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MATHEMATICAL WORKFORCE ALLOCATION PROBLEM SOLVING

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ABSTRACT
Efficient distribution of workforce is one of the preconditions for successful company management. Companies are trying to discover an ideal balance between the number of employees, their skills, and total price of engagement, always having in mind to completely satisfy the given goals. Workforce allocation in big companies presents a serious problem, especially in the recent years, since the number of jobs increased, as well as the number of workers that perform those jobs. Parameters in allocation problem solving are represented by different needs of the employer and the hierarchy of the workers' professional qualifications. Workforce allocation problem solving is defined as the mathematical model of the integer linear programming that includes three parameters: expertise of the workers, work difficulty, and days of the week.
Keywords: Hierarchical location problems, Combinatorial Optimization, NP-hard problems, Linear programming

1. INTRODUCTION

Location problems present special class of optimization problems that usually require distance minimization, minimization of the total travel time, or some other parameter. These problems are very often subject to research, and the main reasons are the practical applications in different areas. Location problems are related to the determination of an object position (location) in the area where other relevant objects already exist. Objects, for whose location the place is required, are usually some type of centers that provide services and are very often called suppliers, whereas service users (already set up objects) are called clients or users. Hierarchical location problems present extension of the location problems that usually imply that objects (suppliers) are set in order to respect the hierarchy among them. Hierarchical location problems, as the original location problems they originated form, are usually NP – hard problems, and their solving requires various heuristics and metaheuristics.

2. HIERARCHICAL WORKFORCE ALLOCATION PROBLEM FORMULATION

Efficiently allocated workforce, with the respect to the hierarchy, is one of the basic preconditions for the successful company management. Companies strive to find an ideal balance between the number of employees, their qualifications, and total engagement price, always having in mind to satisfy the set tasks. Hierarchical workforce allocation (Hierarchical workforce scheduling problem -HCLP) in big companies presents a serious problem, especially in the recent years, since the number of jobs

increased, as well as the number of workers that perform those jobs. The starting problem parameters are the hierarchy of the workforce expert qualifications and different needs of the employers. The basic principle when defining the problem is that more competent workers can perform less demanding jobs, but not vice versa. However, as in practice, more expert workers can be paid more, so the goal is to avoid these situations.
The basic workforce allocation problem model includes three parameters: expertise of the workers, work difficulty, and days of the week. The model should fulfill the following conditions:

- Working week has seven days that are marked from Monday to Sunday with numbers 1 to 7.
- All workers work full time, and are classified as the most qualified from rank 1 to the rank *m*, where different types of workers are paid differently. The jobs of the rank l can each day be executed by the workers of the same or higher rank.
- Each worker has *n* free days in the week (usually $n = 2,3,4$).
- The task is to find the cheapest solution that enables the completion of all weekly assignments.

Index *k* indicates the type of workers, m - number of different type of workers (workers with lower index *k* have a higher rank), index *l* indicates the type of job (as many jobs as the types of workers - *m),* whereas index *j* indicates the day of the week. Having in mind the previously introduced indexes, different variables can be defined:

- c_k - price of workers rank k ,
- w_k - number of workers type k ,
- x_{klj} - number of workers type k , engaged for j days at work l ,
- y_{kj} - number of workers type k that are not working for j days,
- d_{lj} - number of jobs rank l that need to be completed for j. days.

With these indexes and variables, mathematical model of the integer linear programming [1] can be defined in the following way:

To minimize:

$$z = \sum_{k=1,2,...m} c_k w_k \qquad ... (1)$$

in order to get:

$$\sum_{l \geq k} x_{klj} + y_{kj} = w_k \qquad ... (2)$$

where $k = 1,2,...m,\ j = 1,2,...7$

$$\sum_{j} y_{kj} \geq w_k n\,,\ k = 1,2,...,m, n = 2,3,4 \qquad ... (3)$$

$$\sum_{k \leq l} x_{klj} = d_{lj}\,, l = 1,2,...,m,\ j = 1,2,...,7 \qquad ... (4)$$

Condition (2) shows that each day specific number of workers is working, while others are resting, i.e. all are present. Condition (3) provides each worker with certain number of free days, whereas condition (4) guarantees that all the planned jobs will be completed. It is important to take into account the inequality $c_1 > c_2 > ... > c_m$ during problem formulation.
Mathematical model of the linear integer programming for hierarchical workforce allocation problem solving [2] is defined as follows:

To minimize:

$$z = \sum_{b=1,..,B} \ \sum_{k=1,2,...,m} c_{bk} w_{bk} \qquad ... (5)$$

in order to get:

$$\sum_{l>k} x_{bklj} + y_{bkj} = w_{bk}, \quad \text{... (6)}$$

where: $b = 1,2,3,\ k = 1,2,...,m,\ j = 1,2,...,7$

$$\sum_{j} y_{bkj} \geq A_{bn} w_{bk}, \quad \text{... (7)}$$

where: $b = 1,2,3,\ k = 1,2,...,m,\ n = 2,3,4$

$$\sum_{b=1,2,3} \sum_{k \leq l} x_{bklj} = d_{lj}, \quad \text{... (8)}$$

where: $l = 1,2,...,m,\ j = 1,2,...,7.$

Similar to the first model, index *k* indicates the type of workers, number of different types of workers is *m* (workers with lower index *k* have a higher rank), index *l* indicates the type of job (as many jobs as the types of workers - *m*), index *j* indicates the day of the week, whereas index *b* indicates the type of the working hours.

With these indexes, the following marks are introduced:

c_{bk} - price of workers rank k with working hours type b,

w_{bk} - number of workers type k with working hours type b,

x_{bklj} - number of workers type k with working hours type b engaged for j days at work l,

y_{bkj} - number of workers type k with working hours type b, that are not working for j days, and

d_{lj} - number of jobs rank l that need to be completed for j days.

Conditions (1) - (4), that describe the first model, present special case model (5) - (8). By fixing $B = l$ the variety of working hours is lost, so that all workers have eight-hour working day, and thus five-day working week.

3. GENETIC ALGORITHM FOR HIERARCHICAL WORKFORCE ALLOCATION PROBLEM SOLVING

This algorithm enables the application of the real number coding for determination of all elements of the multidimensional row X. Row X presents the number of workers type k with working hours b, engaged for j days at work l. Each member of the row is joined with the real number obtained form the genetic code. Since the problem is numerically stable, each real number can be encoded with the help of 16 bits, thus decreasing the length of the genetic code. Based on the real numbers obtained from the genetic code, for each l and j, integer values of the row x_{bklj} are calculated by using d'Hondt method [3]. This method has minimum sampling error, and it is used for allocating seats in party-list proportional representation. After all the votes have been counted, successive quotients are calculated for each list, using $\frac{V}{s+1}$ formula, where V is the total number of votes that list received, whereas s is the number of seats that list has been allocated to that moment. The next seat is allocated to the list with the highest quotient, and then the quotients are recalculated. The process is repeated until all seats have been allocated.

Usage of d'Hondt method in hierarchical workforce allocation problem solving implies that for fixed l and j, depending on the real numbers that are joined to X s, related x_{bklj} increases, until all jobs d_{lj} are completed. More specific, for each l and each j one d'Hondt allocation method d_{lj} is applied, in accordance with the formula (8). As it has already been described, each step is forming a quotient in which dividend is real number that is sampled (obtained from the genetic code), whereas divisor is obtained integer value of the row X increased for 1. The value of the row X, that is equal to the highest quotient, is increased for 1, thus increasing its divisor. The total number of steps is d_{lj}. Row w_{bk} is obtained based on elements x_{bklj} for each b and k. Value $\frac{\sum_{k>j} \sum_{j} x_{bkj}}{7 - A_{bn}}$ is calculated and for each day of the week j there is $\sum_{k>l} x_{bklj}$. Maximum of the calculated numbers (8 numbers) becomes value w_{bk}. Values $y_{bkj} = -w_{bk} + \sum_{l>k} x_{bklj}$ are obtained based on condition (6). In this way, solution admissibility is provided, in accordance with the conditions (6) - (8). Value z is obtained by adding by the formula (5). The following lemma can be applied:

Lemma: The complexity of the function calculation presented by the genetic algorithm is $O(m^2 \cdot b \cdot MaxD)$, where the highest value in job row is $MaxD$.

Picture 1 shows the genetic algorithm schema.

```
Učitavanje_ulaznih_podataka();
Generisanje_Početne_Populacije();
while (not Kriterijum_Zaustavljanja_GA()) do
    for (i = (Nelit +1) to Npop ) do
        if (Postoji_u_kešu(i)) then
            obj[i] = Pronađi_u_kešu(i);
        else
            obj[i] = Funkcija_Cilja(i);
            Stavi_u_keš(i, obj[i]);
            if (Pun_keš()) then
                Izbaci_LRU_blok_keša()
            endif
        endif
    endfor
    Računanje_funkcije_prilagođenosti();
    Selekcija();
    Ukrštanje();
    Mutacija();
Endwhile
Štampanje_Izlaznih_Podataka();
```

Picture 1 Genetic algorithm

4. CONCLUSION

Hierarchical workforce allocation in big companies presents a serious problem, especially in the recent years, since the number of jobs increased, as well as the number of workers that perform those jobs. This area has been researched thoroughly, and there are many papers that present these problems. Mathematical model of linear integer programming can help in hierarchical workforce allocation problem solving. Experimental results show that genetic algorithm for given instances reaches optimum solution in relatively short implementation period. Since genetic algorithm implementation period increases polynomially with problem dimension, solutions can be reached in instances of higher dimension, for which optimum solution is unknown.

5. REFERENCES

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