

**DETERMINATION OF STRESS CONCENTRATION FACTORS VIA
NUMERICAL METHODS: BAR OF CIRCULAR CROSS SECTION
WITH U-SHAPED GROOVE SUBJECTED TO TENSION AND
BENDING**

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ABSTRACT

This paper deals with the determination of stress concentration factors using numerical methods. The cases of the bar with circular cross section and U-shaped groove subjected to tension and bending are considered. Determination of model parameters is achieved by combining the design-of-experiment, implemented in MATLAB, and Finite Element analysis, as an alternative to standard mechanical tests. The design-of experiment is used in order to set-up a variety of combinations of geometrical parameters, whereas the Finite Element analysis, performed in ABAQUS, is applied using obtained geometry variations in order to calculate corresponding stress concentration factors. MATLAB is also used for subsequent mathematical modeling in order to obtain appropriate expressions for stress concentration factors. Obtained mathematical models agree well with existing data from literature.

Keywords: stress concentration, finite element analysis, design of experiment, tension, bending, U-notch

1. INTRODUCTION

Development of mathematical and finite element analysis software together with improvements in computer technology provide a possibility for improving and simplifying existing mathematical models of stress concentration factors. In this paper this possibility is shown for stress concentration of the bar with circular cross section and U-shaped groove subjected to tension and bending, as given in more detail by Kisija [1]. For this purpose ABAQUS and MATLAB softwares are employed. MATLAB is used for design of experiment and mathematical modeling, whereas ABAQUS is used for determination of stress concentration factors for particular problem. The whole procedure is based on work by Kacmarcik *et.al.* [2].

2. PROBLEM DESCRIPTION

The theoretical stress concentration factor K_t can be defined as the ratio of the peak stress in the body (or stress in the perturbed region) to some other stress (or stresslike quantity) taken as a reference stress, i.e.

$$K_t = \sigma_{max} / \sigma_{nom} \dots\dots\dots(1)$$

In this paper the cases of the bar with circular cross section and U-shaped groove subjected to tension and bending are considered (Figure 1).

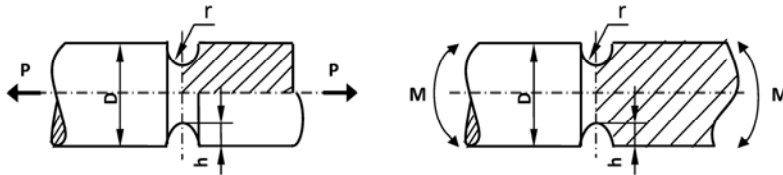


Figure 1. Bar of circular cross section with u-shaped groove subjected to tension and bending.

Nominal stresses for these problems are:

$$\sigma_{nomt} = \frac{4 \cdot P}{\pi \cdot (D - 2h)^2}, \text{ for tension, } \dots\dots\dots (2)$$

$$\sigma_{nomb} = \frac{32 \cdot M}{\pi \cdot (D - 2h)^3}, \text{ for bending } \dots\dots\dots (3)$$

The expressions given in literature [3,4] define the stress concentration as function of the geometrical factors $\sqrt{h/r}$ and h/D in the following form:

$$K = C_1 + C_2 \cdot \left(\frac{2h}{D}\right) + C_3 \cdot \left(\frac{2h}{D}\right)^2 + C_4 \cdot \left(\frac{2h}{D}\right)^3 \dots\dots\dots (4)$$

where C_1, C_2, C_3, C_4 are:

$$C_1 = a_1 + a_2 \cdot \sqrt{h/r} + a_3 \cdot h/r; \quad C_2 = b_1 + b_2 \cdot \sqrt{h/r} + b_3 \cdot h/r \dots\dots\dots (5)$$

$$C_3 = c_1 + c_2 \cdot \sqrt{h/r} + c_3 \cdot h/r; \quad C_4 = d_1 + d_2 \cdot \sqrt{h/r} + d_3 \cdot h/r$$

The constants a_i, b_i, c_i, d_i ($i=1,2,3$) are different in the existing models in literature [3,4], and also differs for the cases of tension and bending. Here, all models are given for two ranges: the first one is $0.25 < h/r < 2$, and the second one is $2 < h/r < 50$. In this work it was decided to examine the former one.

3. DESIGN OF EXPERIMENT

For the design of experiment the optimal design technique is employed. Using this technique, via Model based calibration toolbox, total of 12 minimum experimental points are obtained for chosen 5th order polynomial (Note: in subsequent analysis, chosen polynomial is reduced to fourth order – only these results are presented.). However, 18 more points are added for better accuracy, thus making the total of 30 experimental points. The investigated area is set in the following ranges:

$$0.25 \leq x = h/r \leq 2$$

$$0.1 < y = h/D < 0.25.$$

The design matrix is shown in Figure 2.

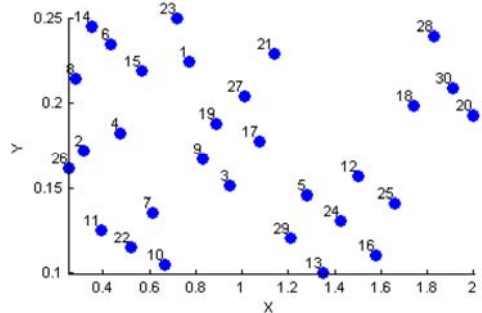


Figure 2. Design matrix.

4. EXPERIMENT - FINITE ELEMENT ANALYSIS

Finite element analysis is applied to the combinations of geometrical parameters, i.e. the experimental points obtain by the design of experiment. Numerical simulation is done in ABAQUS software. Investigated problems subjected in tension can be considered axysymmetric, whereas those subjected to bending must be modelled as 3D problems due to loading asymmetry. As a consequence, for the bending problems, solid hexagonal elements are implemented, whereas for the tension quadrilateral

elements with axisymmetric behavior are used. In order to get more accurate results the mesh in the region of stress concentration was finer than in the rest of domain. Number of elements for tension problems was on average around 3500, whereas for bending cases around 120 000. Example of a mesh and a FEM analysis results for one experimental point is shown in the Figure 3. For each experimental point stress concentration factor is obtained using the maximum principal stress value via Eq. 1.

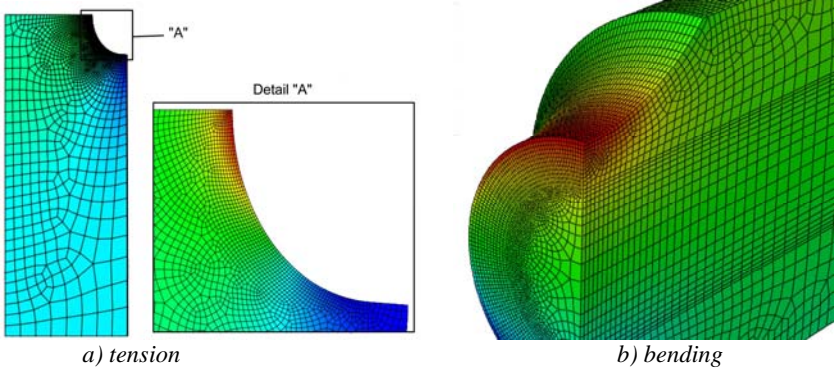


Figure 3. Finite element analysis.

5. MATHEMATICAL MODEL

Based on the results obtained from the FEM analysis, related to the combinations of geometrical factors $(x = \sqrt{h/r}, y = h/D)$, and after minimization of predicted errors, two 4th order linear polynomial response models are calculated with MATLAB software with following expressions (K_t – tension, K_{tb} – bending):

$$K_t = 0.4517 + 2.961 x + 1.84546 y - 0.36788 x^2 - 16.7312 x y + 5.0576 x^2 y + 23.4095 xy^2 - 10.2063 x^2 y^2 \dots (6)$$

$$K_{tb} = 2.4915 + 0.6237 x + 19.9763 y + 87.4932 y^2 - 9.80195 xy^2 - 130.684 y^3 - 0.5618 x^2 y^2 + 19.7936 x^2 y^2 \dots (7)$$

These models show excellent agreement with the experimental data, having PRESS RMSE¹–0.0034068 and RMSE²–0.0030259 for tension, and PRESS RMSE–0.022649 and RMSE–0.020414 for bending.

The expressions 6 and 7 can be transformed into form given by Eq. (5)

$$K_t = C_1 + C_2 \cdot \left(\frac{2h}{D}\right) + C_3 \cdot \left(\frac{2h}{D}\right)^2 + C_4 \left(\frac{2h}{D}\right)^3$$

where C_1, C_2, C_3, C_4 are given in Table 1.

Table 1.

tension		Bending	
C_1	$0.4518 + 2.961 \sqrt{h/r} - 0.368 h/r$	C_1	$1.147 + 1.409 \sqrt{h/r} - 0.203 h/r$
C_2	$0.9227 - 8.3656 \sqrt{h/r} + 2.5288 h/r$	C_2	$-3.303 - 3.396 \sqrt{h/r}$
C_3	$5.8523 \sqrt{h/r} - 2.551 h/r$	C_3	10.554
C_4	-	C_4	$-9.372 - 2.942 h/r$

¹ Root mean squared error of predicted errors
² Root mean squared error

6. MODEL EVALUATION

In order to evaluate the obtained models, they are compared with the exiting models given in literature [4] and with the web application for stress concentration calculation [5]. Figure 4a and b show comparison between these calculated models and data from literature for tension ($D/d=1.3$) and bending ($D/d=1.2$), respectively ($d = D - 2h$). From the given charts a good agreement is clearly noticed, where maximum deviation for tension case is around 4% and for bending 7.4%

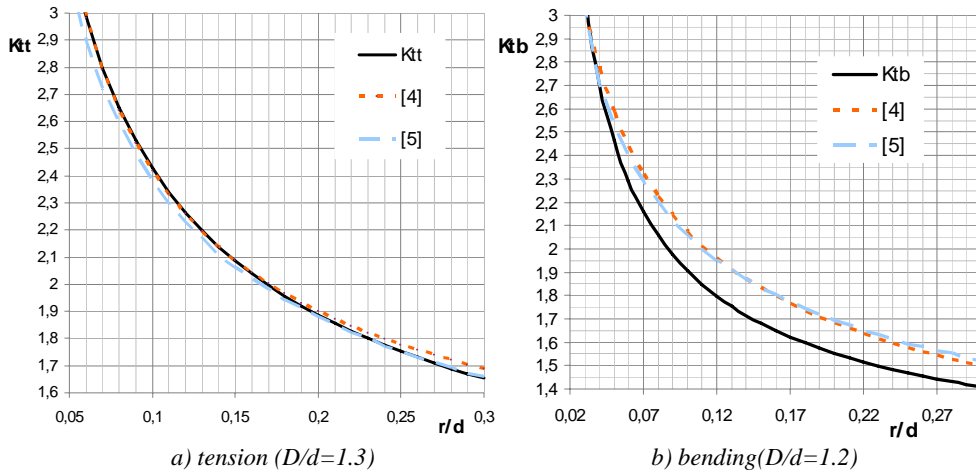


Figure 4. Charts of stress concentration factors.

7. CONCLUSIONS

This paper describes the procedure for determination of stress concentration factors using numerical simulation as alternative to real mechanical tests. In particular, the problems with bar of circular cross section with U-groove subjected to tension and bending are investigated.

Geometrical variations for both tension and bending are obtained using MATLAB, whereas the calculation of stress concentration factors is performed via ABAQUS. Final expressions for stress concentration factors are obtained using regression analysis tools in MATLAB. The obtained values for stress concentration factors over a range of geometrical variations for both tension and bending show a very good agreement with the data from literature.

Further investigation is also possible in order to extend presented investigation for the case of torsion. However, the procedure and software used in this paper could be used for other cases of stress concentration, where data are no available or to simplify and improve existing models.

8. REFERENCES

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