13th International Research/Expert Conference
”Trends in the Development of Machinery and Associated Technology”
TMT 2009 Hammamet, Tunisia, 16 – 21 October 2009

IMAGE COMPRESSION USING THE HAAR WAVELET TRANSFORM

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ABSTRACT
We introduce the simplest wavelet transform, the so- called Haar wavelet transform and explain how it can be used to compress images. Every image can be presented as twodimensional matrix of numbers. This matrix occupies a certain space in hard disc and Haar transform reduce numbers of bites of that matrix. The original images can be reconstructed by inverse Haar transform. We can use software like Mathematica or Mathlab. In those there are built in functions for computing Haar transform. In this paper we present procedure for compression and reconstruction of images using software Mathematica.
Keywords: Haar transform, wavelet, ortogonal matrix

1. INTRODUCTION
Each digital image is represented by a matrix of numbers, ranging from 0 (representing black) to some positive whole number (representing white). Each matrix entry gives to a small square which is called pixel.
The number of distinct colors that can be represented by a pixel depends on the number of bits per pixel (bpp). A 1 bpp image uses 1-bit for each pixel, so each pixel can be either number 1 (white) or number 0 (black). Each additional bit doubles the number of colors available, so a 2 bpp image can have 4 colors, and a 8 bpp image can have 256 colors. For color depths of 15 or more bits per pixel, the depth is normally the sum of the bits allocated to each of the red, green, and blue components. For examples, in the case of a 256x256 pixel gray scale image, the image is stored as a 256x256 matrix with each element of the matrix being a whole number ranging from 0 (for black) to 255 (for white).
The resolution is a pixel count in digital imaging. Higher resolution means more image detail. If image consists of a fairly large number number pixels and high number of bits per pixel, image has more colors and details, but matrix which specify that image has large number elements. This fact poses immediate storage problems.
We are going to describe a scheme for transforming such large arrays of numbers into arrays that can be stored efficiently. Orginal image can then be reconstruted by a computer with relatively little effort.

2. COMPRESSION WITH HAAR TRANSFORM

We will explain process on 8x8 matrix. The process can be generalized to $2^k \times 2^k$ matrix, where is k integer positive number.

One divides corresponding matrix into 8x8 blocks and considers blocks like separate matrices. Process should be repeate on each block matrix.

If there are 2^k elements in row in matrix, then the transformation process row of matrix will consist of k steps. In our case $k = 3$.

First step: Divide elements of each row into four pairs. Calculate the average of each of these pairs. These number will be the first four elements in corresponding row in new matrix. Subtract corresponding average from the first elements of the pairs. Calculated numbers are the last four elements in corresponding row in new matrix.

We will get the same result if we multiply image matrix (or block image matrix) on the right by the matrix:

$$H1 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & -\frac{1}{2} \end{pmatrix}$$

Second step: We consider the first four elements in each row in the last matrix as two pairs, and calculate averages of these pairs as in the first step above. These numbers are the first two elements in correspoding row in new matrix. The third and the fourth elements of correspoding row in new matrix is substract these averages from the first element of each pair. The last four elements in new rows are the same as the last four elements in corresponding rows in MatrixImage·H1.

We will get the same result if we multiply MatrixImage·H1 on the right by the matrix:

$$H2 = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix};$$

Third step: Calculate averages of the two elements of rows in MatrixImage·H1·H2. That numbers is the first numbers in rows in new matrix. The second elements in rows are substract the averages from the first elements. Others elements in rows are the same as in MatrixImage·H1·H2. We will get same result if we multiply MatrixImage·H1·H2 on the right by the matrix:

$$H3 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

We can write RowTransformedMatrix = MatrixImage $\cdot$W, where is $\mathrm{W} = \mathrm{H1}\cdot\mathrm{H2}\cdot\mathrm{H3}$.

$$W = H1.H2.H3 = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{4} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{4} & 0 & -\frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{8} & \frac{1}{8} & -\frac{1}{4} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{8} & \frac{1}{8} & -\frac{1}{4} & 0 & 0 & -\frac{1}{2} & 0 & 0 \\ \frac{1}{8} & -\frac{1}{8} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{8} & -\frac{1}{8} & 0 & \frac{1}{4} & 0 & 0 & -\frac{1}{2} & 0 \\ \frac{1}{8} & -\frac{1}{8} & 0 & -\frac{1}{4} & 0 & 0 & 0 & \frac{1}{2} \\ \frac{1}{8} & -\frac{1}{8} & 0 & -\frac{1}{4} & 0 & 0 & 0 & -\frac{1}{2} \end{pmatrix}$$

Matrix W is invertible and its columns form an ortogonal basis of $\mathbb{R}^8$. Haar transform performs the above operations on each row of the matrix image, and then the same operation on the columns of RowTransformedMatrix. We get matrix $Haar\_Matrix\_image = \mathrm{W}^T \cdot \mathrm{MatrixImage} \cdot \mathrm{W}$.

We can retrieve our MatrixImage, because matrices W^T and W are invertibles, so we have

$$\mathrm{MatrixImage} = \left(\mathrm{W}^{-1}\right)^T \cdot Haar\_Matrix\_image \cdot \mathrm{W}^{-1}$$

This process is called decompressing the compressed image.

Elements with little variation in ImageMatrix will be zero elements in matrix *Haar_Matrix_image*. Matrix that has many zeros takes much less memory to store. If *Haar_Matrix_image* doesn't have many zeros, we choose non-negative integer number what is marked as ε, and we let any elements in *Haar_Matrix_image* whose absolute value is less than ε to be reset to zero. We will mark last obtained matrix as CompressMatrix. If ε is positive, then some detail will be lost when image decompressed, therefore we have to choose ε wisely. The ratio of the number of nonzero elements in matrix *Haar_Matrix_image* and to the number of nonzero elements in matrix CompressMatrix is called compression ratio.

Example: We have matrix of digital image:

$$\mathrm{MatrixImage} = \begin{pmatrix} 241 & 94 & 127 & 174 & 217 & 153 & 4 & 235 \\ 188 & 86 & 3 & 80 & 144 & 143 & 151 & 59 \\ 87 & 203 & 71 & 81 & 228 & 51 & 57 & 181 \\ 170 & 124 & 44 & 12 & 10 & 126 & 98 & 209 \\ 214 & 35 & 218 & 15 & 195 & 59 & 40 & 219 \\ 187 & 151 & 140 & 82 & 182 & 85 & 230 & 66 \\ 4 & 161 & 161 & 250 & 118 & 3 & 211 & 12 \\ 231 & 254 & 101 & 218 & 18 & 169 & 111 & 222 \end{pmatrix}$$

Using software Mathematica 5.0, this matrix is transformed in the matrix:

$$\text{Haar\_Matrix\_image} = \begin{pmatrix} 128. & 3.14 & 20.4 & -6.38 & 13.4 & -2.94 & 20.2 & -18.8 \\ -7.98 & -11.9 & 17.2 & 11.3 & 9. & -9.81 & -4.44 & -27.9 \\ 10.8 & 1.72 & -9.44 & 21.1 & 39.9 & -18.3 & 0.5 & 12. \\ -3.94 & -17.2 & 13.3 & 13.4 & 49.4 & 58.4 & 33.6 & -12.9 \\ 24.4 & 10.4 & -19.6 & 6.75 & 11.3 & 7.5 & 15.8 & -80.8 \\ 10.4 & 1.13 & -12.5 & 26.5 & -40.5 & -10.5 & 73.3 & -3.25 \\ -8. & -1.75 & -12.5 & 3. & 35.8 & 36.3 & 9.75 & -85.8 \\ -25.3 & -3.25 & -51.5 & 5.5 & -33.5 & 7. & 66.5 & 77.5 \end{pmatrix}$$

In the last matrix, there are not any zeros, so we choose positive number $\varepsilon = 15$, and we get Compress_Matrix:

$$\text{CompressMatrix} = \begin{pmatrix} 128.328` & 0 & 20.4063 & 0 & 0 & 0 & 20.1875 & -18.8125 \\ 0 & 0 & 17.1563 & 0 & 0 & 0 & 0 & -27.9375 \\ 0 & 0 & 0 & 21.125 & 39.875 & -18.25 & 0 & 0 \\ 0 & -17.1875 & 0 & 0 & 49.375 & 58.37 & 33.625 & 0 \\ 24.4375 & 0 & -19.625 & 0 & 0 & 0 & 15.75 & -80.75 \\ 0 & 0 & 0 & 26.5 & -40.5 & 0 & 73.25 & 0 \\ 0 & 0 & 0 & 0 & 35.75 & 36.25 & 0 & -85.75 \\ -25.25 & 0 & -51.5 & 0 & -33.5 & 0 & 66.5 & 77.5 \end{pmatrix}$$

In this case compress ratio is 2:1.

3. CONCLUSIONS

The Haar wavelet transform is usually presented using special function called Haar wavelets. Image transforms are very important in digital processing, for example when we call up a WWW adress which contains an image, that image appears in installments, the source computer recalls the Haar transformed matrix from its memory, starting with an aproximation and working up to the final complete image.

We can get better results if matrices H1, H2, H3 are ortogonal. This mean that we divide each column of the matrices H1, H2, H3 by its lentgh. In that case this transform is called normalized wavelet transform.

4. REFERENCES

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