + \frac{1}{2})] . \quad \text{... (8)}$$

The starting thickness of the sample s=(D-d)/2) and the half of the central extrusion angle α are given by the process. According to the process geometry the length l and the angle γ are determined, while the angle δ is calculated by equation (3). The value of the integral in the equation (8) between limits from O to point A can be calculated numerically. For lower precision of results, approximate graphical methods can be used to solve this integral. R. Hill [1] used F. W. Bessel's functions to determine numerical values of coordinates of crossed points of sliding lines α and β. To find the stress values for points O and C, the Mohr's stress circles are drawn, according to equations of plastic flow (3.42a) [2], $(\sigma_1 - \sigma_3 = 2k_s)$ by hypothesis of largest deformation energy spent on shape transformation, for planar deformation state, which can be written as follows:

$$\sigma_{\max} - \sigma_{\min} = 2k_s . \quad \text{... (e)}$$

According to the equation (1) and for $\sigma_c = \sigma_a$; $\sigma_0 = \sigma_b$ $\omega_{ab} = \omega_{0c} = 2\gamma + \alpha$ and for process condition, where $\sigma_a < \sigma_b$ we obtain:

$$\sigma_a - \sigma_b = -2k_s\omega_{ab} . \quad \text{... (f)}$$

At external surface in point B there are no stresses in z direction, i.e.:

$$\sigma_{\max} = \sigma_z = 0 \ and \ \sigma_{\min} = \sigma_x . \quad \text{... (g)}$$

The component stress and the medium normal stress are derived from conditions of plastic flow equation (e):

$$\sigma_x = -2k_s; \ \sigma_z = 0; \ \tau_{xz} = 0; \ \sigma_b = -k_s . \quad \text{... (9)}$$

From figure 2a, the angle of sliding line $a - b$ is:

$$\omega_{ab} = \omega_{oc} . \quad \text{... (10)}$$

This value is seen from double fan field of sliding lines. At the contact surface in point C according to equations (g), from conditions of plastic flow in point C ($\sigma_{\max} = \sigma_\alpha$; $\sigma_{\min} = \sigma_z$), we obtain component stresses and medium normal stress in point C:

$$\left.\begin{aligned} \sigma_x &= -2k_s\omega_{ab} = -2k_s(2\gamma + \alpha) \\ \sigma_z &= -2k_s(1 + \omega_{ab}) = -2k_s(1 + 2\gamma + \alpha) \\ \tau_{xz} &= 0 \\ \sigma_a &= -k_s(1 + 2\omega_{ab}) = -k_s(1 + 4\gamma + 2\alpha) \end{aligned}\right\} . \quad \text{... (11)}$$

The value of angle γ is derived from the relation between tool geometry and the working piece geometry: r, r_a, α and R, from double centred fan field of sliding lines, with frictionless conditions.

The stress at the contact surface is derived by replacing (10) in the second equation of the system (11).

$$\sigma_n = \sigma_z = -2k_s(1 + \omega_{ab}) = -2k_s(1 + 2\gamma + \alpha) . \quad \text{... (12)}$$

The magnitude of working pressure depends on normal stress (11) and the tool shape. The normal pressure of rifled tool onto contact surface with the sample can be derived from (d).

$$p = -\sigma_1 = -(\sigma_c + k_s) = \sigma_n ; \ p = 2k_s(1 + 2\gamma + \alpha) . \quad \text{... (13)}$$

The tangential stresses and angles of crossing between sliding lines and the tool contour can be derived from figure 2b, under the contact friction conditions (for μ_α - friction factor at the contact surface) and from the third equation in system (6.5) [1]:

$$|\tau_{xz}| = k_s \cos 2\omega \qquad \text{or} \qquad \cos 2\omega = \frac{|\tau_{xz}|}{k_s} . \quad \text{... (h)}$$

$$\mu = \mu_\alpha ; \qquad |\tau_{xz}| = \mu_\alpha p' ; \qquad \omega = \varphi_1 . \quad \text{... (i)}$$

From (h) and (i) the angle between the sliding lines and the tool contour φ_1 can be derived:

$$\cos 2\varphi_1 = \frac{\mu_\alpha p'}{k_s} . \quad \text{... (14)}$$

The conditional relation between the most important angles of sliding line fields, can be obtained according to the second condition of sliding line properties. Therefore:

$$\varphi_{0D} = \omega_0 - \omega_D = \varphi_{EC} = \omega_E - \omega_C = \cos t. . \quad \text{... (j)}$$

The relation between angles and the sliding line field with friction is given as:

$$\delta - \gamma = \alpha + \frac{\pi}{4} - \varphi_1 . \quad \text{... (15)}$$

Having in mind affirmative contact conditions in rifling (under assumption of good working surfaces of the tool and with good lubrication, we can achieve $\mu < 0{,}1$), R. Hill [10] determined that normal pressure onto contact surfaces with friction p' only slightly depends on friction, and we can assume that $p' \approx p$, where p is normal tool pressure. Approximate extrusion pressure of tool in friction rifling:

$$\sigma_z^{'} = \sigma_z (1 + \mu_\alpha ctg\alpha) . \qquad \text{... (16)}$$

where σ_z is the stress of friction rifling, calculated by equation (11).
For boundary deformation rate across the wall thickness, the sliding line field is shown in figure 2c.
The angle of the part of centred fan sliding line field from the point A equals $\gamma = 0$, and having in mind (3) we obtain: $\alpha = \delta - \gamma$; $\alpha = \delta$. By using the same conditions as in equation for normal pressure given in (13), we reach the equation for normal pressure for boundary deformation rate across the wall thickness as:

$$p = 2k_s (1 + \alpha) . \qquad \text{... (17)}$$

Minimum ratio between external and internal radius of the sample where rifling can be performed without longitudinal cracks, is given in (18).

$$\overline{BE} = \overline{BC} = \overline{AC} \; ; \; \frac{R - r_a}{\sin 45^0} = \sqrt{2}(R - r_a) ; \; r_a - r = \overline{AB} \sin\alpha \; ; \; \frac{\overline{BC}}{\sin 45^0} \sin\alpha = 2(R - r_a)\sin\alpha \; ;$$

$$\frac{r_a - r}{R - r_a} = 2\sin\alpha \; ; \qquad \frac{R}{r} = \frac{\frac{r_a}{r} \cdot (1 + 2\sin\alpha) - 1}{2\sin\alpha} . \qquad \text{... (18)}$$

2. MAXIMUM DEFORMATION RATE

Figures 3a and 3b show the diagrams of relation between R/r and r_a/r and the angle α as lower limit of the process where longitudinal cracks occur in sample. Experiment was used to determine the lower limit of the process and the above mentioned relation. If we are under the lower limit, the longitudinal cracks occur in sample.

a) b)

Figure 3.The relation between the limit value R/r of r_a/r and α.

3. CONCLUSION

Through application of Method of characteristics and sliding lines in rifling made by conical tool extrusion, the mathematical model was determined in function of: sample geometry, tool geometry and limit deformation rate, when longitudinal cracks occur. The experimental testing of samples made of: AlCu5, C10E and 48CrMo4, the theoretical hypothesis was confirmed.

4. REFERENCES

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