13th International Research/Expert Conference
”Trends in the Development of Machinery and Associated Technology”
TMT 2009, Hammamet, Tunisia, 16-21 October 2009

# CRITICAL ANALYSIS OF NORMAL AND SHEAR CONTACTING STRESSES OF CYLINDRICAL SOLIDS ACCORDING TO SEVERAL RESEARCHES

**Dr.sc. Sadullah Avdiu, Dr.sc. Nijazi Ibrahimi,**
**Faculty of Mechanical Engineering**
**Prishtina, Kosova**

## ABSTRACT
*The normal and tangential contacting stresses of cylindrical solids represent the stresses that depend on several factors, in such a way the shape and magnitude of contacting surface depend on load intensity, the shape and material of solids, and the load distribution law. Many researches to achieve good selection of solving the equations of mention stresses have been studied. Recently, in literature the different results of these stresses obtained from identical input data have been reported. In this paper the critical analysis of normal and tangential contacting stresses of cylindrical solids regarding to optimal expressions of stress calculations have been presented.*
**Keywords:** Contacting stresses, cylindrical solids, Hertzian contacting stresses, Load distribution law.

## 1. INTRODUCTION
Stresses are those that are caused during the reciprocal oppression of two solids. These stresses have local character and very quickly will be eliminated with dismissal from the center of the contact area. Therefore, during solving of these problems differ two phases:
- *The first phase*– in which will be assigned the shape and size of contact surface created in the area of contact under the action of external loads, and
- *The second phase* - in which will be assigned stresses and displacements.

Shape and size of the contact surfaces depends on: physical and mechanical features of materials, shapes of the surface of bodies in the contact area, magnitude, direction and form of action of external loads, direction and relative speed motion of surfaces in contact, surface quality of the contact surfaces, the presence of lubricant and its type, the temperature, etc. When the form and size of contact area and the distribution law of external load through that surface are known, with the implementation of known laws from the theory of elasticity, can be given the differ-rential equations of equilibrium with help of which can be determined the stresses and deformations [5].
So far, exact solutions are found only for limited number of special cases that is for some shapes of the contours of the contact surface and laws of load distribution, while the other more complicated cases are simplified in these cases.

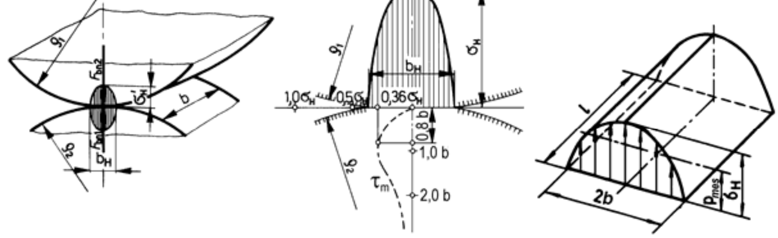


*Figure* 1.1.

If the solids that contacts have an inflection surfaces of the contact (fig. 1.1.) (E.g. rolling and sliding bearings, gears, elements of the cam mechanism, the wheels trolley and rail etc.), assigning of the contacting stresses is a complex problem, which will be solved by methods of theory of elasticity. In these cases, the material, in the vicinity of the contact surfaces, is created a state of spatial strain (three-dimensional strain) [6].
First exact solution of the elementary cases of press of elastic bodies with the help of methods of theory of elasticity is given from the German physicist, *Heinrich Hertz* in 1881.
After this are done several scientific works, in which are elaborated in detail the other cases of the stressed situations in the area of contact of two elastic bodies under oppression, under the assumptions that:

- *stresses in the contact area cause only the elastic deformations in limits of the Hooks law,*
- *dimensions of contact surfaces are small compared to the radii of curvature and dimensions of the bodies who contacts,*
- *press forces, distributed across contact surfaces, acting normal on that surfaces,*
- *in contact surfaces are announced only normal stresses while the bodies are from homogeneous and isotropic material.*

## 2. ANALYSE OF NORMAL STRESSES BY DIFFERENT AUTHORS

### 2.1. Determination of half width of the contact ellipsis

The contact of teeth sides of the gears is replaced by the contact of cylinders, which have the equal radius to the radii of curvature of teeth sides of the gears to the current contact point (fig. 2.1.). During the work the gears are pressed with a certain force, therefore the area of contact is a narrow rectangle of width *2b* and length *l*. Since the size *l* is a constant, half-width *b* is given according to different authors by expressions:

- According to [2]:

$$b=\sqrt{\frac{2q}{\pi}\frac{k_1+k_2}{\lambda}} \tag{2.1}$$

- According to [7]:

$$b=\sqrt{\frac{2F}{\pi l}\frac{\left(1-\mu_1^2\right)/E_1+\left(1-\mu_2^2\right)/E_2}{1/d_1+1/d_2}} \tag{2.2}$$

- According to [3] and [6]:

$$b=1{,}075\sqrt{q\frac{1/E_1+1/E_2}{1/R_1+1/R_2}} \tag{2.3}$$

- According to [1]:

$$b=\sqrt{\frac{8F(1-\mu^2)\rho}{\pi\cdot l\cdot E}} \tag{2.4}$$

### 2.2. Determination of contact stress on the teeth side

Determination of normal stresses or Hertzian contact stresses is a permanent preoccupation of various researchers. Therefore, as a result of this, and expressions for computing this stress, according to the different authors, are:

- According to [3]:

$$\sigma_H=0{,}418\sqrt{\frac{F_{bn}E}{l}\frac{\rho_1+\rho_2}{\rho_1\rho_2}} \tag{2.5}$$

- According to [4]:

$$\sigma_H=\sqrt{\frac{F_{bn}E}{2\pi^2 l}\frac{\rho_1+\rho_2}{\rho_1\rho_2}} \tag{2.6}$$

- According to [2]:

$$\sigma_H=\sqrt{\frac{F_{bn}}{\pi l}\frac{\rho_1+\rho_2}{\rho_1\rho_2}\frac{1}{\left(1-\mu_1^2\right)/E_1+\left(1-\mu_2^2\right)/E_2}} \tag{2.7}$$

- According to [6]:

$$\sigma_H = 0{,}418\sqrt{2q\frac{E_1E_2}{E_1+E_2}\frac{r_1+r_2}{r_1r_2}} \tag{2.8}$$

- According to [1]:

$$\sigma_H = \sqrt{\frac{F_{bn}}{2\pi\rho l}\frac{E}{1-\mu^2}} \tag{2.9}$$

The radii of curvature on the current point of contact ($\rho_1$ and $\rho_2$) (fig. 2.1.) vary depending on the position of gear teeth in meshing, where for the gaining of such relations are used the results of paper [8].

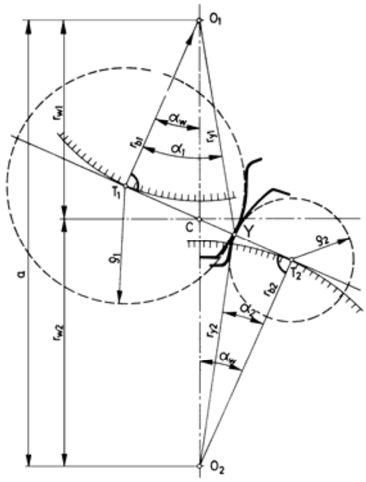



*Fig.* 2.1.

### 3. SHEAR STRESSES OF CYLINDRICAL SOLIDS

The stresses which are spread in tangential plane ($x$, $y$) of the contact surfaces are called *shear stresses*.

In this paper was attempted to verify the assumptions of different authors for shear stresses by using the data from [2] and [7].

Expressions for contact stresses of two cylindrical bodies according to [2] and [7] are:

- According to [2]:

$$\sigma_z = -p_0\left[1-\frac{k^3}{(1+k^2)^{3/2}}\right] \tag{3.1}$$

$$\sigma_c = -\frac{p_0}{2}\left[1{,}6+\frac{k}{\sqrt{1+k^2}}\left(-2{,}6+\frac{k^2}{1+k^2}\right)\right] \tag{3.2}$$

$$\tau_{rz} = \frac{1}{2}(\sigma_c - \sigma_z) \tag{3.3}$$

Where are:

$k = \dfrac{z}{b}$ and $p_0 = \dfrac{b\lambda}{k_1+k_2}$.

- According to [7] the componential stresses are:

$$\sigma_x = -2\nu p_{max}\left(\sqrt{1+\frac{z^2}{b^2}}-\frac{z}{b}\right) \tag{3.4}$$

$$\sigma_y = -p_{max}\left[\left(2-\frac{1}{1+\left(z^2/b^2\right)}\right)\sqrt{1+\frac{z^2}{b^2}}-2\frac{z}{b}\right] \tag{3.5}$$

$$\sigma_z = -\frac{p_{max}}{\sqrt{1+\left(z^2/b^2\right)}} \tag{3.6}$$

Where is $p_{max} = \dfrac{2F}{\pi b l}$.

According to the above expressions, the shear stress can be computed by expression:

$$\tau_{xy} = \frac{\sigma_x - \sigma_y}{2} \tag{3.7}$$

### 4. ANALYZE OF THE GAINED RESULTS

Based on the equations given in section 2, and equations (3.1) to (3.7) is compiled a program for comparative computing of the contact stresses given by different authors. Therefore, based on the compiled program the obtained results will be analyzed as follows.

Based on the gained results and given diagrams in fig. 4.1, 4.2, 4.3 and 4.4, it can be concluded that there are differences between the different authors in the way of determination the maximum contact stresses in the contact surfaces of the cylindrical bodies loaded with normal load. According to the gained results, expressions from to [1] and [3] give identical results, while according to [2] and [4], this stress is significantly smaller.

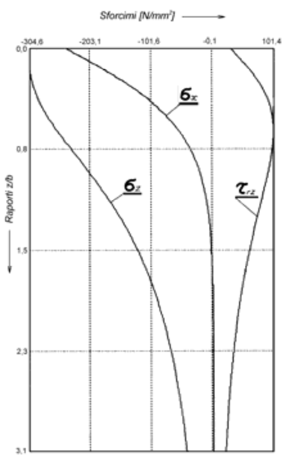



Fig. 4.1. *Contact stresses [2].*

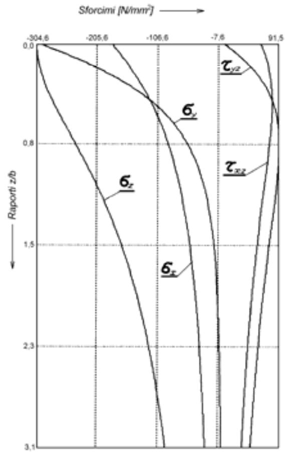
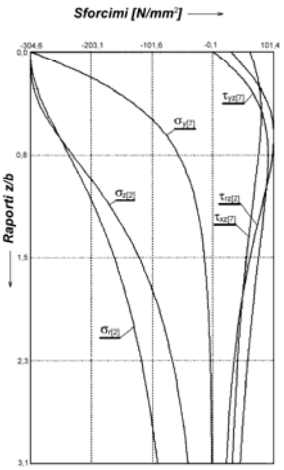



Fig. 4.2. *Contact stresses [7].*



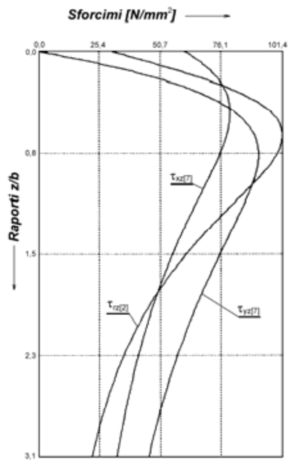




*Fig.*4.3. *Maximal normal,* *Fig.* 4.4. *Maximal shear,*
*stresses according to* [2] *and* [7].

Although the authors [2] and [7], at first view, give different equations for the maximum contact press and rectangle width to the contact surface, equations (2.1), (2.2), (3.5) and (3.6), the obtained results shows that these equations gives fully identical values even with $p_0$=$p_{max}$=304,617 N/mm$^2$, and $b$ = 0,1776932 mm, according to both authors. However, the values of other parameters (sizes) differ significantly and that:

- from fig. 4.4, according to [2] for the case when $z$=0, the shear stress have a value of $\tau_{rz}$=30,462 N/mm$^2$ (that is illogical), while according to the author [7] for $z$=0 the shear stress is $\tau_{yz}$=0.
- the obtained results by computing according to [2] are $\tau_{max}$ = 0,3329095$p_{max}$ and $z$= 0,637925$b$ and do not match with those that given by the author himself ($\tau_{max}$ = 0,304$p_{max}$ and $z$=0,78$b$), while according to [7] obtained values are: $\tau_{max}$ = 0,3002833$p_{max}$ and $z$ = 0,7860525$b$, and they overlap with those that the author gives himself.

## 5. CONCLUSION

With this critical analysis an attempt is made to explain and to enlighten this problem. The results obtained are partial, in the absence of laboratory tests to verify the claims mentioned above. However, with the help of comparative analysis some dilemmas about which expressions are closer to reality are eliminated. Therefore, for practical use can be preferred expressions by the author [7].

## 6. REFERENCES

[1] Eugen Oberšmit: *Ozubljenja i zupčanici*, SNL, Zagreb, 1987.
[2] Danilo Rasković: *Teorija elastičnosti*, Naučna knjiga, Beograd, 1985.
[3] Grupa autora: *IMP* 1 i 2 *deo*, Zavod za izdavanje udžbenika, Beograd, 1992.
[4] St. Fronius: *Maschinenelemente, Antriebselemente*, Veb Verlag Technik, Berlin, 1971.
[5] Vera Krsmanović; *Jedan prilaz načinu rešavanja prostornih kontaktnih zadataka*, Savez društva za mehaniku Jugoslavije, Kupari, 1-5 juna, 1981.
[6] Davorin Bazjanac: *Nauka o čvrstoći*, Tehnička knjiga, Zagreb, 1968.
[7] J. E. Shigley: *Mechanical engineering design*, McGraw Hill, 1986.
[8] Nijazi Ibrahimi, Sadullah Avdiu: *Shpejtësitë e rrëshqitjes së dhëmbëzorëve cilindrik evolvent*, Makineria, Nr. 1, Prishtinë, 1997.