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SOME ASPECTS OF THE SONIC FLOW WITH THE UPPER FREQUENCY

Carmen Bal
Technical University of Cluj Napoca
15 Constantin Daicoviciu Street, Cluj Napoca
Romania

Nicolaie Bal
Technical University of Cluj Napoca
102-103 streets Muncii, Cluj Napoca
Romania

ABSTRACT
Sonicity is the science of transmitting mechanical energy through vibrations. Starting from the theory of the musical accords, Gogu Constantinescu found the laws for transmitting the mechanical power to the distance through oscillations that propagate in continuous environments (liquid or solid) due to their elasticity.
In this paper we are present some aspect about the transmission the sonic flow in the infinity long line transmission, some aspect theoretical and some examples of this effect.
Keywords: sonic flow, sonic pressure, sonic capacity, sonic resistance.

1. THEORETICAL ASPECTS OF THE SONIC FLOW

Considering the general equation of the flow and the pressure of the pipe with variable section:

$$\begin{aligned} \frac{d\overline{p}_s}{dx} &= j \cdot \omega \cdot \overline{L}_s \cdot \overline{Q}_s \\ \frac{d\overline{Q}_s}{dx} &= j \cdot \omega \cdot \overline{C}_s \cdot \overline{p}_s \end{aligned} \tag{1}$$

In which the impedance L_s and the sonic capacity C_s, given by the relations:

$$\overline{L}_s = L_s - j \cdot \frac{C_f}{\omega} \text{ şi } \overline{C}_s = C_s - j \cdot \frac{C_p}{\omega} \tag{2}$$

From the relation $C_f = E \cdot \gamma \cdot l \cdot v_{ef} \cdot \frac{1}{2g \cdot s \cdot d}$, if we note $k = \frac{\varepsilon \cdot v_{ef}}{2d}$ result $C_f = \frac{k \cdot \gamma \cdot l}{g \cdot S}$, for l = 1, capacity $C_f = \frac{k \cdot \gamma \cdot l}{g \cdot S}$ will be[2]:

$$C_f = \frac{k \cdot \gamma}{g \cdot S} \tag{3}$$

Replacing in (2), the relation (3), knowing that $L_s = \frac{\gamma \cdot l}{g \cdot s}$, we obtain:

$$\overline{L}_s = \frac{\gamma}{g \cdot S}(1 - j \cdot \frac{k}{\omega}) \tag{4},$$

Analogue we can write $C_p = k_1 C_s$, where $C_s = \frac{S \cdot 1}{E}$

And for $l = 1$, from the relation (2) we obtain:

$$C_s = \frac{S}{E} \cdot \left(1 - j \cdot \frac{k_1}{\omega}\right) \tag{5}$$

k and k_1 represents constants of which value depend, first of viscosity, and the second one of the *histerezis* or the plasticity of the fluid.

The *viscosity* of any particular form of the material can be defined through the coefficient C_f or k, so the lost energy through the internal forces ell be under the form: $W_c = \frac{C_f \cdot p_s^2}{2} = \frac{k \cdot L_s \cdot p_s^2}{2}$, meaning the *lost of kinetic energy.*

Perditance [3] can be defined through the coefficient C_p or k_1, with the lost of proper energy:

$$W_p = \frac{C_p \cdot Q_s^2}{2} = \frac{k_1 \cdot C_s \cdot Q_s^2}{2}$$, meaning *the lost of potential energy*.

The numerical coefficients k and k_1 represent proportions of kinetics and potential energy, which will transform into heat or in another form of energy, disappearing from the considered flow. The same two coefficients together with the mass and the elasticity coefficient define the type of material.

A material is perfectly elastic when C_p and k_1 zero and perfect fluid when C_f and k are zero.

For normal materials none of the following constants are zero L_s, C_s, C_f şi C_p. The big variety of properties of different materials is due to different values of this constant.

From the pervious relations we observe that the values of the constants depend of the values of the pressure p_s and the flow Q_s.

Friction, represented through the friction coefficient C_f will contain any delayed force in faze with the flow, because of the movement of the corpus.

Perdition represented through the perdition coefficient C_p, will include all the loss of movement in faze with the pressure, because of it.

Following we study the general case of practical energy transmission through longitudinal waves in which the energy will be the one that will cross a column of variable section.

By the multiplication of the relations (4) and (5) we have:

$$\overline{C}_s \cdot \overline{L}_s = \frac{\gamma}{g \cdot E}(1 - j \cdot \frac{k}{\omega})(1 - j \cdot \frac{k_1}{\omega}) = \frac{\gamma}{g \cdot E}[1 - \frac{k \cdot k_1}{\omega^2} - j\frac{k + k_1}{\omega}] \tag{6}$$

From the relation (6) we observe that the multiplication of $\overline{C}_s$ and $\overline{L}_s$ is independent of the area of the transmission line section.

For *high frequencies* the term from developing the series, $\frac{k \cdot k_1}{\omega^2}$ becomes very low, nearly zero:

$$(\overline{\mu}_1) \cong \mu \cdot \sqrt{1 - j \cdot \frac{(k + k_1)}{\omega}} \tag{7}$$

If the frequency is high the upper expression can be simplified taking the following form:

$$\overline{\mu}_1 \cong \mu \cdot \left[1 - j \cdot \frac{(k + k_1)}{2\omega}\right] \tag{8}$$

For simplification we consider a conical pipe, the section S can be expressed by the relation: $S = qx^2$, where „x” is the distance of the section from the peak of the cone and „q” is a numerical constant.

Starting from the relation:

$$\frac{1}{s} \cdot \frac{ds}{dx} = \frac{2}{x},$$

General equation (7) becomes:

$$\frac{d^2 \overline{p}_s}{dx^2} + \mu_1^2 \cdot \overline{p}_s + \frac{2}{x} \cdot \frac{d\overline{p}_s}{dx} = 0$$
$$\frac{d^2 \overline{Q}_s}{dx^2} + \mu_1^2 \cdot \overline{Q}_s - \frac{2}{x} \cdot \frac{d\overline{Q}_s}{dx} = 0 \tag{9}$$

The results will be the same if we consider an elementary knot or a „*vibrating radius”*, formed by every conical pipe, having a very small conic angle that we want to consider, and for studying the phenomenon of every conical pipe, we don’t have any option but to add the obtained results from the consideration of every radius that form the cone.

Considering such a radius in which $\alpha = \mu_1 x$. Replacing in the relation (1) we obtain:

$$\frac{d\overline{p}_s}{dx} = \frac{d\overline{p}_s}{d\alpha} \cdot \frac{d\alpha}{dx} = \mu_1 \cdot \frac{d\overline{p}_s}{d\alpha} = \omega \cdot \sqrt{\overline{L}_s \cdot \overline{C}_s} \cdot \frac{d\overline{p}_s}{d\alpha} \tag{10}$$

$$\frac{d^2 \overline{p}_s}{dx^2} = \frac{dd\overline{p}_s}{dx \cdot (dx)} = \frac{d}{d\alpha} \cdot \frac{d\overline{p}_s}{dx} \cdot \frac{d\alpha}{dx} = \frac{d}{d\alpha} \cdot \left(\mu_1 \cdot \frac{dp_s}{d\alpha} \right) \cdot \mu_1 = \mu_1^2 \cdot \frac{d^2 p_s}{d\alpha^2} \tag{11}$$

(because $\frac{d\alpha}{dx} = \mu_1$):

$$\alpha = \mu_1 x \Rightarrow x = \frac{\alpha}{\mu_1} \Rightarrow \frac{2}{x} = \frac{2}{\frac{\alpha}{\mu_1}} = \frac{2 \cdot \mu_1}{\alpha}. \tag{12}$$

Relation (11) can be written under the following form:

$\frac{d\overline{p}_s}{d\alpha} = \mu_1 \cdot \frac{d\overline{p}_s}{d\alpha}$, so the relation (10) becomes: $\mu_1^2 \frac{d^2 \overline{p}_s}{d\alpha^2} + \mu_1 \cdot \overline{p}_s + \frac{2}{\alpha} \cdot \frac{d\overline{p}_s}{d\alpha} = 0$ (13)

So we can write:

$$\frac{d^2 \overline{p}_s}{d\alpha^2} + \frac{2}{\alpha} \cdot \frac{d\overline{p}_s}{d\alpha} + p_s = 0 \tag{14}$$

similar with the relation (14) we can write:

$$\frac{d\overline{Q}_s}{dx} = \frac{dQ_s}{d\alpha} \cdot \frac{d\alpha}{dx} = j \cdot \omega \cdot \overline{C}_s \cdot p_s \tag{15}$$

but $\alpha = \mu_1 \cdot x$, so $\frac{d\alpha}{dx} = \mu_1$ (16)

Replacing the relation (16) in (15) result:

$$\frac{d\overline{Q}_s}{dx} = \mu_1 \cdot \frac{d\overline{Q}_s}{d\alpha} \tag{17}$$

If we take the friction and the histerrezis equal with zero, we can replace Q_s , $\overline{p}_s, \overline{\psi}$ in this formulas by Q_s, p_s and ψ. If this is not necessary, every quantity of α, β and ψ are complex and for simplification it is necessary to separate the terms containing j in order to get to more explicit formulas.

From the relations mentioned above we observe that α has a form: $\alpha = \mu_1 x$ and using the relation (12):

$$\overline{\alpha} = \mu_1 \cdot x = \mu \cdot x \cdot \left(1 - j \cdot \frac{k + k_1}{2 \cdot \omega} \right)$$

If we represent α şi β without the vectorial symbol, the numerical values of this quantities (their modules neglecting the square of the quantities $\frac{k + k_1}{2 \cdot \omega}$ to the unit). $\left|\overline{\alpha}\right| = \sqrt{R_e^2 + I_m^2}$

For simplifying the demonstration we consider the particular case in which the receptor and the generator are at a considerable distance from the cones peak, so that the terms that contain α and β are retained multiplying the features.

In this case the relations are simplified:

$$p_{s\alpha} = \frac{\beta}{\alpha} \cdot \left[\overline{p}_{s\beta} \cdot \cos\overline{(\overline{\alpha} - \overline{\beta})} + j \cdot \frac{\overline{Q}_{s\beta}}{\overline{\psi}_{\beta}} \cdot \sin\overline{(\overline{\alpha} - \overline{\beta})} \right]$$

$$Q_{s\alpha} = \frac{\alpha}{\beta} \cdot \left[\overline{Q}_{s\beta} \cdot \cos\overline{(\overline{\alpha} - \overline{\beta})} + j \cdot \overline{p}_{s\beta} \cdot \overline{\psi}_{\beta} \cdot \sin\overline{\overline{(\overline{\alpha} - \overline{\beta})}} \right] \tag{18}$$

For $\cos\overline{(\overline{\alpha} - \overline{\beta})}$: $\cos\overline{(\overline{\alpha} - \overline{\beta})} = \cosh(\alpha - \beta) \cdot \frac{k + k_1}{2 \cdot \omega}$ (19)

$$\cos\overline{(\overline{\alpha} - \overline{\beta})} = \cos(\alpha - \beta) \cdot \cosh\alpha_2 - j \cdot \sin(\alpha - \beta) \sinh\alpha_2$$

By replacing the relations (19) in the relation (18), we will have:

$$p_{s\alpha} = \frac{\beta}{\alpha} \left[(p_{s\beta}) \cdot \cosh(\alpha - \beta) \cdot \frac{k + k_1}{2 \cdot \omega} + \frac{\overline{Q}_{s\beta}}{\overline{\psi}_{\beta}} \cdot \sinh(\alpha - \beta) \cdot \frac{k + k_1}{2 \cdot \omega} \right]$$

$$Q_{s\alpha} = \frac{\alpha}{\beta} \left[(Q_{s\beta}) \cdot \cosh(\alpha - \beta) \cdot \frac{k + k_1}{2 \cdot \omega} + p_{s\beta} \cdot \overline{\psi}_{\beta} \cdot \sinh(\alpha - \beta) \cdot \frac{k + k_1}{2 \cdot \omega} \right] \tag{20}$$

As well we can replace (ψ_β) with ψ, the term containing (ψ_β) can be neglected.
Representing the number of wavelengths between the receiver and the generator with *m* and the frequency with f, we have in the case $p_{s\beta}$ and $Q_{s\beta}$ are in phases [3]:

$$\overline{p}_{s\alpha} = \frac{\beta}{\alpha} [p_{s\beta} \cosh\gamma + \frac{\overline{Q}_{s\beta}}{\overline{\psi}_{\beta}} \sinh\gamma]$$

$$\overline{Q}_{s\alpha} = \frac{\alpha}{\beta} [\overline{Q}_{s\beta} \cosh\gamma + \psi_{\beta} \overline{p}_{s\beta} \sinh\gamma] \tag{21}$$

where $\gamma = m \frac{k + k_1}{2f}$. Observing that $m\lambda = l$, where l is the distance between the receiver and the generator, $2f\lambda = v$, so $m = \frac{l}{\lambda}$ and $\frac{1}{2f} = \frac{\lambda}{v}$, where v *represents the speed of the liquid*, by replacing this relations we obtain, the value of γ: $\gamma = \frac{l}{\lambda} \frac{k + k_1}{2fv} 2\lambda f = \frac{l}{v}(k + k_1)$ (22)

The relation (20) help calculated the sonic flow and sonic pressure in the pipe with the upper frequency.

2. REFERENCES

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